



A MODEL TO DEVELOP HOTEL MANAGEMENT SYSTEM TO OPTIMIZE REVENUE

Munnujahan Ara^{1*}, Mahir Foysal¹, Md. Azmol Huda¹, Samen Bairagi²

¹Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh

²Basic Sciences and Humanities Department, University of Asia Pacific, Bangladesh

KUS: 21/16:29092021

Manuscript submitted: September 29, 2021

Accepted: June 29, 2022

Abstract

Hotel provides facilities for customers and earns revenue. In this sector of business, many challenges might be faced by the hotel management since there arise price competition between hotels. Hotel management has to follow some strategies to earn maximum revenue. Though it is difficult to estimate actual profit in any epidemic situations, we showed the optimal revenue is possible even within this kind of epidemic situations. And to do this, a revenue management model was developed in terms of pricing and room allocation capacity. Price is the sole factor in this model. Two types of demand models are used, one is deterministic demand model and another is stochastic demand model and both are used for pricing model and capacity model formulation. This work showed how hotels can gain more revenue by using proposed capacity and pricing model, even in the time of global crisis situation like COVID-19 epidemic.

Keywords: Deterministic demand model, stochastic demand model, capacity model, pricing model, revenue management, optimal capacity

Introduction

Revenue Management (RM) predicts customer behavior at minuscule level of market, optimize availability of product and finally show the price which maximum revenue can be gained. The main aim of RM is firstly categorized the products for different customers and at time basis to sell the right product to the right customer. Both customer and producer can be benefited by the help of RM system. From customer sight, RM help to choose the right product among many products of same category at a low price, on the other hand by the help of RM system, a producer can easily find what types of products should be produce and also the pricing level. Talurri and Ryzin (2004) define RM as: "RM is concerned with such demand-management decisions referred to as either sales decisions and the methodology and systems required to make them."

RM refers to the collection of strategies and tactics, firms use to scientifically manage demand for their products and services, and it is considered as one of the most successful application areas of operations research (OR). The practice of RM has grown as a relatively obscure practice among a handful of major airlines in the post deregulation era in the United States (circa 1978) to its status today as a mainstream business practice with a growing list of industry users, ranging from Walt Disney Resorts to National Car Rental. Professional practice and research in the area is also expanding.

Civil Aviation Board of U.S. started to lose control of the regulation of airlines fare after the deregulation act of 1978. At that time, former Chairman and CEO of American Airlines invented yield management system, as a result the strictly pricing system diminished. Then there occurs more flexible pricing strategy called American Super Saver Fares. Later on, in 1985 a more flexible capacity-controlled model occurred called DINAMO (Dynamic

*Corresponding author: < munnujahan@math.ku.ac.bd >
DOI: <https://doi.org/10.53808/KUS.2022.19.02.2116-se>

Inventory Allocation and Maintenance Optimizer), which is called Ultimate Super Saver Fares. So, this yield management system aimed discounts in those situations of surplus of empty seats.

In 1972 when developing the overbooking first RM system research had been exercised by Littlewood (1972). Necessary conditions like- handling capacity, competitors' rate, is included in this system explicitly as variables. The maximum likelihood method is used to determine the unknown parameters. A dynamical model which helps to finding booking strategy for hotel yield management is made by Badinelli (2000) and which shows relationship among the marketing segments.

Game theory is also used in pricing strategy like-hotel management, hospitality management. Where the strategies (price and player) moving simultaneously. It is a duopoly game segmentation which works in competitive market like-hotel industry. In this case two hotels compete between them by offering same products in this regards price moves simultaneously; which is called '*Bertrand equilibrium*'. On the other hand, Cournot method describes how much revenue gained by a company through a competition among them. In this regards many companies should be produce same product and there exists no cooperation and also no collusion. Companies should be rational in this model and there is no product differentiation. Bertand and Cournot (2004) competition have been used to model the demand of a company that has influenced from the competitors.

For hotel industry RM system helps both the customer and the hotel company. A customer can be categorized the hotel by price basis against of hotel facilities. Then it is easier for the customer to choose the appropriate one to stay. On the other hand, for a hotel company it is easier for them to find the best room allocation system in against of price also they can aware of the customer choice. The main problem they face is the cancellation of a booking room near the booking date, it generally occurs when a customer book a room over phone. The fixed capacity of room is also a problem for them at the time of vacation. Finally, the price factor; is the main factor when the hotel companies compete among them, because of high price, one may lose some customers and as a result there occurs a situation where the number of unbook rooms increase and the cherished revenue level cannot be gained. To avoid the loss, need proper type of management of pricing system and capacity of room allocation.

A tourist spot like-Saint Martin Island in Bangladesh, people go there to spend their leisure and entertain them, and hotel provides those opportunities for the customers. Our main objective is to construct a revenue management system which will be useful in increasing the revenue of the hotels at Saint Martin Island during Covid-19 epidemic environment. In order to do that first we constructed an optimal capacity allocation model and then with the help of game theory a pricing model was developed based on the competition of two hotels.

A brief summary of the upcoming sections is as: an introductory section is given earlier where we discussed about the basic ideas and methods that used for this work, like as, probability distribution of demands, where the Poisson distribution and exponential distribution separately showed, also booking limit and protection levels which are the main theme of our work. We also discussed about RM Duopoly model and Static Bertrand Models. And finally, we discussed about the Bertrand and Cournot competition model. In the subsequent sections we formulate the capacity model including the controls booking limit and protection level, monopoly model, duopoly model with deterministic demand and stochastic demand, where we mainly used deterministic model for analyzing data for both hotels. Based on the collected data, numerical studies were done on optimal capacity allocation and competitive prices for both hotels are also mentioned. In the next section, we discussed about our finding according to the models we developed and showed the earned revenue amount by each hotel. And finally, we summarize our total work as conclusion.

Methodology

In this section various methods and topics are discussed that helped us to understand and formulate the capacity and pricing model. We examined quantity-based revenue management for a single resource, especially allocating capacity of a resource to different classes of demand and the controls known as booking limits and protection levels. Next, revenue management (RM) duopoly games between two firms are discussed. Finally, we talked about pricing-based revenue management using game theory model of pricing by Bertrand and Cournot Competition.

Probability Distributions of Demand

In order to understand and formulate capacity model and pricing model, we need to consider the probability distribution of demand. In our case demand is the customers' arrival rate. According to Talurri and Ryzin (2004), demand follows Poisson distribution which is used to model the booking limit. Also, exponential distribution can be used to calculate the time interval between each demand, which is used to model the stochastic demand model.

Poisson Distribution

Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

When the following assumptions become true, the Poisson distribution becomes an appropriate model:

- k is the number of times an event occurs in an interval and k can take values $0, 1, 2, \dots$
- The occurrence of one event does not affect the probability that a second event will occur. That is, events occur independently.
- The average rate at which events occur is independent of any occurrences. For simplicity, this is usually assumed to be constant, but may in practice vary with time.
- Two events cannot occur at exactly the same instant; instead, at each very small sub-interval exactly one event either occurs or does not occur.

If these conditions are true, then k is a Poisson random variable, and the distribution of k is a Poisson distribution.

Exponential Distribution

This distribution is the probability distribution in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution.

The probability density function (pdf) of an exponential distribution is,

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (1)$$

Here $\lambda > 0$ is the parameter of the distribution, often called the rate parameter. The distribution is supported on the interval $[0, \infty)$.

The cumulative distribution function is given by

$$F(x; \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (2)$$

Booking Limits

Booking limit means the controls which limit the capacity level for a particular item when the company is eager to sell them to its customer at a given point of time. For example, let a hotel has 15 rooms as a booking limit of which the hotel management can allocate them at a fixed price. When they are able to allocate all the rooms, then the fixed price class would be closed for the customer. Generally, most often the room allocation is less than the room shown by them. If they want to maintain capacity level for the future, they can use another class allocation for fixed price. There are two types of booking limit: one is partitioned and other is nested.

For a partitioned booking limit the whole number of rooms can be classified into different blocks. For example, a hotel has total of 40 rooms for allocation. The hotel management is eager to allocate 24 of the total rooms are at a fixed price and other 16 at a discount price. If 24 rooms are allocating at a fixed price then this class would be closed. So, as same for the discount class, then there occurs partitioned booking limit.

