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A MATHEMATICAL MODELING ON THE ENVIRONMENTAL IMPACT BY TRAFFIC JAM: A CASE STUDY IN DHAKA, BANGLADESH

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Abstract

Traffic jams are a common incident in Bangladesh, particularly in major cities like Dhaka. Traffic jams not only cause significant delays and economic losses but also contribute to substantial environmental degradation. This study object to quantify the environmental impact of traffic jam in Bangladesh using mathematical modeling, a case study of Dhaka city is presented, focusing on air pollution and greenhouse gas emissions. The model analyzed by using stability theory of differential equations with equilibrium analysis by convincing stable or unstable equilibrium. Moreover, the analytical results are validated by numerical simulations for describing the complex behaviors of the model. Overall, the model represents an important tool for investigating environmental impact of traffic jam and this model can help inform policy decisions and promote more sustainable practices.

Keywords: Traffic jam, Environmental impact, Air pollution, Emission, Bangladesh, Traffic flow modeling.

Introduction

Mathematical modeling is an interesting and engaging field to do the work of research. It is mainly an eliciting process. Systematic mathematical analysis can densely lead to a better understanding of bio-economic and diseases related models. Bangladesh, with its burgeoning population and rapid urbanization, faces a significant challenge in managing vehicular traffic, particularly in major cities like Dhaka. The resulting traffic jams not only impede economic activities but also cast a shadow over environmental sustainability. This study delves into the often-overlooked environmental repercussions of traffic congestion, employing advanced mathematical modeling techniques to quantitatively assess and comprehend the intricate dynamics at play. The World Bank's comprehensive report on urbanization in Bangladesh highlights the exponential growth of cities, with Dhaka standing out as one of the fastest-growing megacities globally [23]. This urban expansion, coupled with a surge in vehicular traffic, has transformed traffic jams into a critical issue, affecting not only the flow of daily life but also posing significant threats to the country's environmental equilibrium. While the economic ramifications of traffic congestion are well-established, the environmental impact remains a critical yet underexplored facet. Traffic-related air pollution, including particulate matter and greenhouse gas emissions, not only jeopardizes public health but also contributes substantially to environmental degradation. The World Health Organization (WHO) underscores the global implications of air pollution, estimating that it leads to millions of premature deaths annually (WHO, 2018). In the context of Bangladesh, where the impacts of climate change are becoming increasingly evident, understanding and mitigating these environmental effects is paramount [24]. The rapid pace of global urbanization has led to an unprecedented surge in vehicular traffic, resulting in widespread traffic congestion. This phenomenon poses multifaceted challenges to urban environments, impacting economic productivity, air quality, and overall sustainability. Studies by Wang and Zhou (2015) emphasize the complexity of urban traffic dynamics, highlighting the interconnectedness of traffic flow, congestion patterns, and road geometry [25]. Understanding these dynamics is crucial for developing accurate mathematical models that can assess the environmental impact of traffic jams. Traffic congestion is a major contributor to air pollution, releasing pollutants such as particulate matter (PM),

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nitrogen oxides (NO_2), and carbon monoxide (CO) into the atmosphere. These emissions not only degrade air quality but also have adverse effects on public health and the environment [15]. The transportation sector significantly contributes to greenhouse gas emissions, with vehicles being a primary source of carbon dioxide (CO_2) emissions. Understanding the link between traffic congestion and increased emissions is crucial for developing strategies to mitigate environmental impact [4].

The Cell Transmission Model (CTM) proposed by Daganzo (1995) provides a comprehensive framework for understanding traffic flow on highways. CTM captures the dynamics of traffic density, speed, and congestion, serving as a fundamental basis for traffic modeling [8]. Recent advancements in traffic modeling include the integration of environmental factors. Traffic volume, road conditions, driver behavior, and enforcement of traffic lows affect the environment and increase road accidents [6]. Chen et al. (2019) proposed a model that incorporates emissions into traffic flow models, offering a more holistic understanding of the environmental consequences of traffic congestion [7]. Such integrated models are essential for accurately assessing the environmental impact. International case studies, such as those conducted in Beijing and Los Angeles provide valuable insights into the complex relationship between traffic congestion and environmental quality. These studies highlight the need for context-specific solutions to address the environmental challenges posed by traffic jams. Ahmed et al. (2020) conducted a case study on traffic-related air pollution in Dhaka, emphasizing the unique challenges faced by the city. The study underscores the importance of considering local factors, including traffic patterns, road infrastructure, and emission sources, for developing effective strategies to mitigate environmental impact [1].

Traffic-induced air pollution extends its reach beyond the immediate surroundings. The pollutants released during congestion have a broader environmental impact, affecting soil quality, water bodies, and contributing to overall environmental degradation. Using various data sources and analytical techniques to understand the impact of alcohol consumption on drinking and traffic jams in Bangladesh informs effective policy decisions to reduce traffic jams and improve road safety [13]. The consequences of traffic congestion on biodiversity and ecosystem health are often overlooked. The disruption caused by pollutants can have cascading effects on flora and fauna, emphasizing the need for a holistic approach to urban planning. For increasing carbon emission the Earth's temperature is rise. Studies by Sajib Mandal the depletion of atmospheric oxygen on global scale (which, if happens, obviously can kill most of life on Earth) is another possible catastrophic consequence of the global warming, a global ecological disaster that has been overlooked [10]. Carbon emissions contribute significantly to global warming. When fossil fuels like coal, oil, and natural gas are burned for energy, they release carbon dioxide (CO2) and other greenhouse gases into the atmosphere. Global warming is increasing at an alarming rate due to the extreme emission of GHGs [5]. While existing literature provides valuable insights into the global and national context of traffic-related environmental issues, there is a noticeable research gap concerning the specific dynamics of Bangladesh, particularly in Dhaka [22]. This study aims to address this gap by developing a mathematical model that considers the unique traffic and environmental conditions of Dhaka, providing insights for sustainable urban planning.

