



## BLACK HOLES: THE ULTIMATE FATE OF THE COLLAPSING STARS

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**Abstract:** Black holes are among the most intriguing objects in modern physics (Alwis, 1992). It is one of the most fascinating objects in the universe, and it can be understood on the basis of Einstein's general theory of relativity. Black hole is nothing but the ultimate destiny of massive stars which undergo a dramatic gravitational collapse (Horowitz and Teukolsky, 1999). In the present study, we investigate the nature of the collapsing stars, the observational evidence for black holes and some recent developments in the black hole physics.

**Key words:** General relativity, Schwarzschild metric, event horizon, gravitational collapse, singularity, black hole

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### Introduction

A black hole is an object — typically a collapsed star — whose gravity is so strong that its escape velocity exceeds the speed of light. Since nothing is known to exceed the speed of light, nothing can escape from a black hole. A black hole is defined simply as a region of space time that cannot communicate with the external universe. The boundary of this region is called the surface of the black hole, or the event horizon. A black hole is a super dense object that has an intense gravitational pull (Anon, 2006a).

A black hole is an object predicted by general relativity with a gravitational field so strong that nothing can escape it, not even light. One of the most interesting phenomena, which is entirely an outcome of the general theory of relativity, is that of the black hole. Although the mathematical existence of the black hole, as embodied in the exact solution of Schwarzschild, dates back almost to the advent of general relativity itself, most of the relevant aspects related to it were discovered essentially after the mid-sixties. These include the geometric structure of the black hole, the physical phenomena associated with this structure, the differences between static and rotating black holes, perturbation effects, black hole thermodynamics, quantum field theory in the black hole gravitational fields and the uniqueness of the black hole solutions (Iyer *et al.*, 1988). These discoveries, incorporating several diverse areas of physics and mathematics, have revealed an extraordinary rich variety of phenomena associated with black holes. There are two parts to a black hole, a singularity and an event horizon. If we were to take a slice of a black hole right through its center it would look like the Fig.1.

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## Event Horizon

The 'surface' of a black hole is the so-called event horizon, an imaginary surface surrounding the mass of the black hole. Stephen Hawking proved that the topology of the event horizon of a non-spinning black hole is a sphere. At the event horizon, the escape velocity is equal to the speed of light. Anything inside the event horizon, including a photon, is prevented from escaping across the event horizon by the extremely strong gravitational field. Particles from outside this region can fall in, cross the event horizon, and will never be able to leave. Since external observers cannot probe the interior of a black hole, according to classical general relativity, black holes can be entirely characterized according to three parameters: mass, angular momentum, and electric charge. This principle is summarized by the saying, coined by John Wheeler, "black holes have no hair" meaning that there are no features that distinguish one black hole from another, other than mass, charge, and angular momentum.

The event horizon is where the force of gravity becomes so strong that even light is pulled into the black hole. Although the event horizon is part of a black hole, it is not a tangible object. If we were to fall into a black hole, it would be impossible for us to know when we hit the event horizon. A mathematical derivation of the radius of an event horizon is shown below in the section mathematical formulation.

## Singularity

The singularity is not really a tangible object either. According to the General Theory of Relativity the Singularity is a point of infinite space time curvature. This means that the force of gravity has become infinitely strong at the center of a black hole. Everything that falls into a black hole by passing the event horizon, including light, will eventually reach the singularity of a black hole. Before something reaches the singularity it is torn apart by intense gravitational forces. Even the atoms themselves are torn apart by the gravitational forces.

## Historical Background of Black Holes

The concept of a black hole was first proposed by John Michell in 1783. At that time, the Newtonian theory of gravity and the concept of escape velocity were well known. Michell computed that a body with 500 times the radius of the Sun and of the same density would have, at its surface, an escape velocity equal to the speed of light and therefore would be invisible. Although he thought it unlikely, Michell considered the possibility that many such objects that cannot be seen might be present in the cosmos. Using Newton's Laws in the late 1790s John Michell of England and Pierre-Simon LaPlace of France independently suggested the existence of an 'invisible stars'. Michell and LaPlace calculated the mass and size- which is now called the 'event horizon' – that an object needs in order to have an escape velocity greater than the speed of light. In 1915, Albert Einstein developed the theory of gravity called general relativity and also showed that gravity does not influence light. But within a month, Karl Schwarzschild (Schwarzschild, 1916) gave the solution for the gravitational field of a point mass, showing that something we now call a black hole could theoretically exist. Such a black hole is called the Schwarzschild black hole and the Schwarzschild radius is now known to be the radius of the event horizon of a non rotating black hole. In another coincidence, only a few months after Schwarzschild, a student of Lorentz, Johannes Droste, independently gave the same solution for the point mass as Schwarzschild had and wrote even more extensively about its properties. In the 1920s, Subrahmanyan Chandrasekhar argued that special relativity demonstrated that a non-

