Khulna University Studies 4(2): 737-741

HALL EFFECTS ON AN MHD FREE CONVECTION AND MASS TRANSFER FLOW IN A ROTATING POROUS MEDIUM WITH TIME DEPENDENT WALL TEMPERATURE

M. M. Alam[a]*, M. A. Sattar[b] and M. D. Hossain[a]

[a]*Mathematics Discipline, Khulna University, Khulna 9208, Bangladesh.*
[b]*Department of Mathematics, Dhaka University, Dhaka-1000, Bangladesh.*

KUS-03/12-270303

Manuscript received: March 27, 2003; Accepted: March 18, 2004

Abstract: The effects of Hall current on an unsteady MHD free convection and mass transfer flow in an electrically conducting incompressible fluid past an infinite vertical porous plate in a rotating porous medium having a constant heat source and a variable suction have been analyzed. Similarity equations are derived by introducing a time dependent length scale. These equations are then solved numerically. The effects of various parameters entering into the problem on the primary and secondary velocities are shown graphically. Finally the values of τ_x, τ_y of the skinfriction coefficient and the Nusselt number N_u are tabulated.

Key words: Hall effect; Free convection; Mass transfer flow and Wall temperature

Introduction

Recently interest has grown considerably on the study of magnetohydrodynamic (MHD) viscous flows with Hall currents due to its engineering applications. The unsteady MHD free convection flow with Hall currents was first studied by Pop (1972). Rapties and Kafousias (1982) studied free convection and mass transfer flow through a porous medium in the presence of transverse magnetic field. Comprehensive studies have also been carried out on MHD free convection and mass transfer flows in a rotating system by many workers. Some of them are Debnath (1972; 1973), Rapties and Perdikis (1982) and Geogantopoulos *et al.*(1981), Alam and Sattar (1998; 1999). Recently Mahato and Maiti (1988) have studied the unsteady free convective and mass transfer flow in a rotating porous medium when the plate temperature is an oscillatory function of time. More recently Jha (1991) studied the effects of Hall current and wall temperature oscillation on free convective and mass transfer flow in a rotating porous medium with constant heat source. Jha (1991) employed perturbation technique to solve the corresponding momentum and energy equations. Most of these studies, however, considered constant temperature or constant heat flux at the plate. However, Elliott (1969) and Pop (1972) considered time dependent plate temperature for unsteady flows near an infinite flat horizontal plate and near a infinite vertical plate, respectively.

Following the work of Jha (1991), we propose to study the unsteady MHD free convection and mass transfer flow with Hall current in an incompressible electrically conducting fluid past an infinite vertical porous plate in a rotating porous medium having a constant heat source and a variable suction with time dependent temperature at the plate. A uniformly applied magnetic field is taken into consideration along with a time dependent suction velocity. Unlike the work of Jha (1991) the momentum and energy equations in the present study have been solved numerically by the method of superposition.

Mathematical Analysis

Consider an Unsteady MHD free convective and mass transfer flow of an electrically conducting viscous incompressible fluid through a porous medium along an infinite vertical porous plate (z=0) with the effects of Hall current. The flow is subjected to a time dependent suction velocity. The flow is also assumed to be in the x-direction which is taken along the plate in the upward direction and z-axis is normal to it. At time t>0, the temperature and the species concentration at the plate are raised to $T_w(\neq T_\infty)$ and $C_w(\neq C_\infty), T_\infty$ and C_∞ being the temperature and the species concentration of the uniform velocity, and thereafter maintained constant. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected (Shercliff, 1965). As in the case of Jha (1991), the fluid is permeated by a strong magnetic field **B** such that $\mathbf{B}=(0,0,B_0)$. Using the relation $\nabla.J=0$ for the current density $\mathbf{J}=(J_x, J_y, J_z)$ we obtain J_z =constant. Since the plate is non conducting $J_z=0$ at the plate and hence zero everywhere. The generalized Ohm's law in the absence of electric field (Meyer, 1958), is of the form

*Corresponding author: Tel: 88-041-720171-3, Ext.265; Fax: 88-041-731244

DOI: https://doi.org/10.53808/KUS.2002.4.2.0312-se

$$\mathbf{J}+\frac{\omega_e\tau_e}{B_0}\mathbf{J}\wedge\mathbf{B}=\sigma'(\mu_e\mathbf{q}\wedge\mathbf{B}+\frac{1}{en_e}\nabla p_e)$$

where $\mathbf{q}=(u,v,w)$ is the velocity vector and $\sigma', \mu_e, \omega_e, \tau_e, e, n_e$ and p_e are respectively the electric conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time ,the electric charge ,the number density of electron and electron pressure . We consider a weakly ionized gas for which the electron pressure is negligible. Also under usual assumptions the thermoelectric pressure and ion slip are negligible. Then we have from Ohm's law

$$J_x=\frac{\sigma'\mu_e B_0}{1+m^2}(mu-v)$$

$$J_y=\frac{\sigma'\mu_e B_0}{1+m^2}(u+mv)$$

where $m=\omega_e\tau_e$ is the Hall parameter.