For nested booking limit, let the total number of rooms is divided into two prices. For example, a hotel has total of 40 rooms for allocation. The hotel management is eager to allocate 24 of the total rooms are at a fixed price of 3000 Bangladeshi taka (BDT) per room and other 16 at a discount price of 2000 BDT per room, when there is no refund available for 2000 BDT per room. Then if a third-party business group takes all the 40 rooms at a fixed price, then the hotel management can increase their revenue by using nested booking limit. Let us consider the nested booking limit for class j denoted by b_j

Protection Levels

A protection level means the amount of capacity which a company can reserve the items for some particular people or some group of people. There are also two types of protection levels, one is partitioned and other is nested. A partitioned protection level is relative to a partitioned booking level. Suppose that, a hotel has a booking limit of 15 rooms as a fixed price is equivalent to reserve (protect) 15 rooms for fixed price. In nested protection level, protection level can be defined for sets of different classes ordered in a hierarchical manner. For example, let a hotel has fixed price rooms for 3000 BDT each and discount price rooms for 2000 BDT each. Then let the protection level denoted by y_j which is defined as the capacity level for classes $j, j-1, \dots, 1$.

Let for nested protection level, let a hotel has set a protection level of 24 rooms as a fixed price class that means the hotel management is eager to allocate 24 out of the whole rooms at a fixed price. On the other hand, a protection level of total 40 rooms in which there is a combination of fixed price class and discount price class. In the last case there is no specified protection level, it is clear that there occurs all the availability of capacity level. Let the booking limit is j , C be the total capacity, y_{j-1} denotes the protection level for classes $j-1$ and higher. Then we write,

$$b_j = c - y_{j-1}, j = 2, \dots, n \tag{3}$$

When, $b_j = C$, then occurs highest class of booking limit which is equal to the capacity and when $y_n = C$, then occurs combination of all classes' protection level is equal to capacity.

Littlewood's Two-Class Model

In Littlewood's model it deals with prices. Let us assume for two products with their associated prices p_1 and p_2 , where $p_1 > p_2$, the total capacity is C and let there are no overbooking and cancellation. D_j be the j^{th} class demand, where $F_j(\cdot)$ is its distribution. In this model, demand for class 2 come first. The main goal for this problem is to find out how much demand of class 2 can be accepted without seeing the demand of class 1. Suppose that a hotel has x units of remaining capacity and they get a booking for class 2. Then they earn revenue p_2 by collecting the order. If they do not accept the request for class 2, they will sell the remaining x unit at p_1 (higher price) if and only if there occurs a high demand for class 1. That is $D_1 \geq x$. Thus, the hotel's gain from reserving x unit class 2 rooms for class 1 rooms is denoted by $p_1(D_1 \geq x)$. Therefore, the hotel will accept class 2 request for class 1 as long as price exceeds the marginal value if and only if $p_2 \geq p_1 P(D_1 \geq x)$. It shows that right hand side is decreasing in x . Let y_1^* denotes an optimal protection level. So, the company will accept when the remaining capacity y_1^* exceeds and on the other hand they will not accept if the remaining capacity is y_1^* or less, such that y_1^* satisfies

$$p_2 < p_1 P(D_1 \geq y_1^*) \text{ and } p_2 \geq p_1 P(D_1 \geq y_1^* + 1) \tag{4}$$

If $F_1(x)$ denotes a continuous distribution to model demand then the optimal protection level y_1^* is given by

$$p_2 = p_1 P(D_1 > y_1^*) \text{ equivlently } y_1^* = F_1^{-1} \left(1 - \frac{p_2}{p_1} \right) \tag{5}$$

This is known as Littlewood's rule. By Littlewood's rule, if we set a protection level y_1^* for class 1, then it is an optimal policy or a booking limit of $b_2^* = c - y_1^*$, then class 2 demand is optimal. Otherwise, we use $\pi(x) = p_1 P(D_1 > x)$ as a bid price control. Suppose, D_1 is normal distribution, μ be the mean, σ be the standard deviation. Then by Littlewood's rule, $F_1(y_1^*) = 1 - p_2/p_1$ and the optimal protection level can be denoted by $y_1^* = \mu + z\sigma$, where $z = \Phi^{-1}(1 - p_2/p_1)$ and the inverse of standard normal cumulative distribution function (c.d.f) is $\Phi(\cdot)^{-1}$. Meeting the mean demand for class 1, the hotel should reserve enough capacity. μ is plus or minus, a factor which depends both on revenue ratio and demand variation σ . If $p_2 / p_1 > 0.5$, the optimal protection level is less than the mean demand. If $p_2 / p_1 < 0.5$, it is greater than the mean demand. If the value of ratio p_2 / p_1 is low, then the more capacity for class 1 occurs.

RM Duopoly Games

Let us consider a duopoly case where two firms have their fixed capacity level but both of them want to sell them for different price group, for example, H for high and L for low, where the identical prices be $p_H > p_L$. Customer choice among the firms within the same class and strategic decision refers to the capacity allocation for the low (L) class. Customers demand for class (class H and class L) and firms (firm 1 and firm 2) which is modeled D_{kj} as random variable. Let $k = HL$ (high class and low class) and $i = 1, 2$ (firm 1 and firm 2), then the reallocation of demand can be happening as follows. If the high class H of firm 1 is closed then the remaining customers cherishing class H for firm 1 goes to firm 2. As the same way if for firm 1, class L is closed, all the customers cherishing L goes to firm 2.

If there is the possibility of no buy-up then a customer of demanding L class is assumed to never buy a H class and a customer demanding H never buy L class. It is assumed that all H demand customers appear after L demand customers. There is also possibility of existence of pure-strategy Nash equilibrium by using Littlewood's rule. Let y_1 be the protection level for firm 1 for high fare class over the integer set between $[0, C]$, on the other hand $C - y_2$ be the strategy space for firm 2 for high fare class over the integer set between $[0, C]$ and y_2 be the protection level for firm 2. By Littlewood's rule, the optimal protection level for H class is independent for L class customer. As there is possibility of not buying-up L class demand, then this duopoly RM games can be defined for only H class demand protection level. By Littlewood's rule, it is assumed that two classes demand cannot be correlated. That shows that for firm 1, H class demand is correlated with firm 2, H class demand so as for the L class. In this case by using Littlewood's rule each firm is able to set its best response. For different firm for a fixed protection level both L and H class demand is uncorrelated. By the above assumption it is assumed that there is still a guaranteed equilibrium. In case of correlation between L class and H class demand it needs not always existence of an equilibrium.

A firm's (say firm 1) objective function can be caused by the correlation between the demands of L class and H class. At the same time as firm 2 increases their booking limit for L class then the optimal solution for firm 1 can jump from one step of the multimodal function to another step. For correlation matrix the existence of pure-strategy equilibrium is one of the main results in this case.

Static Bertrand Models

Bertrand price competition model is the second fundamental model of oligopoly competition. The main assumption of this model in case of perfect competition is firms produce similar product and customers buy from the firm offering the lowest price. Firms produce enough product to satisfy all its demand. Each firm competes on prices and prices of the product changes non-cooperatively and simultaneously. We consider the model for duopoly but it can be extended to n firm's case. Let us consider two firms, firm-1 and firm-2. We consider that each firm has same marginal cost. Let the market demand denoted by the function $d(p)$.

Demand for firm-1 at price p_1 is given by the function $d_{p_1}(\cdot)$, i.e.,

$$d_1(p_1, p_2) = \begin{cases} d(p_1) & \text{if } p_1 < p_2 \\ d(p_1)/2 & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 < c_2 \end{cases} \quad (6)$$

Similarly, demand function for firm-2. Firm-1 will lose its demand if firm-2 prices are slight less than firm-1. Then, firm-1 profit function is given by

$$V_1 = (p_1 - c)d_1(p_1, p_2) \quad (7)$$

Let (p_1^*, p_2^*) denote the Nash equilibrium in prices. In the Bertrand model the equilibrium is very straight forward, if both firms price of marginal cost C is same then both will obtain unique equilibrium which is similar to the perfect competition. If we explain this, from firm-1 perspective it will lose all the demand if its price is less than firm-2's price p_2 . If firm-2's price p_2 is greater than the marginal cost, i.e., $p_2 > C$ than it helps the firm-1 to gain demand at price $p_1 = p_2 - \delta$ where $\delta > 0$. Firm-1 gets all the demand and make an increasing profit. Similarly, reverse situation for

firm-2 also gets the upper hand and gets all the demand and can make a profit. This price competition continues until both firms reach their marginal cost C , which is the only equilibrium.

So, in Bertrand's model, firms do not make any profit even if they face only one competition. This result tells us that, if price is the only difference between the firms, the outcome of their competition will gain nothing for them. That's why firms try to have some difference between them and often avoid direct price competition as far as they can.