Materials and Method

Mathematical Formulation of the Model

The environmental impact of traffic jams can be modeled using a system of differential equations that describe the interactions between traffic flow(Q), traffic density(ρ), road geometry(G), and carbon emission rate(E).

The proposed model consists of four interconnected differential equations:

$$\frac{dQ}{dt} = \alpha Q(1-\frac{Q}{k_1}) - \beta\rho Q \tag{2.1}$$

This equation describes the change in traffic flow(Q) over time(t). The first term, $\alpha Q(1-\frac{Q}{k_1})$, represents 'the intrinsic growth of traffic flow with a carrying capacity k_1 and α is the growth rate of traffic flow. The second term $-\beta\rho Q$ reflects the negative impact of high traffic density(D) on traffic flow where β is the decreasing rate of traffic flow by the traffic density.

$$\frac{d\rho}{dt} = a\rho(1-\frac{\rho}{k_2}) - (bQ + cG)\rho \tag{2.2}$$

$\frac{d\rho}{dt}$ represents the rate of change of traffic density over time. a is the growth rate of traffic density in the absence of traffic flow and road geometry influence. k_2 is the carrying capacity, representing the maximum traffic density that the road can sustain. The second term represent negative impact of traffic density by the traffic flow and road geometry where b and c represents the decreasing rate of traffic density by the traffic flow and road geometry respectively.

$$\frac{dG}{dt} = \frac{\eta G}{m+Q} - \frac{\varphi G}{\rho} - \delta G \tag{2.3}$$

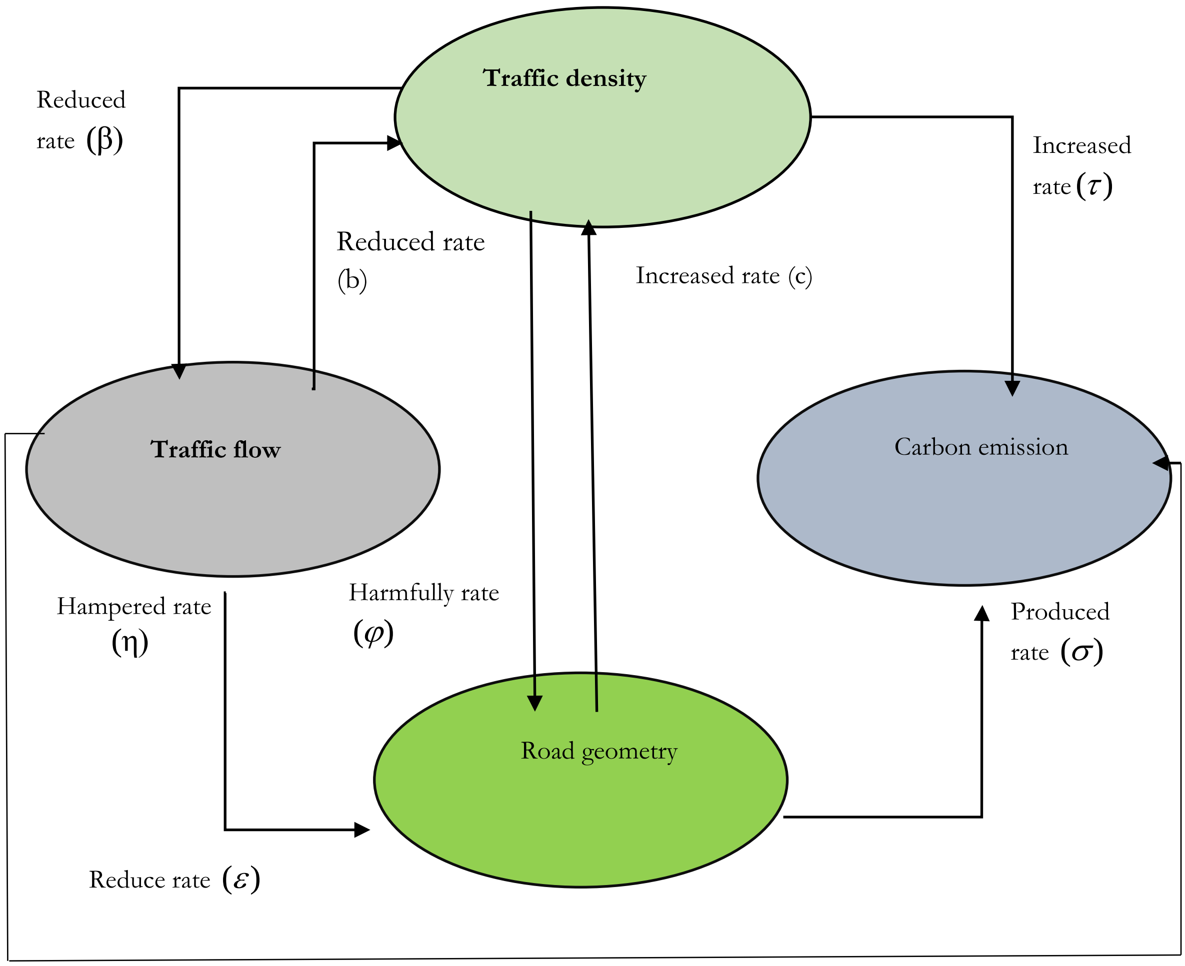

Figure 1. Impact of traffic density on the traffic flow, road geometry and carbon emission in the transportation system.