In accordance with the above assumptions and Boussineq's approximation that variation of density of the field is taken in the body force term, the basic equations (Alam and Sattar, 1998; 1999) relevant to the problem are momentum equations

$$\frac{\partial u}{\partial t}+w\frac{\partial u}{\partial z}-2\Omega v=\upsilon\frac{\partial^2 u}{\partial z^2}+g_0\beta(T-T_\infty)+g_0\beta^*(C-C_\infty)-\frac{\upsilon u}{K}-\frac{\sigma' B_0^2\mu_e^2}{\rho(1+m^2)}(u+mv) \quad (1)$$

$$\frac{\partial v}{\partial t}+w\frac{\partial v}{\partial z}+2\Omega u=\upsilon\frac{\partial^2 v}{\partial z^2}-\frac{\upsilon v}{K}+\frac{\sigma' B_0^2\mu_e^2}{\rho(1+m^2)}(mu-v) \quad (2)$$

continuity equation:

$$\frac{\partial w}{\partial z}=0 \quad (3)$$

energy equation:

$$\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2}+\frac{Q}{\rho C_p}(T_\infty-T) \quad (4)$$

concentration equation:

$$\frac{\partial C}{\partial t}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2} \quad (5)$$

where u and v are the velocity components along x and y directions, w is the suction velocity, T and C are respectively the temperature and concentration of the fluid, υ is the kinematic viscosity of the fluid, ρ is the density of the fluid, k is the thermal conductivity, C_p is the specific heat at constant pressure, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, g_0 is the acceleration due to gravity, σ' is the electric conductivity, Ω is the constant angular velocity, Q is the heat source, K is the permeability of the porous medium and D is the molecular diffusivity.

The boundary conditions for the problem are

$$u=0, v=0, w=w(t), T=T_w, C=C_w \quad at \quad z=0$$

$$u=0, v=0, w=0, T=T_\infty, C=C_\infty \quad as \quad z\to\infty \quad (6)$$

In order to obtain a similarity solution to the problem considered, we now introduce a similarity parameter σ as

$$\sigma=\sigma(t), \qquad t>0 \quad (7)$$

such that σ is a length scale.

In terms of this length scale, a convenient solution of equation (3) is considered to be

$$w = -w_0 \frac{\upsilon}{\sigma} \tag{8}$$

where w_0 is the suction parameter.

We now introduce the following dimensionless variables

$$\eta = \frac{z}{\sigma}$$
$$u = U_0 f(\eta) \tag{9}$$
$$v = U_0 g(\eta)$$

$$T = T_\infty + (T_w - T_\infty)\sigma_*^n \theta(\eta)$$
$$C = C_\infty + (C_w - C_\infty)\phi(\eta). \tag{10}$$

where $\sigma_* = \dfrac{\sigma}{\sigma_0}, \sigma_0$ being the value of σ at $t = t_0, U_0$ is the uniform velocity and n is a positive integer. Introducing the equations (8)-(10) in equation (1),(2), (4) and (5), we obtain the following dimensionless equations respectively

$$-\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}\eta f' - w_0 f' - 2Rg = f'' - \frac{1}{K'}f + G_r\theta + G_m\phi - \frac{M}{(1+m^2)}(f + mg) \tag{11}$$

$$\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}g'\eta - w_0 g + 2Rf = g'' - \frac{1}{K'}g + \frac{M}{1+m^2}(mf - g) \tag{12}$$

$$\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}(n\theta - \eta\theta') - w_0\theta' = \frac{1}{P_r}\theta'' - \frac{\alpha}{P_r}\theta \tag{13}$$

$$-\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}\eta\phi' - w_0\phi' = \frac{1}{S_c}\phi'' \tag{14}$$