Bertrand and Cournot Competition

In Ultraphone game, which is like as Bertrand competition, two firms compete just on cost without separate their indistinguishable produce items. It very well may be shown that if demand is a linear function of price and assuming the two firms have a similar marginal cost, then the only Nash equilibrium is for the two firms to set a value equivalent to their marginal cost. As in the model above, cost may be down when competition between two firms happens, and in a Bertrand competition, costs are driven down to their most reduced limit.

With no matter charged price, both Bertrand model and UltraPhone example assume that it is always possible for the firms to produce a sufficient quantity to supply the market, no matter what price is charged. Therefore, the Bertrand model does not fit industries with capacity constraints that are expensive to adjust. Another alternative model is Cournot competition, in which the firms' strategies are quantities rather than prices. If the firms choose higher the quantities, then the price should be lower the prices. Under a cournot competition between two firms, the resulting Nash equilibrium price is higher than the marginal cost (the Bertrand solution) but lower than the monopoly price. As the number of firms competing in market increase, however, the Cournot equilibrium price falls, in the theoretical limit (an infinite number of competitors), the price drops down the marginal cost. Products are identical is another important assumption of Bertrand competition, besides the assumption that there are no capacity constraints, so that the firm offering the lowest price attracts all demand.

Competitors have two choices: High and Low prices. In practice, firms may choose from a variety of prices, so now assume that each firm chooses a price over a continuous range. Let firm's price be $p_i; i = A \text{ or } B$. In this game, a player's strategy is the choice of a price, and now we have to define payoffs, given the strategies. First, we describe how each competitor's price affects demand for the product. In the last section, all customers chose the lowest price. Here we assume that the products are differentiated, so that some customers receive different utilities from the products and are therefore loyal; they will purchase from A or B , even if the other firm offers lower price. Specifically, let $d_i(p_i, p_j)$ be the demand for product i , given that the competitors charge p_i and p_j . We define the linear demand function,

$$d_i(p_i, p_j) = a - bp_i + cp_j \quad (8)$$

If $C > 0$, we say that the products are substitutes: a lower price charged by firm A leads to more demand for A's products and less demand for B's (although if $p_B < p_A$), B does not steal all demand. If $C < 0$, then the products are complements: a lower price charged by A raises demand for both products. In the UltraPhone example, the products are substitutes; product A and a specialized accessory (e.g., a docking station) would be complements. If $C = 0$, then the two products are independent, there is no interaction between the firms, and both firms choose their optimal monopoly price.

Now let m be the constant marginal cost to produce either product. Firm i 's margin is therefore $(p_i - m)d_i(p_i, p_j)$. This function is concave with respect to p_i , and therefore the optimal price for firm i given that firm j charges p_j can be found by taking the derivative the function with respect to p_i , setting that first derivative equal to 0, and solving algebraically for p_i . Let $p_i(p_j)$ be the resulting optimal price, then

$$p_i(p_j) = \frac{1}{2b}(a + bm + cp_j) \quad (9)$$

Here $p_i(p_j)$ is the best response function of i to j , which is the optimal price function of the competitor's price.

Model Formulation

Capacity Model

The main goal of this model is to fix up capacity for customers who are willing to stay at high price rooms or low-price rooms. Hotel management fix up some rooms for customers with low price which is called booking limit. When they gain their targeted limit then rest of the rooms (protection limit) will be offered to the next customers who want high price rooms and they are always ready to pay the room's rent for full price. Hotel management could not gain optimal point if they set more rooms for booking limit, because there is possibility that the number of demands for high price rooms are more than prediction, on the other hand, if booking limit is to low then number of unallocated rooms may occur and they could not reach their optimal point. So, it is necessary to develop both booking limit and protection limit model.

Here both booking limit and protection level are influenced by low price (p_L) and full price (p_F) ratio (r). We assume that the arrival of demand follows Poission distribution. The CDF of Poission distribution is given by

$$F = \sum_{k=0}^y \frac{\lambda^k e^{-\lambda}}{k!} \quad (10)$$

Here,

- k is the number of times an event occurs in an interval and k can take values $0, 1, 2, \dots$
- e is Euler's number ($e = 2.71828\dots$)
- λ is equal to the expected value of a random variable and also to its variance

According to Littlewood's two class model is given by

$$y^* = F^{-1} \left(1 - \frac{p_L}{p_{H_i}} \right) \quad (11)$$

Here,

- p_L is the low priced room of hotel i
- p_{H_i} is the high priced room of hotel i
- F^{-1} is the inverse poission distribution

The booking limit can be obtained using total capacity C . Subtracting protection level from total capacity we get the booking limit b :

$$b = C - y^* \quad (12)$$

Using this capacity model optimal booking limit (number of rooms fixed for low price) and protection level (number of rooms fixed for high price) can be obtained.

Monopoly Model

There are two types of prices offered for a room and they are high price p_H and low price p_L and $p_H > p_L$. Customers are categorized by two types such as young customers who come as group and customers who come as families. There exist two different cases: for young customer, they have no fixed schedule and in case of pricing system they are more sensitive than the families.

The highest number of rooms allocated for low price is defined as booking limit b . When the hotel is able to reach its booking limit, then the next demand will be set to high price. The protection level y^* is defined as the number of rooms that is protected and always set to high price.

The revenue R depends on price, booking limit, protection level and the expected demand. Let C_i is the total capacity of hotel i and D_i is the demand of hotel i . If the hotel management does not change their capacity, then we use occupancy rate of rooms to model the demand. Demand is always highly correlated with price, if the price of rooms is higher, there occurs lower demand. So, it can be said that the demand function only depends on the lower price. Then the room occupancy rate can be formulated as:

$$d(p_L) = \alpha - \beta p_L, \alpha, \beta > 0 \quad (13)$$

Demand for low priced room (booking limit) comes first and after fulfilling the booking limit next demand will go for high priced room (protection level). In this way demand for high priced room depends on low price. So, our demand function depends only on low price,

$$D(p_L) = Cd(p_L) \tag{14}$$

The revenue for Hotel A can be formulated as:

$$\max R_A = \begin{cases} p_{L_A}b_A + p_{H_A}(C_A - b_A) & \text{if } D_A \geq C_A \\ p_{L_A}b_A + p_{H_A}(D_A - b_A) & \text{if } b_A < D_A < C_A \\ p_{L_A}D_A & \text{if } D_A < b_A \end{cases} \tag{15}$$

Similarly, the revenue model for Hotel B can be written as:

$$\max R_B = \begin{cases} p_{L_B}b_B + p_{H_B}(C_B - b_B) & \text{if } D_B \geq C_B \\ p_{L_B}b_B + p_{H_B}(D_B - b_B) & \text{if } b_B < D_B < C_B \\ p_{L_B}D_B & \text{if } D_B < b_B \end{cases} \tag{16}$$

To gain the maximum revenue limit, the hotel management should set the optimum booking limit and protection level, for estimating demand we have to consider the ratio between the high price and low price. It is assumed that the demand follows Poission process, hence the number of demands is Poission distributed with cumulative distributed function $F(y)$. Optimum protection level is determined based on Littlewood’s Rule that is discussed in the above section.

Duopoly Model with Deterministic Demand

Deterministic demand means that demand is known. Hotel A has rooms fixed for low price known as booking limit denoted by b_A and rooms fixed for full price known as protection level denoted by y_A^* . This model limits on duopoly competition between two hotels called Hotel A and Hotel B . Which means only the competition between two hotels at a time is considered. As the demand is a function of price, in order to maximize the revenue, each hotel management will set competitive price, where the demand is influenced by price of competitor, we will use game theory to analyze the model.

First, we need to construct an occupancy rate for the duopoly model. Here, we will consider both hotels’ prices since one hotel’s price may affect the other hotels demand. Since demand is a function of price, the higher the price of Hotel A , the demand of Hotel A will be decreasing and for the higher the price of Hotel B , the demand of Hotel A will be increasing. Thus, the occupancy rate for Hotel A is given by,

$$d_A(p_{L_A}, p_{L_B}) = \alpha_A - \beta_A p_{L_A} + \gamma_A p_{L_B} \tag{17}$$

where, d_A is the occupancy rate (ratio of demand and capacity) and $\alpha_A, \beta_A, \gamma_A$ are the coefficient of occupancy rate model of Hotel A towards Hotel B . The demand function which can be found by multiplying the occupancy rate with the total capacity of Hotel A :

$$D_A = C_A d_A(p_{L_A}, p_{L_B}) \tag{18}$$

So, the revenue for Hotel A is given by:

$$\max R_A = \begin{cases} p_{L_A}b_A + p_{H_A}(C_A - b_A) & \text{if } D_A \geq C_A \\ p_{L_A}b_A + p_{H_A}(D_A - b_A) & \text{if } b_A < D_A < C_A \\ p_{L_A}D_A & \text{if } D_A < b_A \end{cases} \tag{19}$$

Now the revenue when demand for Hotel A is less than the total capacity of Hotel A but greater than the booking limit can be written as:

$$R_A = p_{L_A}b_A + p_{H_A}(D_A - b_A) \tag{20}$$

The payoff function is:

$$R_A(p_{L_A}, p_{L_B}) = p_{L_A} b_A + \frac{p_{L_A}}{r_A} (C_A \alpha_A - C_A \beta_A p_{L_A} + C_A \gamma_A p_{L_B} - b_A)$$

$$R_A(p_{L_A}, p_{L_B}) = p_{L_A} b_A + \frac{C_A p_{L_A} \alpha_A}{r_A} - \frac{C_A p_{L_A}^2 \beta_A}{r_A} + \frac{C_A \gamma_A p_{L_B} p_{L_A}}{r_A} - \frac{b_A}{r_A} p_{L_A}$$

Here, r_A is the ratio between low price and high price of Hotel A.