$\frac{dG}{dt}$ represents the rate of change of road geometry over time. The first term, $\frac{\eta G}{m+Q}$ represents that road geometry tends to adjust in response to traffic flow where *m* is the saturation constant of traffic flow and η is the growth rate of road geometry. $-\dfrac{\varphi G}{\rho}$ the term that represents the interaction between traffic density and road geometry where φ is the decreasing rate of road geometry by the traffic density. The last term $-\delta G$ represent how road geometry naturally tends to decrease over time.

$$\frac{dE}{dt} = \mu E - \varepsilon QE + (\tau\rho + \sigma G)E \tag{2.4}$$

The first term μE represents the natural growth of carbon emissions over time [2]. The second term $-\varepsilon QE$, indicates that the negative impact of carbon emissions by traffic flow where ϵ is the decreasing rate of emissions by traffic flow. If traffic flow (Q) increases, it tends to decrease the rate of change of emissions due to the negative sign. The third term represent positive impact of carbon emissions by the traffic density and road geometry where τ and σ increasing rate of carbon emissions by traffic density and road geometry respectively.

$$\frac{dQ}{dt} = \alpha Q(1 - \frac{Q}{k_1}) - \beta\rho Q \tag{2.5}$$

$$\frac{d\rho}{dt} = a\rho(1 - \frac{\rho}{k_2}) - (bQ + cG)\rho \tag{2.6}$$

$$\frac{dG}{dt} = \frac{\eta G}{m + Q} - \frac{\varphi G}{\rho} - \delta G \tag{2.7}$$

$$\frac{dE}{dt} = \mu E - \varepsilon QE + (\tau\rho + \sigma G)E \tag{2.8}$$

Analytical Analysis

Now it is going to be demonstrated the positivity test of all variables, boundedness of the system, stability analysis at equilibrium points, and numerical simulation.

Positivity Test

Now we are going to prove that all the variables of the system of differential equations (2.5) - (2.8) are positive through Lemma.

Lemma:

Let $Q(0) > 0, \rho(0) > 0, E(0) > 0, G(0) > 0$ and $G(t), F(t), C(t), T(t) \in R_4^+$ then the solutions $G(t), F(t), C(t), T(t)$ of the model are non-negative.

Proof:

To verify the lemma for the model, we have used the system (2.5) - (2.8) First, we consider Equation (2.5) given as

$$\frac{dQ}{dt} = \alpha Q(1 - \frac{Q}{k_1}) - \beta\rho Q \tag{2.9}$$

In order to find the positivity, Equation (2.9) can be written as,

$$\frac{dQ}{dt} \geq \alpha Q(1 - \frac{Q}{k_1}) - \beta\rho Q$$

$$\Rightarrow \frac{dQ}{dt} \geq [\alpha(1 - \frac{Q}{k_1}) - \beta\rho]Q$$

$\Rightarrow \frac{dQ}{dt} \geq A_1 Q$ [where, $A_1 = \alpha(1-\frac{Q}{k_1}) - \beta\rho$]

$\Rightarrow \frac{dQ}{Q} \geq A_1 dt$

$\Rightarrow \ln Q \geq A_1 t + \ln d_1$ [Where d_1 is an integrating constant.]

$$\therefore Q(t) \geq d_1 e^{A_1 t} \tag{2.10}$$

Now applying the initial condition at $t = 0, Q(0) = Q_0 > 0$ then from Eq. (2.10), we have

$Q(0) = Q_0 \geq d_1$

Putting the value of d_1 in equation (2.10). We have

$Q(0) \geq Q_0 e^{A_1 t}$

When $t \to \infty, Q(t) > 0$

$Q(t)$ positive for all $t \geq 0$

$Q \geq 0$

we consider Eq. (2.6) given as

$$\frac{d\rho}{dt} = a\rho(1-\frac{\rho}{k_2}) - (bQ + cG)\rho \tag{2.11}$$

In order to find the positivity, Eq. (2.11) an be written as,

$\Rightarrow \frac{d\rho}{dt} \geq a\rho(1-\frac{\rho}{k_2}) - (bQ + cG)\rho$

$\Rightarrow \frac{d\rho}{dt} \geq [a(1-\frac{\rho}{k_2}) - (bQ + cG)]\rho$

$\Rightarrow \frac{d\rho}{dt} \geq B_1 \rho$ [where, $B_1 = a(1-\frac{\rho}{k_2}) - (bQ + cG)$]

$\Rightarrow \frac{d\rho}{\rho} \geq B_1 dt$

$\Rightarrow \ln \rho \geq B_1 t + \ln d_1$ [where d_1 is integrating constant.]

$$\therefore \rho(t) \geq d_1 e^{B_1 t} \tag{2.12}$$

Now applying the initial condition at $t = 0, \rho(0) = \rho_0 > 0$ then from Eq. (2.12), we have

$\rho(0) = \rho_0 \geq d_1$

Putting the value of d_1 in equation (2.12). We have

$\rho(0) \geq \rho_0 e^{B_1 t}$

When $t \to \infty, \rho(t) > 0$

$\rho(t)$ is positive for all $t \geq 0$

$\therefore \rho \geq 0$

we consider Eq. (2.7) given as,

$$\frac{dG}{dt}=\frac{\eta G}{m+Q}-\frac{\varphi G}{\rho}-\delta G \qquad (2.13)$$

In order to find the positivity, Eq. (2.13) an be written as,

$$\frac{dG}{dt}+\left[\frac{\eta}{m+Q}-\frac{\varphi}{\rho}-\delta\right]G\geq 0$$

$$\Rightarrow \frac{dG}{dt}+c_1 G\geq 0 \qquad (2.14)$$

[where, $c_1=\frac{\eta}{m+Q}-\frac{\varphi}{\rho}-\delta$]