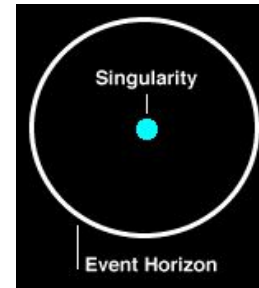


Fig. 1. A black hole with its singularity and event horizon.

radiating body above 1.44 solar masses, now known as the Chandrasekhar limit, would collapse since there was nothing known at that time that could stop it from doing so. Chandrasekhar (Chandrasekhar, 1931) discovered in 1930 the existence of an upper limit to the mass of a completely degenerate configuration. In 1935, Arthur Eddington (Eddington, 1935) opposed Chandrasekhar's arguments and believed that something would inevitably stop the collapse. Both were correct, since a white dwarf more massive than the Chandrasekhar limit will collapse into a neutron star. However, a neutron star above about three solar masses (the Tolman-Oppenheimer-Volkoff limit) will itself become unstable against collapse due to similar physics. In 1939, Robert Oppenheimer and H. Snyder (Oppenheimer and Snyder, 1939) predicted that massive stars could undergo a dramatic gravitational collapse. Black holes could, in principle, be formed in nature. Such objects for a while were called frozen stars since the collapse would be observed to rapidly slow down and become heavily redshifted near the Schwarzschild radius. Oppenheimer and Snyder (1939) found, by calculating the collapse of a homogeneous sphere of pressure less gas in general relativity that the sphere eventually becomes cut off from all communication with the rest of the Universe. This was the first rigorous calculation demonstrating the formation of a black hole. Black holes and the problems of gravitational collapse were not the topic of much interest until the 1960s. However, in the late 1950s, Wheeler and his collaborators began a serious investigation of the problem of collapse and it is Wheeler (Wheeler, 1968) who coined the name 'black hole' in 1968. In 1963, Kerr discovered an exact family of charge-free solutions to Einstein's vacuum field equations (Kerr, 1963). The charged generalization was subsequently found as a solution to Einstein-Maxwell field equations by Newman *et al.* (Newman *et al.*, 1965). The Kerr-Newman geometry described by these solutions provides a unique and complete description of the external gravitational and electromagnetic fields of a stationary black hole.

Interest in black holes was rekindled in 1967 because of theoretical and experimental progress. Stephen Hawking and Roger Penrose proved that black holes are a generic feature in Einstein's theory of gravity, and cannot be avoided in some collapsing objects. Interest was renewed in the astronomical community with the discovery of pulsars. The older Newtonian objects of Michell and Laplace are often referred to as 'dark stars' to distinguish them from the 'black holes' of general relativity.

A number of important properties of black holes were discovered and several powerful theorems concerning black holes were proved during this period. The discovery of compact X-ray sources in 1962, quasars in 1963 and pulsars in 1968 helped motivate this intensive theoretical study of black holes. Observations of the binary X-ray source Cygnus X-1 in the early 1970s provided the first plausible evidence that black holes might actually exist in space.

### Mathematical Formulation

Black holes are predictions of Albert Einstein's theory of general relativity. There are many known solutions to the Einstein field equations which describe black holes, and they are also thought to be an inevitable part of the evolution of any star of a certain size. In particular, they occur in the Schwarzschild metric, one of the earliest and simplest solutions to Einstein's equations, found by Karl Schwarzschild in 1915. This solution describes the curvature of space time in the vicinity of a static and spherically symmetric object, where the metric is,

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (1)$$

where  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$  is a standard element of solid angle.