According to Bertrand and Cournot competition, this payoff function is concave, therefore the optimal price for Hotel A given that Hotel B charges p_{L_B} can be found by taking the derivative of the function with respect to p_{L_A} , setting the first derivative equal to 0 and solving algebraically for p_{L_A} , we have

$$\begin{aligned} \frac{dR_A}{dp_{L_A}} &= b_A + \frac{C_A \alpha_A}{r_A} - \frac{2C_A p_{L_A} \beta_A}{r_A} + \frac{C_A \gamma_A p_{L_B}}{r_A} - \frac{b_A}{r_A} \\ &\Rightarrow b_A + \frac{C_A \alpha_A}{r_A} - \frac{2C_A p_{L_A} \beta_A}{r_A} + \frac{C_A \gamma_A p_{L_B}}{r_A} - \frac{b_A}{r_A} = 0 \\ &\Rightarrow \frac{2C_A p_{L_A} \beta_A}{r_A} = b_A + \frac{C_A \alpha_A}{r_A} + \frac{C_A \gamma_A p_{L_B}}{r_A} - \frac{b_A}{r_A} \\ &\Rightarrow 2C_A p_{L_A} \beta_B = b_A r_A + C_A \alpha_A + C_A \gamma_A p_{L_B} - b_A \\ &\Rightarrow p_{L_A} = \frac{b_A r_A + C_A \alpha_A + C_A \gamma_A p_{L_B} - b_A}{2C_A \beta_A} \\ &\therefore p_{L_A} = \frac{b_A r_A + C_A \alpha_A - b_A}{2C_A \beta_A} + \frac{\gamma_A}{2\beta_A} p_{L_B} \end{aligned} \quad (21)$$

This is the pricing model for Hotel A which is influenced by the price of Hotel B.

Similarly, Hotel B has rooms fixed for low price known as booking limit denoted by b_B and rooms fixed for full price known as protection level denoted by y_B^* . The occupancy rate for Hotel B is given by,

$$d_B(p_{L_A}, p_{L_B}) = \alpha_B - \beta_B p_{L_B} + \gamma_B p_{L_A} \quad (22)$$

where, d_B is the occupancy rate (ratio of demand and capacity) and $\alpha_B, \beta_B, \gamma_B$ are the coefficient of occupancy rate model of Hotel B towards Hotel A.

The higher the price of Hotel B the demand of Hotel B will be decreasing and the higher the price of Hotel A the demand of Hotel B will be increasing. The demand function for Hotel B is,

$$D_B = C_B d_B(p_{L_A}, p_{L_B}) \quad (23)$$

The revenue for Hotel B is

$$\max R_B = \begin{cases} p_{L_B} b_B + p_{H_B} (C_B - b_B) & \text{if } D_B \geq C_B \\ p_{L_B} b_B + p_{H_B} (D_B - b_B) & \text{if } b_B < D_B < C_B \\ p_{L_B} D_B & \text{if } D_B < b_B \end{cases} \quad (24)$$

Now the revenue when demand for Hotel B is less than the total capacity of Hotel B but greater than the booking limit, can be written as

$$R_B = p_{L_B} b_B + p_{H_B} (D_B - b_B) \quad (25)$$

The payoff function is

$$R_B(p_{L_A}, p_{L_B}) = p_{L_B} b_B + \frac{p_{L_B}}{r_B} (C_B \alpha_B - C_B \beta_B p_{L_B} + C_B \gamma_B p_{L_A} - b_B)$$

$$R_B(p_{L_A}, p_{L_B}) = p_{L_B} b_B + \frac{C_B p_{L_B} \alpha_B}{r_B} - \frac{C_B p_{L_B}^2 \beta_B}{r_B} + \frac{C_B \gamma_B p_{L_B} p_{L_A}}{r_B} - \frac{b_B}{r_B} p_{L_B}$$

Here r_B is the ratio between low price and high price of Hotel B .

According to Bertrand and Cournot competition, this payoff function is concave, therefore the optimal price for Hotel B given that Hotel A charges p_{L_A} can be found by taking the derivative of the function with respect to p_{L_B} , setting the first derivative equal to 0 and solving algebraically for p_{L_B} , we have

$$\begin{aligned} \frac{dR_B}{dp_{L_B}} &= b_B + \frac{C_B \alpha_B}{r_B} - \frac{2C_B p_{L_B} \beta_B}{r_B} + \frac{C_B \gamma_B p_{L_A}}{r_B} - \frac{b_B}{r_B} \\ &\Rightarrow b_B + \frac{C_B \alpha_B}{r_B} - \frac{2C_B p_{L_B} \beta_B}{r_B} + \frac{C_B \gamma_B p_{L_A}}{r_B} - \frac{b_B}{r_B} = 0 \\ &\Rightarrow \frac{2C_B p_{L_B} \beta_B}{r_B} = b_B + \frac{C_B \alpha_B}{r_B} + \frac{C_B \gamma_B p_{L_A}}{r_B} - \frac{b_B}{r_B} \\ &\Rightarrow 2C_B p_{L_B} \beta_B = b_B r_B + C_B \alpha_B + C_B \gamma_B p_{L_A} - b_B \\ &\Rightarrow p_{L_B} = \frac{b_B r_B + C_B \alpha_B + C_B \gamma_B p_{L_A} - b_B}{2C_B \beta_B} \\ &\therefore p_{L_B} = \frac{b_B r_B + C_B \alpha_B - b_B}{2C_B \beta_B} + \frac{\gamma_B}{2\beta_B} p_{L_A} \end{aligned} \tag{26}$$

This is the pricing model for Hotel B which is influenced by the price of Hotel A .

Duopoly Model with Stochastic Demand

Stochastic demand means that demand is uncertain, i.e., demand is a random variable. This model takes into account the stochastic behavior of the demand. Since demand is affected by price, stochastic demand model becomes a function of price. Thus, CDF of price is used to model occupancy rate or demand. $F(p_L)$ is the probability function of demand which is less than or equal to low price p_L . So, $1 - F(p_L)$ represents the probability of demand who are able to pay more than the low price p_L . Talluri and Van Rayzin (2004) developed demand model as a function $d(p_L)$ of price given by

$$d(p_L) = N(1 - F(p_L)) \tag{27}$$

where N is market size, F_{p_L} is CDF of lower price. However, this model can only be applied for monopoly case. According to (27), the occupancy rate for two hotels namely Hotel A and Hotel B are as follow

$$\begin{aligned} d_A(p_{L_A}) &= N(1 - F(p_{L_A})) \\ d_B(p_{L_B}) &= N(1 - F(p_{L_B})) \end{aligned} \tag{28}$$

Thus, for duopoly model using (28) the occupancy rate for Hotel A is

$$d_A(p_{L_A}, p_{L_B}) = N(1 - F(p_{L_A}) + 1 - F(p_{L_B})) \tag{29}$$

To determine the value of occupancy rate, let us consider two cases, when $p_{L_A} > p_{L_B}$ and $p_{L_A} < p_{L_B}$ $p_{L_A} < p_{L_B}$.

Let CDF for Hotel A is $F_A^+(p_{L_A})$ when $p_{L_A} < p_{L_B}$ and $F_A^-(p_{L_A})$ when $p_{L_A} > p_{L_B}$. Then, CDF of Hotel A can be formulated as:

$$F_A(p_{L_A}) = \begin{cases} F_A^+(p_{L_A}) & \text{if } p_{L_A} < p_{L_B} \\ F_A^-(p_{L_A}) & \text{if } p_{L_A} \geq p_{L_B} \end{cases}$$

Substituting these values to the previous occupancy rate the new occupancy rate is

$$d_A(p_{L_A}, p_{L_B}) = \begin{cases} N[1 - F_A^+(p_{L_A}) + 1 - F_B^-(p_{L_B})] & \text{if } p_{L_A} < p_{L_B} \\ N[1 - F_A^-(p_{L_A}) + 1 - F_B^+(p_{L_B})] & \text{if } p_{L_A} \geq p_{L_B} \end{cases}$$

We assume that, demand that is a CDF of price follows exponential distribution. Where the rate parameter is denoted by λ_A which is the average rate of time between each demand of Hotel A and each price is the random variable.