$\therefore$ Integrating factor $=e^{\int c_1 dt}=e^{c_1 dt}$

Now multiplying both sides of the equation (2.14) by $e^{c_1 dt}$, then we get

$$e^{c_1 dt}\frac{dG}{dt}+e^{c_1 dt}c_1 G\geq 0$$

Integrating the above equation, we have

$$e^{c_1 dt}c_1 G\geq c_2 \qquad (2.15)$$

Now applying the initial condition $t=0$, then from equation (2.15), we have

$$G\geq c_2$$

$$\Rightarrow c_2\leq G$$

Putting the value of c_2 in equation (2.15), we get

$$e^{c_1 dt}G\geq -G$$

$$e^{c_1 dt}G+G\geq 0$$

$$(1+e^{c_1 dt})G\geq 0$$

$$\therefore G\geq 0$$

we consider Eq. (2.8) given as

$$\frac{dE}{dt}=\mu E-\varepsilon QE+(\tau\rho+\sigma G)E \qquad (2.16)$$

In order to find the positivity, Eq. (2.16)an be written as,

$$\frac{dE}{dt}+[-\mu+\varepsilon Q-(\tau\rho+\sigma G)]E\geq 0$$

$$\Rightarrow \frac{dE}{dt}+d_1 E\geq 0 \qquad (2.17)$$

[where, $d_1=-\mu+\varepsilon Q-(\tau\rho+\sigma G)$]

Integrating factor $=\mathrm{e}^{\int \mathrm{d_1 dt}}=\mathrm{e}^{\mathrm{d_1 dt}}$

Now multiplying both sides of the equation (2.17) by $\mathrm{e}^{\mathrm{d_1 dt}}$ then we get

$$e^{d_1 dt}\frac{dE}{dt}+e^{d_1 dt}d_1 E\geq 0$$

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$$\Rightarrow \frac{d}{dt}(e^{d_1 dt}E) \geq 0$$

$$d(e^{d_1 dt}E) \geq 0 \qquad (2.18)$$

Integrating the above equation, we have

$$e^{d_1 dt}E \geq d_2$$

Now applying the initial condition $t = 0$, then from equation (2.18), we have

$$E \geq d_2$$

Putting the value of d_2 in equation (2.18), we get

$$e^{d_1 dt}E \geq -E$$

$$(1+e^{d_1 dt})E \geq 0$$

$$\therefore \mathrm{E} \geq 0$$

Therefore, it is clear that $Q \geq 0, \rho \geq 0, G \geq 0, E \geq 0$

Hence, Lemma is proved.

Boundedness of the Equation

Now, we will establish that the system (2.5) - (2.8) is bounded. By the following lemma, we begin to prove it.

Lemma

The set $\Psi = \left\{ \left(\mathrm{Q}, \rho, \mathrm{G}, \mathrm{E} \right) \in \mathrm{R}_4^+ : \mathrm{h}\left(\mathrm{t}\right) = \mathrm{Q}\left(\mathrm{t}\right) + \rho\left(\mathrm{t}\right) + \mathrm{G}\left(\mathrm{t}\right) + \mathrm{E}\left(\mathrm{t}\right), \mathrm{h}\left(\mathrm{t}\right) \right\} \leq \frac{\omega}{\theta}$ is a region of attraction for each solution and initially all the variables are positive, where θ is a constant.

Proof: Let $\mathrm{h}\left(\mathrm{t}\right) = \mathrm{Q}\left(\mathrm{t}\right) + \rho\left(\mathrm{t}\right) + \mathrm{G}\left(\mathrm{t}\right) + \mathrm{E}\left(\mathrm{t}\right), \quad \theta > 0$ be a constant. Then we can write

$$\frac{dh}{dt} = \frac{dQ}{dt} + \frac{d\rho}{dt} + \frac{dG}{dt} + \frac{dE}{dt}$$

$$\frac{dh}{dt} + \theta h = \alpha Q\left(1 - \frac{Q}{k_1}\right) - \beta\rho\theta + a\rho\left(1 - \frac{\rho}{k_2}\right) - \left(b\theta + c\theta\right)\rho + \frac{\eta G}{m+\theta} - \frac{\varphi G}{\rho} - \delta G + \mu E - \varepsilon\theta E + .$$

$$\left(\tau\rho + \sigma G\right)E + \theta\rho + \theta G + \theta E + \theta a$$

$$= (\alpha+\theta)Q + (a+\theta)\rho + (\mu+\theta)E + (\theta-\delta)G + \left(\frac{n}{m+\theta} - \frac{\theta}{\rho} + \sigma E\right)G + \left(\frac{-\alpha\theta}{k_1} - \beta\rho - b\rho - c\rho - \varepsilon E\right)\theta +$$

$$\left(\frac{a\rho}{k_2} + \psi E\right)\rho + (\sigma G - \varepsilon Q)E$$

Applying the theory of inequality,

$$h \leq \frac{\omega}{\theta} + C_0 e^{-\theta t}.$$

For $t \to \infty$, we have $\theta \leq h \leq \frac{w}{\theta}$

Hence the solution of the system is bounded in ψ

Equilibrium Points

To determine the equilibrium, point of the equation (2.5), (2.6), (2.7) and (2.8) can be obtained by setting,

$$\frac{dQ}{dt}=\frac{d\rho}{dt}=\frac{dG}{dt}=\frac{dE}{dt}=0$$

Then the four equilibrium points are required for the model. These are

i) $E_1=(0,0,0,0)$, which is the trivial equilibrium point

ii) $E_2=(0,0,0,0)$, which is semi-trivial equilibrium E_2 can be obtained by neglecting the non-linear terms.