where $G_r (= \dfrac{\sigma^2 g_0 \beta \alpha_*^2 (T_w - T_\infty)}{U_0 \upsilon})$ is the Grashof number, $R (= \dfrac{\Omega\sigma^2}{\upsilon})$ is the rotational parameter, $G_m (= \dfrac{g_0 \beta^* \sigma^2 (C_w - C_\infty)}{U_0 \upsilon})$ is the modified Grashof number, $M (= \dfrac{\sigma' B_0^2 \sigma^2 \mu_e^2}{\upsilon\rho})$ is the magnetic parameter, $P_r (= \dfrac{\rho C_p \upsilon}{k})$ is the Prandtl number, $S_c (= \dfrac{\upsilon}{D})$ is the Schmidt number, $\alpha = (\dfrac{Q\sigma^2}{k})$ is the heat source parameter and $K' = \dfrac{K}{\sigma^2}$ is the permeability parameter. In equations (11)-(14) primes denote differentiation with respect to $\eta \left[()' = \dfrac{d()}{d\eta} \right]$

The boundary conditions (6) relevant to the equations (11)-(14) now transform to

$$f = 0, g = 0, \theta = 1, \phi = 1 \quad at \quad \eta = 0$$
$$f = 0, g = 0, \theta = 0, \phi = 0 \quad as \quad \eta \to \infty. \tag{15}$$

The equations (11)-(14) are similar except for the term $\dfrac{\sigma}{\upsilon}\dfrac{\partial\sigma}{\partial t}$ where time t appears explicitly.

Thus the similarity condition requires that $\dfrac{\sigma}{\upsilon}\dfrac{\partial\sigma}{\partial t}$ in the equations (11)-(14) must be a constant quantity.

Hence following the work of Sattar & Hossain (1999) one can try a class of solutions of the equations (11)-(14) by assuming that

$$\frac{\sigma}{\upsilon}\frac{\partial \sigma}{\partial t}=\text{C(a constant).} \tag{16}$$

Now integrating (16), one obtains

$$\sigma=\sqrt{2C\nu t} \tag{17}$$

where the constant of integration is determined through the condition that $\sigma = 0$ when t=0. It thus appears from (17) that, by making a realistic choice of C to be equal to 2 in (16) the length scale σ becomes equal to $\sigma = 2\sqrt{\nu t}$ which exactly corresponds to the usual scaling factor considered for various nonsteady boundary layer flows (Schlichting, 1968). Since σ is a scaling factor as well as a similarity parameter, any other value of C in (16) would not change the nature of the solution except that the scale would be different. Now making a realistic choice of C to be equal to 2 in equation (16), the equations (11)-(14) finally become

$$f''+2f'\zeta+2Rg-\frac{1}{K'}f+G_r\theta+G_m\phi-\frac{M}{(1+m^2)}(f+mg)=0 \tag{18}$$

$$g''+2g'\zeta-2Rf-\frac{1}{K'}g+\frac{M}{(1+m^2)}(mf-g)=0 \tag{19}$$

$$\theta''+2P_r\theta'\zeta-\theta(2nP_r+\alpha)=0 \tag{20}$$

$$\phi''+2S_c\phi'\zeta=0 \tag{21}$$

where $\zeta=\eta+\frac{w_0}{2}$.

Among the equations (18)-(21), equation (21) is simple and non-coupled whose solution is straight forward and is obtained as

$$\phi(\eta)=\frac{erfc(\sqrt{S_c}\zeta)}{erfc(\sqrt{S_c}\frac{w_0}{2})} \tag{22}$$

Then using the solution (21) and applying the method of superposition (Na, 1979) the equations (18)-(20) have been solved numerically. The essence of this method of superposition is to reduce the boundary value problem to an initial value problem, which can easily be integrated out, without any iteration, by any initial value solver. For this purpose the well-known Runge Kutta Merson Integration Scheme has been used. Since for weakly ionized gas the value of m (Hall parameter) is less than unity (Sherman and Sutton, 1961 calculations have been made for m=0.3, 0.5, 0.7. As for other parameters values are taken as those of Jha[10] for consistency. Now denoting τ_x and τ_y as the components of the skinfriction and N_u as the Nusselt number, the numerical values of τ_x, τ_y and N_u are obtained from the process of numerical integration. These values are then sorted in Tables 1 and 2.