Thus, the CDF of Hotel A is

$$F_A(p_{L_A}) = \begin{cases} 1 - e^{-\lambda_A p_{L_A}} & \text{if } p_{L_A} < p_{L_B} \\ 1 - e^{-\lambda_A p_{L_A}} & \text{if } p_{L_A} \geq p_{L_B} \end{cases}$$

Hence, the occupancy model becomes as

$$\begin{aligned} d_A(p_{L_A}, p_{L_B}) &= N[1 - 1 + e^{-\lambda_A p_{L_A}} + 1 - 1 + e^{-\lambda_B p_{L_B}}] \\ d_A(p_{L_A}, p_{L_B}) &= N[e^{-\lambda_A p_{L_A}} + e^{-\lambda_B p_{L_B}}] \end{aligned}$$

We know that for optimal value, $\frac{\partial d_A}{\partial p_{L_A}} = 0$ and $\frac{\partial d_A}{\partial p_{L_B}} = 0$. Thus, we have

$$\begin{aligned} -N\lambda_A e^{-\lambda_A p_{L_A}} &= 0 \\ \Rightarrow e^{-\lambda_A p_{L_A}} &= 0 \\ \therefore p_{L_A} &= 0 \end{aligned}$$

Similarly, $p_{L_B} = 0$.

So, the extreme value is

$$\begin{aligned} d_A(p_{L_A}, p_{L_B}) &= 0 \\ \Rightarrow N(e^{-\lambda_A p_{L_A}} + e^{-\lambda_B p_{L_B}}) &= 0 \\ \Rightarrow -e^{-\lambda_A p_{L_A}} &= e^{-\lambda_B p_{L_B}} \end{aligned}$$

Since, $p_{L_A} < p_{L_B}$ the left-hand side becomes as

$$\begin{aligned} e^{-\lambda_A p_{L_A}} &= -\lambda_A p_{L_A} \\ \Rightarrow -(-\lambda_A p_{L_A}) &= e^{-\lambda_B p_{L_B}} \\ \Rightarrow \ln(\lambda_A p_{L_A}) &= -\lambda_B p_{L_B} \\ \therefore \ln(p_{L_A}) &= -\ln(\lambda_A) - \lambda_B p_{L_B} \end{aligned} \tag{30}$$

This is the pricing model for Hotel A with stochastic demand.

Similarly, for duopoly model the occupancy rate for Hotel B is

$$d_B(p_{L_A}, p_{L_B}) = N(1 - F(p_{L_B}) + 1 - F(p_{L_A}))$$

To determine the value of occupancy rate let us consider two cases, when $p_{L_A} > p_{L_B}$ and $p_{L_A} < p_{L_B}$.

Let CDF for Hotel-B is $F_B^+(p_{L_B})$ when $p_{L_B} < p_{L_A}$ and $F_B^-(p_{L_B})$ when $p_{L_B} > p_{L_A}$. Then, CDF of Hotel B can be formulated as:

$$F_B(p_{L_B}) = \begin{cases} F_B^+(p_{L_B}) & \text{if } p_{L_B} < p_{L_A} \\ F_B^-(p_{L_B}) & \text{if } p_{L_B} \geq p_{L_A} \end{cases}$$

Substituting these values to the previous occupancy rate the new occupancy rate is

$$d_B(p_{L_A}, p_{L_B}) = \begin{cases} N[1 - F_B^+(p_{L_B}) + 1 - F_A^-(p_{L_A})] & \text{if } p_{L_B} < p_{L_A} \\ N[1 - F_B^-(p_{L_B}) + 1 - F_A^+(p_{L_A})] & \text{if } p_{L_B} \geq p_{L_A} \end{cases}$$

We assume that, demand for Hotel B is a CDF of price follows exponential distribution. Where the rate parameter is denoted by λ_B which is the average rate of time between each demand of Hotel B and each price is the random variable. Thus, the CDF of Hotel B is

$$F_B(p_{L_B}) = \begin{cases} 1 - e^{-\lambda_B p_{L_B}} & \text{if } p_{L_B} < p_{L_A} \\ 1 - e^{-\lambda_B p_{L_B}} & \text{if } p_{L_B} \geq p_{L_A} \end{cases}$$

Hence, the occupancy model becomes

$$\begin{aligned} d_B(p_{L_A}, p_{L_B}) &= N[1 - 1 + e^{-\lambda_B p_{L_B}} + 1 - 1 + e^{-\lambda_A p_{L_A}}] \\ d_B(p_{L_A}, p_{L_B}) &= N[e^{-\lambda_B p_{L_B}} + e^{-\lambda_A p_{L_A}}] \end{aligned}$$

We know that for optimum value, $\frac{\partial d_B}{\partial p_{L_A}} = 0$ and $\frac{\partial d_B}{\partial p_{L_B}} = 0$. Thus, we have

$$\begin{aligned} -N\lambda_B e^{-\lambda_B p_{L_B}} &= 0 \\ \Rightarrow e^{-\lambda_B p_{L_B}} &= 0 \\ \therefore p_{L_B} &= 0 \end{aligned}$$

Similarly, $p_{L_A} = 0$.

So, the extreme value is

$$\begin{aligned} d_B(p_{L_B}, p_{L_B}) &= 0 \\ \Rightarrow N(e^{-\lambda_A p_{L_A}} + e^{-\lambda_B p_{L_B}}) &= 0 \\ \Rightarrow -e^{-\lambda_B p_{L_B}} &= e^{-\lambda_A p_{L_A}} \end{aligned}$$

Since, $p_{L_B} < p_{L_A}$ the left-hand side becomes

$$\begin{aligned} e^{-\lambda_B p_{L_B}} &= -\lambda_B p_{L_B} \\ \Rightarrow -(-\lambda_B p_{L_B}) &= e^{-\lambda_A p_{L_A}} \\ \Rightarrow \ln(\lambda_B p_{L_B}) &= -\lambda_A p_{L_B} \\ \therefore \ln(p_{L_B}) &= -\ln(\lambda_B) - \lambda_A p_{L_B} \end{aligned} \tag{31}$$

This is the pricing model for Hotel B with stochastic demand.

Model Analysis

According to Wikipedia (2021) the best weather in Saint Martin's Island to visit is usually between November to February, this is the peak season. March to July is off-season for tourists. Due to COVID-19 restrictions, all the ships carrying people to Saint Martin's Island were halted until 11 November, 2020 (Prothom Alo 2021). So fewer people traveled in November 2020 and we were able to collect data of two months (December 2020, January 2021) from two hotels in St. Martin's Island, Ocean Blue Resort and Restaurant (Hotel A) and Rupashi Bangla Resort and Restaurant (Hotel B), within the peak season.

Though here we formulated pricing model for both deterministic and stochastic demand, most of the hotels in this region follows deterministic demand model, i.e., number of rooms are fixed for low and high price. So, data analysis is done for the deterministic demand model. We also calculated the optimal booking limit and protection level for both hotels based on our capacity model. Then, based on our pricing model we found the competitive price for each hotel and evaluated the previous and new revenues for both hotels.

Capacity Model

In this section we considered the monopoly situation. This model provides how to allocate capacity for low price rooms and high price rooms. Based on data collected from Hotel *A*, it has mean of 6 customers per day, 6 rooms fixed for low price (1500 BDT), 8 rooms fixed for high price (2500 BDT) and total capacity (*C*) of 14 rooms. The ratio of high price and low price of Hotel *A* is 0.6. If the demand of Hotel *A* follows poisson distribution then according to capacity model the protection level can be found using the ratio. The inverse CDF poisson distribution is calculated by using MATLAB's inverse cumulative distribution function (iCDF) function. So, the protection level for Hotel *A* is,

$$\begin{aligned}
 y^* &= F^{-1}\left(1 - \frac{P_{L_A}}{P_{L_B}}\right) \\
 &\Rightarrow y^* = F^{-1}(1 - 0.6) \\
 &\Rightarrow y^* = F^{-1}(0.4) \\
 \therefore y_A^* &= 5
 \end{aligned}$$

The booking limit for Hotel *A* is,

$$\begin{aligned}
 b_A &= C - y^* \\
 &\Rightarrow b_A = 14 - 5 \\
 \therefore b_A &= 9
 \end{aligned}$$

Table 1 shows optimal protection level and booking limit for different price ratio of Hotel *A*. From figure 1, we can see that when price ratio gets higher, number of rooms allocated for low price (Booking Limit) increases and number of rooms allocated for high price (Protection Level) decreases because the difference between prices is low. Similarly, when price ratio gets lower, number of rooms allocated for low price (Booking Limit) decreases (and number of rooms allocated for high price (Protection Level) increases because the difference between prices is high.