iii) $E_3=\left(0,\frac{x_2}{a-b\overline{Q}+c\overline{G}},\frac{x_3}{\frac{\eta\overline{G}}{m+\overline{Q}}-\delta},\frac{x_4}{\mu+\tau\overline{\rho}+\sigma\overline{G}}\right)$ which is the dynamical equilibrium point obtained by neglecting traffic flow.

iv) $E_4=\left(\frac{-(b\overline{Q}+c\overline{G})k_1}{2\alpha},\frac{-(b\overline{Q}+c\overline{G}-a)k_2}{2a},\frac{y_3}{\frac{\eta\overline{G}}{m+\overline{Q}}-\frac{\varphi}{\overline{\rho}}-\delta},\frac{y_4}{\mu-\varepsilon\overline{Q}+\tau\overline{\rho}+\sigma\overline{G}}\right)$, which is dynamical positive (existence) equilibrium point when all dynamic variables are present.

Stability Analysis:

For stability analysis, we use the Jacobean technique for which we defined four functions of like A, B, C, D be the function of Q, ρ , G, E as the form,

$$A(Q,\rho,G,E)=\alpha Q\left(1-\frac{Q}{k_1}\right)-\beta\rho Q \quad (2.19)$$

$$B(Q,\rho,G,E)=a\rho\left(1-\frac{\rho}{k_2}\right)-(bQ+cG)\rho \quad (2.20)$$

$$C(Q,\rho,G,E)=\frac{\eta G}{m+Q}-\frac{\varphi G}{\rho}-\delta G \text{-} \quad (2.21)$$

$$D(Q,\rho,G,E)=\mu E-\varepsilon QE+(\tau\rho+\sigma G)E \quad (2.22)$$

Evaluating the equation (2.19) to (2.22) through Jacobean Matrix as,

$$J(Q,\rho,G,E)=\begin{bmatrix}\frac{\partial A}{\partial Q} & \frac{\partial A}{\partial \rho} & \frac{\partial A}{\partial G} & \frac{\partial A}{\partial E}\\ \frac{\partial B}{\partial Q} & \frac{\partial B}{\partial \rho} & \frac{\partial B}{\partial G} & \frac{\partial B}{\partial E}\\ \frac{\partial C}{\partial Q} & \frac{\partial C}{\partial \rho} & \frac{\partial C}{\partial G} & \frac{\partial C}{\partial E}\\ \frac{\partial D}{\partial Q} & \frac{\partial D}{\partial \rho} & \frac{\partial D}{\partial G} & \frac{\partial D}{\partial E}\end{bmatrix}$$

$$J(Q,\rho,G,E)=\begin{bmatrix} \alpha-\frac{2Q}{k_1}-\beta\rho & -\beta Q & 0 & 0 \\ -b\rho G & a-\frac{2\rho}{k_2}-bQG & -b\rho Q & 0 \\ -\frac{\eta G}{(m+Q)^2} & -\frac{\varphi G}{\rho^2} & \frac{\eta}{m+Q}-\frac{\varphi}{\rho}-\delta & 0 \\ -\varepsilon E & -\tau E & \sigma E & \mu-\varepsilon Q+\tau\rho+\sigma G \end{bmatrix}. \tag{2.23}$$

where, i = 1,2,3,4

(i) At the equilibrium point $E_1(0,0,0,0)$ equation (2.23) becomes

$$J_1=\begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & \mu \end{bmatrix}$$

Now to find the eigen values from the above matrix, we have to solve

$$|J_1-\lambda I|=0$$

$$\begin{vmatrix} \alpha-\lambda_1 & 0 & 0 & 0 \\ 0 & a-\lambda_2 & 0 & 0 \\ 0 & 0 & -\delta-\lambda_3 & 0 \\ 0 & 0 & 0 & \mu-\lambda_4 \end{vmatrix}=0$$

$$(\alpha-\lambda_1)(a-\lambda_2)(-\delta-\lambda_3)(\mu-\lambda_4)=0 \tag{2.24}$$

Hence, the required eigenvalues of equation (2.24) are,

$$\lambda_1=\alpha,\ \lambda_2=a,\ \lambda_3=-\delta,\ \lambda_4=\mu$$

Here only one value is negative and three values are positive then three cases arise that are given below:

Case I: If $\lambda_1<0, \lambda_2<0$, and $\lambda_4<0$ then

$\lambda_1<0$, i.e. $\alpha<0$

$\lambda_2<0$, i.e. $a<0$

$\lambda_4<0$, i.e. $\mu<0$

Then equilibrium point E_1 will be stable.

Case II: If $\lambda_1>0, \lambda_2>0$, and $\lambda_4<0$ then

Or $\lambda_1>0, \lambda_4>0$, and $\lambda_2<0$

Or $\lambda_2>0, \lambda_4>0$, and $\lambda_1<0$

Then equilibrium point E_1 will be asymptotically stable.