According to general relativity, a gravitating object will collapse into a black hole if its radius is smaller than a characteristic distance, known as the Schwarzschild radius. (Indeed, Buchdahl's theorem in general relativity shows that in the case of a perfect fluid model of a compact object, the true lower limit is somewhat larger than the Schwarzschild radius.) Below this radius, space time is so strongly curved that any light ray emitted in this region, regardless of the direction in which it is emitted, will travel towards the centre of the system. Because relativity forbids anything from traveling faster than light, anything below the Schwarzschild radius – including the constituent particles of the gravitating object – will collapse into the centre. A gravitational singularity, a region of theoretically infinite density, forms at this point. Because not even light can escape from within the Schwarzschild radius, a classical black hole would truly appear black. The Schwarzschild radius is given by,

$$r_s = \frac{2GM}{c^2} \quad (2)$$

where  $G$  is the gravitational constant,  $M$  is the mass of the object, and  $c$  is the speed of light. For an object with the mass of the Earth, the Schwarzschild radius is a mere 9 millimeters — about the size of a marble. The mean density inside the Schwarzschild radius decreases as the mass of the black hole increases, so while an earth-mass black hole would have a density of  $2 \times 10^{30} \text{ kg m}^{-3}$ , a super massive black hole of  $10^9$  solar masses has a density of around  $20 \text{ kg m}^{-3}$ , less than water! The mean density is given by

$$\rho = \frac{3c^6}{32\pi M^2 G^3} \quad (3)$$

Since the Earth has a mean radius of 6371 km, its volume would have to be reduced  $4 \times 10^{26}$  times to collapse into a black hole. For an object with the mass of the Sun, the Schwarzschild radius is approximately 3 km, much smaller than the Sun's current radius of about 696,000 km. It is also significantly smaller than the radius to which the Sun will ultimately shrink after exhausting its nuclear fuel, which is several thousand kilometers. More massive stars can collapse into black holes at the end of their lifetimes.

The formula also implies that any object with a given mean density is a black hole if its radius is large enough. If the visible universe has a mean density equal to the critical density, then it is a black hole. More general black holes are also predicted by other solutions to Einstein's equations, such as the Kerr metric for a rotating black hole, which possesses a ring singularity. Then we have the Reissner-Nordström metric for charged black holes. Last the Kerr-Newman metric is for the case of a charged and rotating black hole. There is also the Black Hole Entropy formula:

$$S = \frac{A k c^3}{4\eta G} \quad (4)$$

Where  $A$  is the area of the event horizon of the black hole,  $\eta$  is Dirac's constant (the 'reduced Planck constant'),  $k$  is the Boltzmann constant,  $G$  is the gravitational constant,  $c$  is the speed of light and  $S$  is the entropy. A convenient length scale to measure black hole processes is the 'gravitational radius', which is equal to

$$r_G = \frac{GM}{c^2} \quad (5)$$

When expressed in terms of this length scale, many phenomena appear at integer radii. For example, the radius of a Schwarzschild black hole is two gravitational radii and the radius of a maximally rotating Kerr black hole is one gravitational radius. The location of the light

circularization radius around a Schwarzschild black hole (where light may orbit the hole in an unstable circular orbit) is  $3r_G$ . The location of the marginally stable orbit, thought to be close to the inner edge of an accretion disk, is at  $6r_G$  for a Schwarzschild black hole.

### **Formation of Black Holes**

General relativity (as well as most other metric theories of gravity) not only says that black holes *can* exist, but in fact predicts that they will be formed in nature whenever a sufficient amount of mass gets packed in a given region of space, through a process called gravitational collapse. For example, if we compressed the Sun to a radius of three kilometers, about four millionths of its present size, it would become a black hole. As the mass inside the given region of space increases, its gravity becomes stronger — or, in the language of relativity, the space around it becomes increasingly deformed. Eventually gravity gets so strong that nothing can escape; an event horizon is formed, and matter and energy must inevitably collapse to a density beyond the limits of known physics.

Imagine a star which is much more massive than our sun, and which has a mass, called the critical mass, which is large enough to cause a black hole to form. What keeps this star from collapsing onto itself and becoming a black hole? The answer is that there is an intense pressure caused by nuclear reactions within the sun. When the fuel that feeds the nuclear reactions gets used up the massive star cannot support itself anymore. It then collapses to form a black hole.

It is interesting to note that when a black hole is formed by a collapsing star it is actually impossible to watch the final steps of the formation of the black hole from a stationary external reference frame. An external reference frame is a place where one watches the formation of the black hole from far away, like an astronomer on Earth. In addition, it is impossible to see any object fall into a black hole. This is not to say that everything appears to freeze just before entering a black hole. As an object falls into a black hole it gets increasingly dimmer and dimmer from the point of view of an outside observer. By the time an object gets to the edge of a black hole, it will be completely black. This effect, called a gravitational redshift, is caused by the immense gravity near the outside of a black hole

### **Evidence for Existence of Black Holes**

Astronomers have found convincing evidence for a super massive black hole in the center of the giant elliptical galaxy M87, as well as in several other galaxies. The discovery is based on velocity measurements of a whirlpool of hot gas orbiting the black hole. In 1994, Hubble Space Telescope data produced an unprecedented measurement of the mass of an unseen object at the center of M87. This is an interesting problem. How do we prove the existence of something that cannot be observed by definition? There are actually many methods used to see if black holes really exist in our universe (Fig. 2).