Also, booking limit and protection level, i.e., number of rooms allocated for low price and high price are same for price ratio 0.2 and 0.3. From the capacity allocation chart (Table 1) as the number of room allocation at high price (Protection Level) decreases (9 to 5) and number of rooms allocated for low price (Booking Limit) increases (5 to 11), as a result then there must be an intersection of two curves which is shown in figure 1 and the intersection ratio exists between the point 0.2 and 0.3.

Table 1. Capacity allocation for different price ratio of Hotel-*A*

Ratio	Protection Level	Booking Limit
0.1	9	5
0.2	7	7
0.3	7	7
0.4	6	8
0.5	5	9
0.6	5	9
0.7	4	10
0.8	3	11
0.9	3	11

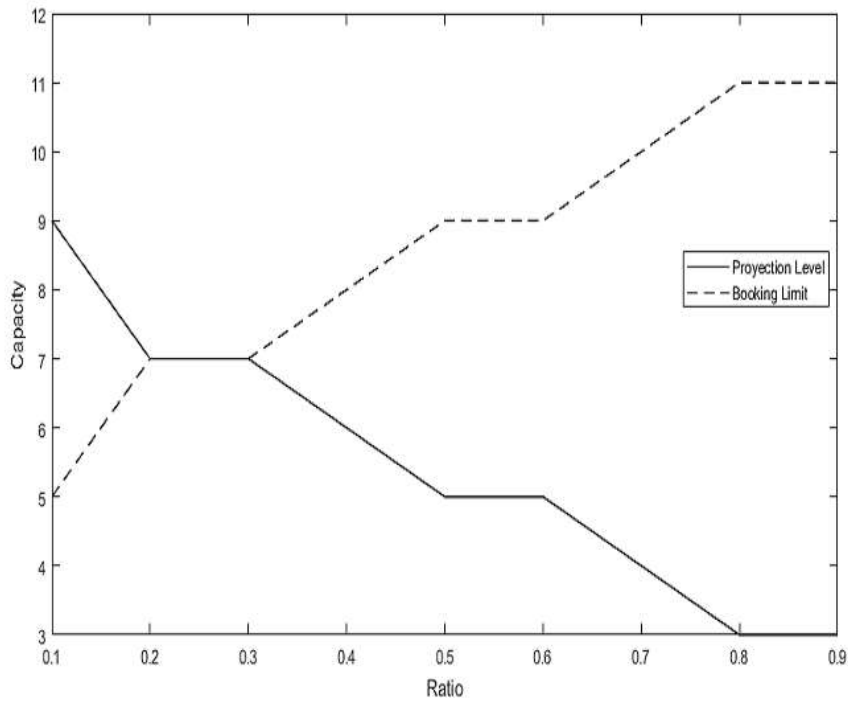


Figure 1. Optimal capacity allocation based on price ratio for Hotel *A*

Table 2. Capacity allocation for different price ratio of Hotel-*A*

Ratio	Protection Level	Booking Limit
0.1	11	7
0.2	10	8
0.3	9	9
0.4	8	10
0.5	7	11
0.6	7	11
0.7	6	12
0.8	5	13
0.9	4	14

Hotel *B* has mean of 8 customers per day, 8 rooms fixed for low price (2000 BDT), 10 rooms fixed for high price (2500 BDT) and total capacity (*C*) of 18 rooms. The ratio of high price and low price of Hotel *A* is 0.7. If the demand of Hotel *B* follows poisson distribution then according to capacity model the protection level can be found using the ratio. The inverse CDF of poisson distribution is calculated using MATLAB's *iCDF* function. So, the protection level for Hotel *B* is,

$$\begin{aligned}
 y^* &= F^{-1}\left(1 - \frac{P_{L_B}}{P_{H_B}}\right) \\
 &\Rightarrow y^* = F^{-1}(1 - 0.7) \\
 y^* &= F^{-1}(0.3) \\
 \therefore y^* &= 6
 \end{aligned}$$

The booking limit for Hotel *B* is,

$$\begin{aligned}
 b_B &= C - y^* \\
 &\Rightarrow b_B = 14 - 5 \\
 \therefore b_B &= 9
 \end{aligned}$$

This is the optimal booking limit for Hotel *B*.

Table 2 shows optimal protection level and booking limit for different price ratio of Hotel *B*. From figure 2 we can see that when price ratio gets higher number of rooms allocated for low price (Booking Limit) increases and number of rooms allocated for high price (Protection Level) decreases because the difference between prices is low. Similarly, when price ratio gets lower number of rooms allocated for low price (Booking Limit) decreases and number of rooms allocated for high price (Protection Level) increases because the difference between prices is high.

Also, booking limit and protection level, i.e., number of rooms allocated for low price and high price are same for price ratio 0.3. From the capacity allocation chart (Table 2) as the number of room allocation at high price (Protection Level) decreases (11 to 4) and number of rooms allocated for low price (Booking Limit) increases (7 to 14), as a result then there must be an intersection of two curves which is shown in figure 2 and the intersection ratio exists on the point 0.3.

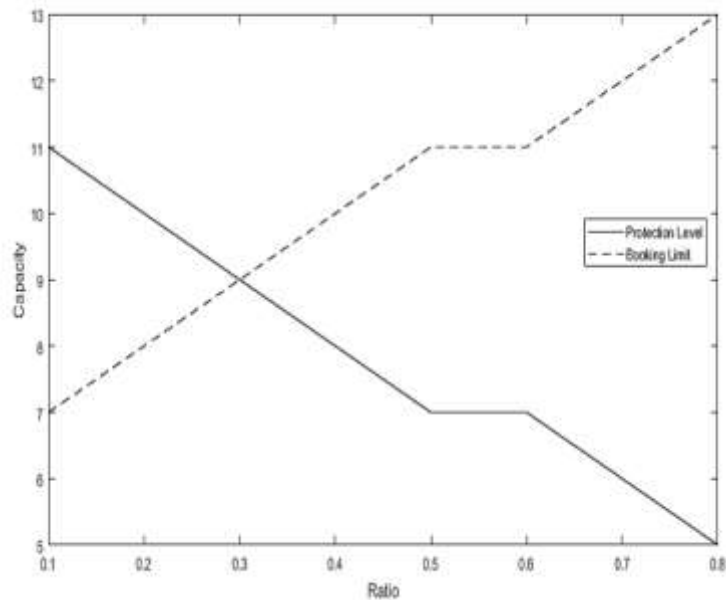


Figure 2. Optimal capacity allocation based on price ratio for Hotel *B*

Pricing Model

Occupancy rate depends on the low price of Hotel A as well as Hotel B. Occupancy rate is calculated by dividing daily demand by total demand of Hotel A. Occupancy rate of Hotel-A is given by,

$$d_A(p_{L_A}, p_{L_B}) = \alpha_A - \beta_A p_{L_A} + \gamma_A p_{L_B} \tag{32}$$

Figure 3 shows that Hotel-A did not have full occupancy on most of the days. Full occupancy occurred during the starting of January due to New Year and last few days of January because of peak season. Occupancy rate decreased after few days of New Year and increased after that.