(ii) At the equilibrium point $E_3 = \left(0, \bar{\rho}, \bar{G}, \bar{E}\right)$ equation (2.23) becomes

$$J_2 = \begin{bmatrix} \alpha - \beta\bar{\rho} & 0 & 0 & 0 \\ -b\bar{\rho}\bar{G} & a - \frac{2\bar{\rho}}{k_2} & 0 & 0 \\ -\frac{\eta\bar{G}}{m^2} & -\frac{\varphi\bar{G}}{\bar{\rho}^2} & \frac{\eta}{m} - \delta & 0 \\ -\varepsilon\bar{E} & -\tau\bar{E} & \sigma\bar{E} & \mu + \tau\bar{\rho} + \sigma\bar{G} \end{bmatrix} \tag{2.25}$$

$$J_2 = \begin{bmatrix} \alpha - \beta\bar{\rho} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & c_{33} & 0 \\ 0 & 0 & 0 & d_{44} \end{bmatrix} R_4^{'} = R_4 - \omega_6 R_3$$

where,

$$\omega_1 = \frac{b\bar{\rho}\bar{G}}{\alpha - \beta\bar{\rho}}, \omega_2 = \frac{\eta\bar{G}}{m^2(\alpha - \beta\bar{\rho})}, \ \omega_3 = \frac{\varepsilon\bar{E}}{(\alpha - \beta\bar{\rho})}, \omega_4 = \frac{b_{32}}{b_{22}}, \omega_5 = \frac{b_{42}}{b_{22}} \ and \ \omega_6 = \frac{c_{43}}{c_{33}},$$

$$c_{33} = \frac{\eta}{m} - \delta, \ d_{44} = \mu + \tau\bar{\rho} + \sigma\bar{G}, \ b_{22} = a - 2\bar{\rho}$$

Now to find the eigenvalues from the above matrix, we have to solve

$$|J_2 - \lambda I| = 0$$

$$\begin{vmatrix} \alpha - \beta\bar{\rho} - \lambda_1 & 0 & 0 & 0 \\ 0 & b_{22} - \lambda_2 & 0 & 0 \\ 0 & 0 & c_{33} - \lambda_3 & 0 \\ 0 & 0 & 0 & d_{44} - \lambda_4 \end{vmatrix} = 0$$

$$(\alpha - \beta\bar{\rho} - \lambda_1)(b_{22} - \lambda_2)(c_{33} - \lambda_3)(d_{44} - \lambda_4) = 0 \tag{2.26}$$

Hence, the required eigen values of equation (2.26) are,

$$\lambda_1 = \alpha - \beta\bar{\rho}$$

$$b_{22} = a - 2\bar{\rho} = \lambda_2$$

$$c_{33} = \frac{\eta}{m} - \delta = \lambda_3$$

$$d_{44} = \mu + \tau\bar{\rho} + \sigma\bar{G} = \lambda_4$$

Here only λ_4 is positive and other three values can be other positive or negative. Then cases arise that are given below:

Case I: If $\lambda_1 < 0, \lambda_2 < 0,$ and $\lambda_3 < 0$ then

$$\lambda_1 < 0 \ \ i.e. \ \ \alpha - \beta\bar{\rho} < 0$$

$$\alpha < \beta\bar{\rho}$$

$$\lambda_2 < 0 \ \ i.e. \ \ a - 2\bar{\rho} < 0$$

$$a < 2\bar{\rho}$$

$$\lambda_3 < 0 \quad .i.e.\ \frac{\eta}{m} - \delta < 0$$

$$\frac{\eta}{m} < \delta$$

Then equilibrium point E_3 will be stable.

Case II: If $\lambda_1 > 0, \lambda_2 > 0$, and $\lambda_3 > 0$ then

$$\lambda_1 > 0 \quad i.e.\ \alpha - \beta\bar{\rho} > 0$$

$$\alpha > \beta\bar{\rho}$$

$$\lambda_2 < 0 \quad i.e.\ a - 2\bar{\rho} > 0$$

$$a > 2\bar{\rho}$$

$$\lambda_3 > 0 \quad .i.e.\ \frac{\eta}{m} - \delta > 0$$

$$\frac{\eta}{m} > \delta$$

Then equilibrium point E_3 will be asymptotically stable.

(iii) At the equilibrium point $E_4 = (\bar{Q}, \bar{\rho}, \bar{G}, \bar{E})$ equation (2.23) becomes

$$J_3 = \begin{bmatrix} \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho} & -\beta\bar{Q} & 0 & 0 \\ -b\bar{\rho}\bar{G} & a - \frac{2\bar{\rho}}{k_2} - b\bar{Q}\bar{G} & -b\bar{\rho}\bar{Q} & 0 \\ -\frac{\eta\bar{G}}{\left(m+\bar{Q}\right)^2} & -\frac{\varphi G}{\bar{\rho}^2} & \frac{\eta}{m+\bar{Q}} - \frac{\varphi}{\bar{\rho}} - \delta & 0 \\ -\varepsilon\bar{E} & -\tau\bar{E} & \sigma\bar{E} & \mu - \varepsilon\bar{Q} + \tau\bar{\rho} + \sigma\bar{G} \end{bmatrix} \tag{2.27}$$

$$J_3 = \begin{bmatrix} \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho} & -\beta\bar{Q} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 \\ 0 & 0 & c_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix} R_4' = R_4 - \omega_6 R_3$$

where,

$$\omega_1 = \frac{b\bar{\rho}\bar{G}}{\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}}, \omega_2 = \frac{\eta\bar{G}}{(m+\bar{Q})^2\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)}, \ \omega_3 = \frac{\varepsilon\bar{E}}{\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)}, \ \omega_4 = \frac{b_{32}}{b_{22}}, \quad \omega_5 = \frac{b_{42}}{b_{22}},$$

$$\omega_6 = \frac{c_{43}}{c_{33}}, \ b_{44} = \mu + \tau\bar{\rho} + \sigma\bar{G}, \ b_{23} = -b\bar{\rho}\bar{Q}, \ b_{33} = \frac{\eta}{m+\bar{Q}} - \frac{\varphi}{\bar{\rho}} - \delta, \quad b_{43} = \sigma\bar{E}, b_{42} = \frac{-\varepsilon\bar{E}\beta\bar{Q}}{\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)}\tau\bar{E}$$

$$b_{32} = \frac{-\eta\overline{\bar{G}\beta\bar{Q}}}{(m+\bar{Q})^2\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)} - \frac{\varphi\bar{G}}{\bar{\rho}^2}, \quad b_{22} = \frac{-b\bar{\rho}\beta\overline{\bar{G}\bar{Q}}}{\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)} + a - \frac{2\bar{\rho}}{k_2} - b\bar{Q}\bar{G}$$