Results and Discussion

The velocity profiles for the x and y components of velocity, commonly known as primary and secondary velocities, are shown in Figs. 1-4 for different values of m, R, K', w_0 and α and for fixed values of M, P_r, G_r, G_m and S_c. In the calculations the value of n is taken to be 2.0, which define the case of linear dependence of the plate temperature on time. Further, the values of M and G_r are taken to be large, since these values respectively correspond to a strong magnetic field and to a cooling problem that is generally encountered in nuclear engineering in connection with the cooling of reactors. The value of S_c, (the Schmidt number) is taken to be .6, which corresponds to water vapour and represents a diffusing chemical species of most common interest in air (for example in air because of the presence of H_2 or H_2O the value of S_c is .6). As for Prandtl number we take $P_r = 0.71$ which corresponds to air at $20^o c$. With these flow parameters, it is thus observed from Figs.1 and 2 that the primary and secondary velocities decrease owing to the increase of α (Heat source parameter), which means that the generation of higher heat in the flow reduces the velocity field. It is also observed from these Figures that the rotation parameter R has a minor effect on the primary velocity but comparatively larger decreasing effect on the secondary velocity. As for the effects of Hall parameter, it is also observed from these Figures that Hall current has a minor increasing effect on the primary velocity whereas there is a considerable increase in the secondary velocity which indicates and also supports the fact that the Hall current induces a cross flow in a free convection boundary layer. In Figs. 3 and 4 the effects of the permeability parameter (K') and the suction parameter (w_0) on the primary and secondary velocities are shown respectively. It appears from these Figures that both the primary and secondary velocities increase with the increase of K'. An opposite effect on the primary and secondary velocities is observed as the suction parameter increases, which is usually expected. Since the energy equation is independent of all parameters except the heat source and suction parameters, the temperature profiles are shown only for α and w_0 in Fig. 5.The effects α and w_0 on the temperature profiles are, however, very low as seen from the Fig. 5.Finally, the effects of various parameters on τ_x, τ_y and *Nu* are shown in Tables 1 and 2. The conclusions and discussion regarding the behavior of the parameters on the skinfriction and Nusselt number are evident from the tables.

Table 1. Numerical values of τ_x, τ_y and N_u for $M = 10, G_r = 10, G_m = 4, K' = 1, P_r = .71$ $S_c = .6$ and $w_0 = .5$

M	R	α	τ_x	τ_y	N_u
0.3	0.2	1.0	2.5539	0.1518	2.44009
0.5	0.2	1.0	2.6076	0.2532	2.44001
0.7	0.2	1.0	2.6763	0.3337	2.44001
0.3	0.4	1.0	2.5587	0.1264	2.44001
0.3	0.6	1.0	2.5626	0.1008	2.44001
0.30.	0.2	1.5	2.5230	0.1484	2.54558
3	0.2	2.0	2.4943	0.1452	2.64688

Table 2. Numerical values of τ_x, τ_y and N_u for M=10, $G_r = 10, G_m = 4, P_r = .71, \alpha = 1.0, R = .2, m = .3$ and $S_c = .6$

K'	w_0	τ_x	τ_y	N_u
1.0	0.5	2.5539	0.1518	2.44009
2.0	0.5	2.5861	0.1585	2.44009
3.0	0.5	2.5971	0.1609	2.44010
1.0	1.0	2.5221	0.1446	2.65436
1.0	1.5	2.4797	0.1363	2.88054

References

Alam, M. M. and Sattar, M A. 1998, Journal of Energy Heat and Mass Transfer, **20**, 77.

Alam, M. M. and Sattar, M. A. 1999, Journal of Energy Heat and Mass Transfer, **21**, 09.

Debnath, L. 1972, ZAMM, **52**, 623.

Debnath, L. 1973, Acta Mech., **18**, 333.

Elliott, V. 1969, ZAMM, **49**, 647.

Georgantopoulos, G. A., Koullias, J., Goudas, C. L. and Courogenis, C. 1981, Astrophys. Space Sci., **74**, 359.

Jha. B. K. 1991, Int. J. of Energy Research, **15**, 79.

Mahato, J. P. and Maiti, M. K. 1988, Indian J.Technol, **26**, 255.

Pop, I. 1972, Indian J. Pure Appl. Math., 3,1216.

Meyer, R. C. 1958, J. Aerospace Sci., **25**, 561.

Na, T. Y. 1979, Computational Method in Engineering Boundary Value Problems, Academic Press, New York.

Raptis, A. and Kafousias, N. 1982, Can. J. Phys. **60**, 1275.

Raptis, A. A. and Perdikis, C. P. 1982, J.Astrophys. Space Sci., **84**, 457.

Sattar, M. A. and Hossain, M. M. 1992, Can. J. Phy., 70, 369.

Schlichting, H. 1968, Boundary Layer Theory, McGrow Hill Book comp. New York.

Shercliff, J. A. 1965, A Text book of Magnetohydrodynamics Pergamon Press Inc., New York.

Sherman, A. and Sutton, G. W. 1961, Magnetohydrodynamics, Evenston, Illinois.