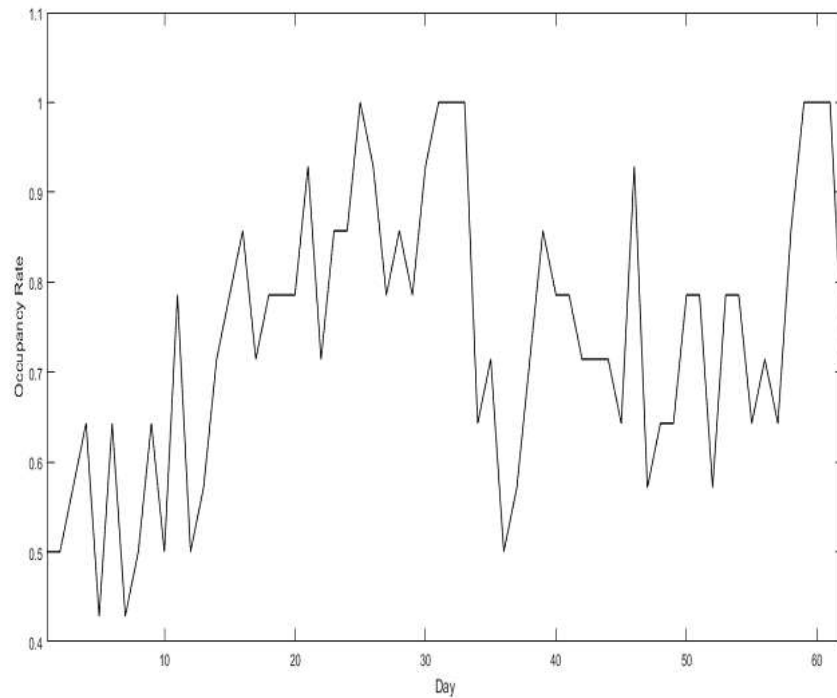


Figure 3. Occupancy rate of Hotel A

Regression analysis was used to find the coefficients for the pricing model as well as we used the occupancy rate model (33). We considered occupancy rate d_A as dependent variable and low price of Hotel A and Hotel B (p_{L_A}, p_{L_B}) as independent variable in regression analysis and calculated the values of coefficients $\alpha_A = -0.0506, \beta_A = 2.9802 \times 10^{-4}, \gamma_A = 2.9822 \times 10^{-4}$,

$$\therefore p_{L_A} = \frac{b_A r_A + C_A \alpha_A - b_A}{2 C_A \beta_A} + \frac{\gamma_A}{2 \beta_A} p_{L_B} \tag{33}$$

Finally, using the values of booking limit, total capacity, price ratio for Hotel A and daily low price of Hotel B in (33) we calculated the low price of Hotel A for each day for two months. Each price we got for Hotel A is based on the price of Hotel B, which is the competitive price for Hotel A in the presence of Hotel B. To understand better, we divided two months of data in 9 weeks.

Table 3 shows the weekly revenue of Hotel A in Taka, based on the data collected and the new revenue per week using the new price we got from the pricing model.

Table 3. Weekly previous and new revenue for Hotel A

Week	Previous Revenue	New Revenue
1	26,250	34,245
2	30,850	36,045
3	39,200	46,395
4	45,300	43,695
5	49,600	51,345
6	43,800	43,695
7	38,000	40,995
8	41,100	38,745
9	37,200	33,660

Figure 4 compares between the previous and new weekly revenue of Hotel A. It's clearly shown that the revenue increases by 7,995 BDT in first week, 5,195 BDT in second week, 7,195 BDT in third week, 1,745 BDT in fifth week and 2,995 BDT in seventh week. Revenue decreases by 1,605 BDT in fourth week, 105 BDT in sixth week, 2,355 BDT in eighth week and 3,540 BDT in ninth week.

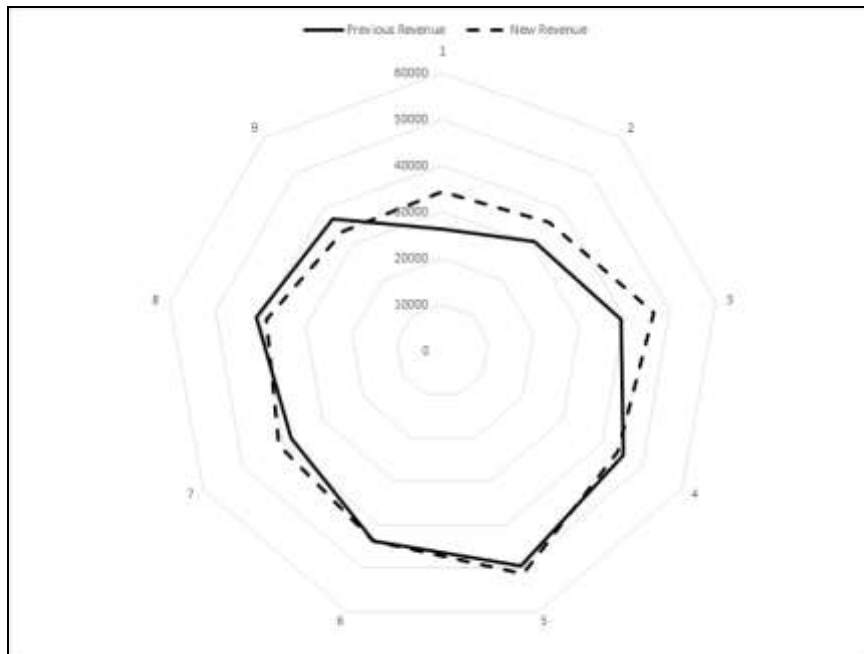


Figure 4. Weekly revenue comparison of Hotel A

Occupancy rate for Hotel B is given by the following equation

$$d_B(p_{L_A}, p_{L_B}) = \alpha_B - \beta_B p_{L_B} + \gamma_B p_{L_A} \quad (34)$$

Figure 5 shows that Hotel B had average room occupancy. Full occupancy occurred during the last few of December due to New Year. Occupancy rate decreased after few days of New Year and increased after that.

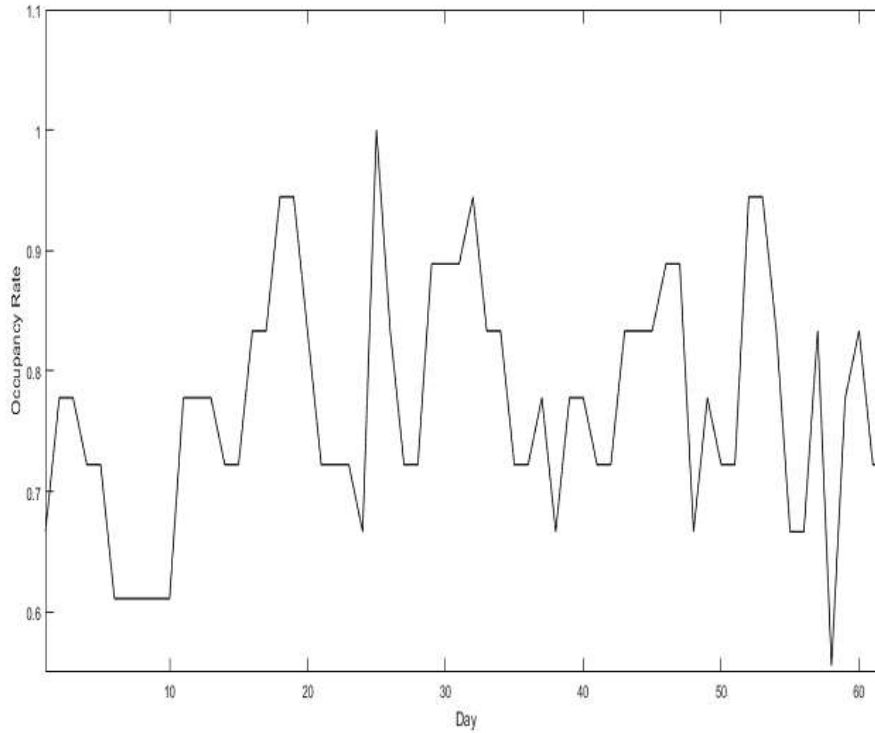


Figure 5. Occupancy rate of Hotel B

Regression analysis was used to find the coefficients for the pricing model (35). We used the occupancy rate model (35). We considered occupancy rate d_B as dependent variable and low price of Hotel A and Hotel B (p_{L_A}, p_{L_B}) as independent variable in regression analysis and calculated the values of coefficients $\alpha_B = 0.3212, \beta_B = 2.436 \times 10^{-4}, \gamma_B = 7.544 \times 10^{-5}$,

$$\therefore p_{L_B} = \frac{b_B r_B + C_B \alpha_B - b_B}{2C_B \beta_B} + \frac{\gamma_B}{2\beta_B} p_{L_A} \quad (35)$$

Using the values of booking limit, total capacity, price ratio for Hotel B and daily low price of Hotel A in (35) we calculated the low price of Hotel B for each day for two months. Each price we got for Hotel B is based on the price of Hotel A, which is the competitive price for Hotel B in the presence of Hotel A.

Table 4. Weekly previous and new revenue for Hotel-B

Week	Previous Revenue	New Revenue
1	53,200	69,048
2	54,400	70,152
3	81,000	71,004
4	72,200	70,800
5	90,400	72,492
6	72,000	70,620
7	60,300	68,760
8	53,250	70,248
9	47,000	59,904

Table 4 and Figure 6 compares between the previous and new weekly revenue of Hotel B. Blue line indicate the previous weekly revenue and red line indicates the new weekly revenue. Revenue increases by 15,848 BDT in first week, 15,752 BDT in second week, 8,460 BDT in seventh week, 16,998 BDT in eighth week and 12,904 BDT in ninth week. Revenue decreases by 9,996 BDT in third week, 1,400 BDT in fourth week, 17,908 BDT in fifth week and 1,380 BDT in sixth week.