Now to find the eigenvalues from the above matrix, we have to solve

$$|J_3 - \lambda I| = \begin{vmatrix} \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho} - \lambda & -\beta\bar{Q} & 0 & 0 \\ 0 & b_{22} - \lambda & b_{23} & 0 \\ 0 & 0 & c_{33} - \lambda & 0 \\ 0 & 0 & 0 & d_{44} - \lambda \end{vmatrix} = 0 \qquad (2.28)$$

Hence, the required eigen values of equation (2.28) are,

$$\lambda_1 = \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}$$

$$\lambda_2 = b_{22} = \frac{-b\bar{\rho}\beta\overline{\bar{G}\bar{Q}}}{\left(\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}\right)} + a - \frac{2\bar{\rho}}{k_2} - b\bar{Q}\bar{G}$$

$$\lambda_3 = c_{33} = b_{33} - \frac{b_{23}b_{32}}{b_{22}}$$

$$\lambda_4 = b_{44} = \mu + \tau\bar{\rho} + \sigma\bar{G}$$

Here only λ_4 is positive and other three values can be other positive or negative. Then cases arise that are given below:

Case I: If $\lambda_1 < 0, \lambda_2 < 0$, and $\lambda_3 < 0$ then

$$\lambda_1 < 0 \text{ i.e. } \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho} < 0$$

$$\alpha < \frac{2\bar{Q}}{k_1} + \beta\bar{\rho}$$

$$\lambda_2 < 0 \text{ i.e.} \frac{-b\beta\bar{\rho}\overline{G}\overline{\overline{Q}}}{\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}} + a - \frac{2\bar{\rho}}{k_2} - b\overline{Q}\overline{G} < 0$$

$$a < \frac{b\beta\bar{\rho}\overline{G}\overline{\overline{Q}}}{\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}} + \frac{2\bar{\rho}}{k_2} + b\overline{Q}\overline{G} < 0$$

$$\lambda_3 < 0 \ \text{ i.e. } b_{33} - \frac{b_{23}b_{32}}{b_{22}} < 0$$

$$b_{33} < \frac{b_{23}b_{32}}{b_{22}}$$

Then equilibrium point E_4 will be stable.

Case II: If $\lambda_1 > 0, \lambda_2 > 0, \text{ and } \lambda_3 > 0$ then

$$\lambda_1 > 0 \text{ i.e. } \alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho} > 0$$

$$\alpha > \frac{2\bar{Q}}{k_1} + \beta\bar{\rho}$$

$$\lambda_2 > 0 \text{ i.e.} \frac{-b\beta\bar{\rho}\overline{G}\overline{\overline{Q}}}{\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}} + a - \frac{2\bar{\rho}}{k_2} - b\overline{Q}\overline{G} > 0$$

$$a > \frac{b\beta\bar{\rho}\overline{G}\overline{\overline{Q}}}{\alpha - \frac{2\bar{Q}}{k_1} - \beta\bar{\rho}} + \frac{2\bar{\rho}}{k_2} + b\overline{Q}\overline{G} > 0$$

$$\lambda_3 > 0 \ \text{ i.e. } b_{33} - \frac{b_{23}b_{32}}{b_{22}} > 0$$

$$b_{33} < \frac{b_{23}b_{32}}{b_{22}}$$

Then equilibrium point E_4 will be asymptotically stable.

Numerical Simulation

Now, we have solved the formulated model numerically based on the initial values of the dynamics $Q_0 = 0.95$, $\rho_0 = 0.90$, $G_0 = 5.67$, $E_0 = 2.97$ and the respective parametric values shown in **Table 1** which are used in the system. The simulations have been completed by using ode45 solver in MATLAB programming language. The purpose of the simulations is to perform of the dynamical behavior of the proposed model. The parametric values are shown in the **Table 1**

Table 1. Description of the Model's Parameter with Their Corresponding Values

Parameter	Description	Values	References
α	Intrinsic growth rate of traffic flow	0.10 vh^{-1}	[Calculated]
β	Decrease rate of traffic flow by traffic density	0.0164 vh^{-1}	[estimated]
a	Growth rate of traffic density in the absence of traffic flow and road geometry influence.	0.1079 vkm^{-1}	[21]
b	Decrease rate of traffic density by traffic flow	0.0495 vkm^{-1}	[Calculated]
c	Decrease rate of traffic density by Road Geometry	0.0290 vkm^{-1}	[Calculated]
η	od stay rate of Road Geometry by traffic flow	0.0645 km	[Calculated]
φ	rmfully rate of Road Geometry by traffic density	9.969 km	[estimated]
δ	tural decrease rate of road geometry	0.063495 km	[estimated]
μ	owth rate of carbon emission	1.741 kg/year	[19]
ε	crease rate of carbon emission by traffic flow	2.60 $kgkm^{-2}$	[estimated]
τ	creasing rate of carbon emission by traffic density	0.0001021 $kgkm^{-2}$	[21]
σ	creasing rate of carbon emission by road geometry	6.56 $kgkm^{-2}$	[Calculated]
m	Saturation constant	0.008	[estimated]
k_1	Carrying capacity of traffic flow	10000	[estimated]
k_2	Carrying capacity of traffic density	1000	[estimated]