The first method is to look for objects in our universe that have a lot of mass, but are very small. For example we can prove that there exists a black hole in an astronomical object called M87. This object weighs three billion times more than our sun, but takes up a volume no larger than our solar system. Another method of finding black holes is to look for an acceleration of matter. Since black holes have such strong gravitational fields, they accelerate anything that gets near them to great speeds. Rapid acceleration of an object can be observed by looking for Doppler shifts in the light given off by an accelerating object. Stellar-mass black holes may be involved in gamma ray bursts (GRBs); short duration GRBs are believed to be caused by colliding neutron stars, which form a black hole on merging (Fig. 3). Observations of long GRBs in association with supernovae (Peratt, 1993; Anon, 2006a) suggest that long GRBs are caused by collapsars; a massive star whose core collapses to form a black hole, drawing in the surrounding material. Therefore, a GRB could

possibly signal the birth of a new black hole, aiding efforts to search for them. Another observational evidence of black hole of ten solar masses as seen from a distance of 600 km with the Milky Way is shown in the following Fig.4. The above pictures represent the existence of black holes and super massive black holes in the space or in the center of our galaxy.



Fig. 2. The jet emitted by the galaxy M87 in this image is thought to be caused by super massive black hole at the galaxy's centre.

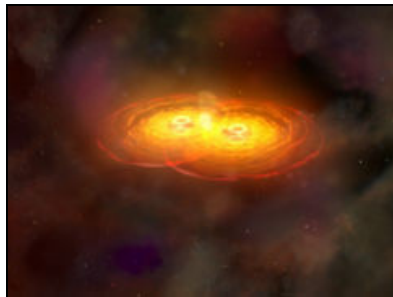


Fig. 3. An artist depiction of two black holes merging.

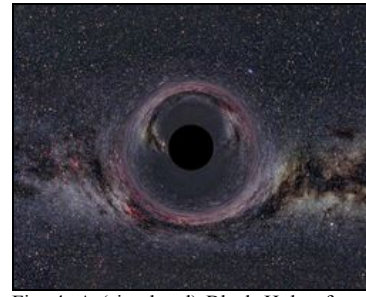


Fig. 4. A (simulated) Black Hole of ten solar masses as seen from a distance of 600 km with the Milky Way in the background (horizontal camera opening angle: 90°).

### Recent Developments

In the recent years, there have been tremendous progresses in the discovery of the new black holes. Many physicists and astronomers have already discovered some new black holes in our Galaxy.

In 2004, astronomers found 31 candidate super massive black holes from searching obscured quasars. The lead scientist said that there are from two to five times as many super massive black holes as previously predicted (Anon, 2006b).

In June 2004 astronomers found a super-massive black hole, Q0906+6930, at the centre of a distant galaxy about 12.7 billion light years away. This observation indicated rapid creation of super-massive black holes in the early universe (Malik, 2006).

In November 2004 a team of astronomers reported the discovery of the first intermediate-mass black hole in our Galaxy, orbiting three light-years from Sagittarius A\*. This medium black hole of 1,300 solar masses is within a cluster of seven stars, possibly the remnant of a massive star cluster that has been stripped down by the Galactic Centre (Anon, 2006c; Anon, 2006d). This observation may add support to the idea that super massive black holes grow by absorbing nearby smaller black holes and stars.

In February 2005, a blue giant star SDSS J090745.0+24507 was found to be leaving the Milky Way at twice the escape velocity (0.0022 of the speed of light), having been catapulted out of the galactic core which its path can be traced back to. The high velocity of this star supports the hypothesis of a super-massive black hole in the centre of the galaxy.

### Discussion

From the above context it is genuinely true that black hole is due to compact objects and compact objects logically begins where normal stellar evolution leaves off. Compact objects consisting of white dwarfs, neutron stars and black holes are 'born' when normal stars 'die', that is, when most of their nuclear fuel has been consumed (Islam, 1992).