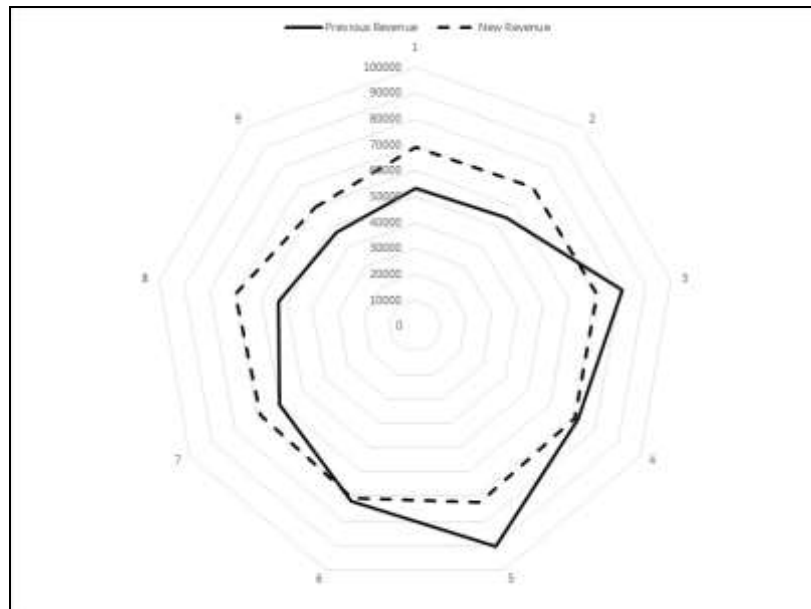


Figure 6. Weekly revenue comparison of Hotel B

Results and Discussion

We formulated capacity model for both hotels which gives the optimal capacity allocation for booking limit and protection level. According to the data that we collected from Hotel *A*, it had total 14 rooms, 6 rooms for low price (Booking Limit) and 8 rooms for high price (Protection Level). Using the capacity model, we found the optimal capacity allocation for Hotel *A* be 9 rooms for low price and 5 rooms for high price to earn optimal revenue.

Hotel *B* had total 18 rooms, 8 rooms for low price (Booking Limit) and 10 rooms for high price (Protection Level). Similarly, optimal capacity allocation for Hotel *B* by using the capacity model should be as 12 rooms for low price and 6 rooms for high price to achieve its optimal revenue.

By using deterministic demand, we also formulated a pricing model which gives the optimal room price for both hotels considering their duopoly competition. Hotel *A* had set their low price at 1500 BDT. But from the occupancy rate we saw that most of the times it did not get this price. Hotel *A*'s total revenue for two months was 3,51,300 BDT. Using the daily occupancy, price of rooms and competitors (Hotel *B*) price as the input, pricing model determined the new optimal price for Hotel *A*, which increases the total revenue by 17,520 BDT.

Similar way, we saw that Hotel *B* had set their low price at 2000 BDT and its total revenue for considering two months was 5,83,750 BDT. After using the daily occupancy, price of rooms and competitors (Hotel *A*) price as the input, pricing model determined the new optimal price for Hotel *B*, which increases the total revenue by 39,278 BDT.

Table 5 shows the previous total revenue for both hotels and their revenue after applying capacity and pricing model. We notice that revenue of Hotel *A* and Hotel *B*, increase 4.9% and 6.7% respectively by using our capacity and pricing model.

Table 5. Total revenue increase percentage for Hotel-*A* and Hotel-*B*

Hotels	Previous Total Revenue	New Total Revenue	Percent Increase
Hotel-A	3,51,300Tk.	3,68,820Tk.	4.9%
Hotel-B	5,83,750Tk.	6,23,028Tk.	6.7%

Based on our capacity model, Hotel *A* should change their booking limit, i.e., 9 rooms for low price and protection level, i.e., 5 rooms for high price. Hotel *B* also should change the booking limit to 12 rooms for low price and 6 rooms for high price. This will increase both hotels' occupancy rate and they will be able to profit more.

Pricing model shows, Hotel *A* and Hotel *B* both should lower their prices. In order to increase demand, Hotel *A* should set their low price between 900 BDT to 1000 BDT and Hotel *B* should set their low price between 800 BDT to 900 BDT. Both hotels revenue will increase eventually by using the optimal capacity allocation for low price and high price and lowering their base price.

Conclusion

In this section, summery of our whole work is discussed. We wanted to construct a revenue management model for hotels in Saint Martin's Island. During this COVID-19 pandemic, hotels could not earn as they used to be. Our main objective was to formulate a model that could increase the revenue of those hotels from what they were earning.

There are certain things we needed to consider, like capacity, price and competition between hotels. Hotels have fixed capacity and they divide their rooms according to prices. Most hotels in tourism sector have two types of prices- high and low. Also, hotels compete with each other based on price. We considered price as the soul factor so demand depends on price of both hotels. We considered two types of demand: stochastic and deterministic. Demand was considered to follow poisson distribution in stochastic case and exponential distribution in deterministic case.

After collected required data, we developed a capacity model for both hotels. We used Littlewood's two-class model to find the optimal booking limit (number of rooms for low price) and protection level (number of rooms for high price) for both hotels. We found that to get the optimal capacity allocation both hotels should change their booking limit and protection level. This will result in a higher occupancy rate and help both hotels to gain more demand. We then developed a pricing model based on each hotels demand. With the help of Bertrand and Cournot competition which is a part of game theory, we constructed our pricing model. Pricing model for both hotels is based on stochastic and deterministic demand. Since the nature of the data, we collected was deterministic, data analysis for pricing model was done for deterministic case only. The result we found using our pricing model showed that both hotels should lower their room price for high and low class to increase their revenue which is higher than what they are earning currently.

In summary of our findings, we can say that both hotels should follow the recommendations given below:

- Hotels should increase their booking limit, i.e., number of rooms fixed for low price.
- Protection level, i.e., number of rooms for high price must be decreased.
- According to our pricing model both hotels have to lower the price of low-cost rooms in order to gain more demand as well as earn more revenue.

Acknowledgements

We are grateful to the providers of two well-known hotels in Saint-Martin, Cox's Bazar, Bangladesh, for their cooperation and gave the primary data to complete this work.

Conflict of Interests

The author declares no conflict of interest.

References

- Badinelli, R.D. (2000). An optimal dynamic policy for hotel yield management, *European Journal of Operational Research*, March, 121(3): 476-503.
- Bitran, G., Caldentey, R. (2002). An overview of pricing models for revenue management, *Manufacturing & Service Operations Management*, 5(3): 203-229.
- Bitran, G., Gilbert, S.M. (1996). Managing hotel reservation with uncertain arrivals, *Operations Research*, 44(1): 35-49.
- CEIC 2021. "Bangladesh Tourism Revenue", Last modified on December 2019. <https://www.ceicdata.com/en/indicator/bangladesh/tourism-revenue>.
- Dai, Y., Chao, X., Fang, S.C., Nuttle, H.L. (2005). Pricing in revenue management for multiple firms competing for customers, *International Journal of Production Economics*, 98: 1-16.
- El-Gayar, N., Hendawi, A.T., Zakhary, A., El-Shishiny, H. (2008). A proposed decision support model for hotel room revenue management, *ICGST Journal of Artificial Intelligence and Machine Learning*, 8(1): 23-28.
- Kopalle, P., Shumsky, R.A. (2010). *Game Theory Models of Pricing*, Handbook of Pricing Management, Oxford University Press, Oxford.
- Littlewood, K. (1972). Forecasting and control of passenger bookings, *Proceedings of the Twelfth Annual AGIFORS Symposium*, Nathanya, Israel.
- The Daily Star, 2021. "Covid-19: Movement of tourist ships on Teknaf-St Martin's route halted", 1 April 2021. <https://www.thedailystar.net/travel/news/covid-19-movement-tourist-ships-teknaf-st-martins-route-halted-2070361>.

Ara et al. (2022). A Model to Develop Hotel Management System to Optimize Revenue. *Khulna University Studies*, Volume 19 (2): 167-188: 2022

Talluri, K.T., Van Ryzin, G.J. (2004). *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, New York.

Wikipedia, 2021. St. Martin's Island, https://en.wikipedia.org/wiki/St._Martin%27s_Island.