All three species of compact object differ from normal stars in two fundamental ways. First, since they do not burn nuclear fuel, they cannot support themselves against gravitational collapse by generating thermal pressure. Instead, white dwarfs are supported by the pressure of degenerate electrons, while neutron stars are supported largely by the pressure of degenerate neutrons (Shapiro and Teukolsky, 1983). On the other hand, black holes are collapsed stars- that is, stars that could not find any means to hold back the inward pull of gravity and therefore collapsed to singularities. With the exception of the spontaneous radiating ‘mini’ black holes with masses  $M$  less than  $10^{15}$  g and radii smaller than a Fermi, all three compact objects are essentially static over the lifetime of the Universe. They represent the final stage of stellar evolution.

The second characteristic distinguishing compact objects from normal stars is their exceedingly small size. Relative to normal stars of comparable mass, compact objects have much smaller radii and hence, much stronger surface gravitational fields. This fact is dramatically illustrated in Table 1.

White dwarfs can be observed directly in optical telescopes during their long cooling epoch. Neutron stars can be observed directly as pulsating radio sources (‘pulsars’) and indirectly as gas-accreting periodic X-ray sources (‘X-ray pulsars’). Black holes can only be observed indirectly through the influence they exert on their environment. For example, they could be observed as gas-accreting aperiodic X-ray sources under appropriate circumstances.

Table 1. Distinguishing traits of compact objects.

Objects	Mass <sup>a</sup> ( $M$ )	Radius <sup>b</sup> ( $R$ )	Mean density ( $\text{g cm}^{-3}$ )	Surface potential ( $GM/Rc^2$ )
Sun	$M_{\odot}$	$R_{\odot}$	1	$10^{-6}$
White dwarfs	$\leq M_{\odot}$	$\sim 10^{-2} R_{\odot}$	$\leq 10^7$	$\sim 10^{-4}$
Neutron stars	$\sim 1-3 M_{\odot}$	$\sim 10^{-5} R_{\odot}$	$\leq 10^{15}$	$\sim 10^{-1}$
Black hole	Arbitrary	$\frac{2GM}{c^2}$	$\sim \frac{M}{R^3}$	$\sim 1$

<sup>a</sup>  $M_{\odot} = 1.989 \times 10^{33}$  g ; <sup>b</sup>  $R_{\odot} = 6.9599 \times 10^{10}$  cm . Source: Cambridge University, England.

Compact objects are the end products of stellar evolution. The primary factor determining whether star ends up as a white dwarf, neutron star or black hole is thought to be the star’s mass. White dwarfs are believed to originate from light stars with masses  $M \leq 4M_{\odot}$ , where  $M_{\odot}$  is the mass of our sun. There is a maximum allowed mass for white dwarfs, which is around  $1.4M_{\odot}$ . White dwarfs progenitor stars probably undergo relatively gentle mass ejection (forming ‘planetary nebulae’) at the end of their evolutionary lifetimes before becoming white dwarfs (Misner *et al.*, 1973).

Neutron stars and black holes are believed to originate from more massive stars. However, the dividing line between those stars that form neutron stars and those that form black holes is very uncertain because the final stages of evolution of massive stars are poorly understood. Neutron stars also have a maximum mass (in the range of  $1.4 - 3 M_{\odot}$ ). Thus the fate of a star with mass  $M \geq 4M_{\odot}$  is not clear at the present time. Further uncertainty is that, one generally assumes that factors other than mass (e.g. magnetic field, rotation, binary star effects) are less important than mass in determining the ultimate fate of a star.

Thus, total gravitational collapse leading to a black hole can, in principle, occur by routes other than by the direct collapse of an evolved, massive star. For example, since there is a definite maximum mass above which a white dwarf or neutron star can no longer support itself against collapse, the accretion of gas by either of these objects (e.g. in a binary system) can lead to black hole formation.

## Conclusion

The identification of an object as a black hole requires the further assumption that no other object (or bound system of objects) could be so massive and compact. Most astrophysicists accept that this is the case, since according to general relativity; any concentration of matter of sufficient density must necessarily collapse into a black hole. So, we conclude that collapsing star is a consequence or concept of a star which finishes all its nuclear burning. In the same process all the stars may have the chance to go under collapse. This collapsing stage is called the black hole. Again if the process continues for all the stars in a galaxy then the whole galaxy may go under a super massive black hole. Thus black holes connect a wide variety of fields of physics. Finding absolutely incontrovertible evidence for a black hole would be the capstone of one of the most remarkable discoveries in the history of science.

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