Now, we are going to describe the effect of traffic density changes to traffic flow, road geometry and carbon emission on the dynamic of the model shown in **Figure 2.1 - 2.3**

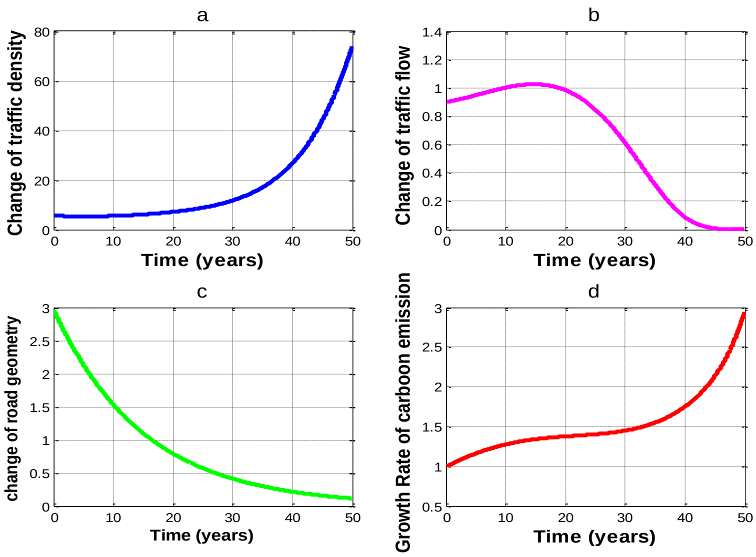

Figure 2.1. Effect of increasing traffic density changes to traffic flow, road geometry and carbon emission when a =0.1079, b = 0.0495 and all other values are the same.

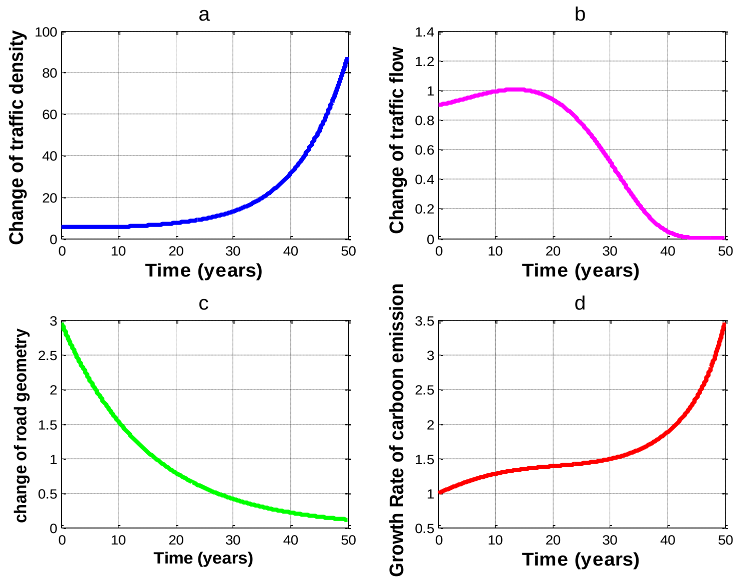

Figure 2.2. Effect of increasing traffic density changes to traffic flow, road geometry and carbon emission when a = 0.1089, b = 0.0485 and all other values are the same.

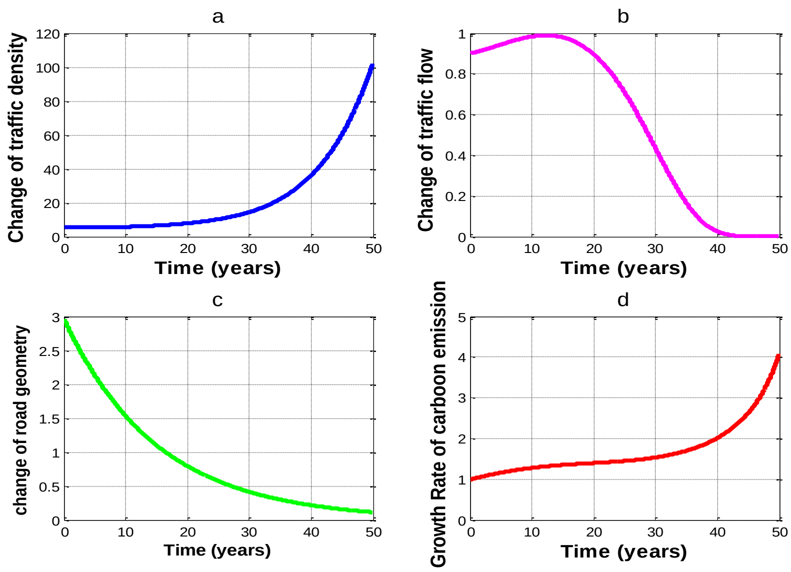

Figure 2.3. Effect of increasing traffic density changes to traffic flow, road geometry and carbon emission when a = 0.1099, b = 0.0475 and all other values are the same.

Due to increases of traffic density, the traffic flow on the road decreases. Therefore the carbon emission from the vehicle gradually increases for stop-and-go traffic and frequent acceleration and deceleration. On the other hand, more vehicles on the road put greater pressure on the pavement, leading to cracks, potholes, and faster deterioration day by day. **Figures 2.1-2.3** separately shows the changing rate of all dynamical variables of the model when a increases from 0.1079 to 0.1099 and b decreases from 0.0495 to 0.0475.

Now, we are going to describe the increasing changing rate of carbon emission by the effect of traffic jam due to increasing traffic density to the other dynamics of the model shown in **Figures** 2.4-2.7

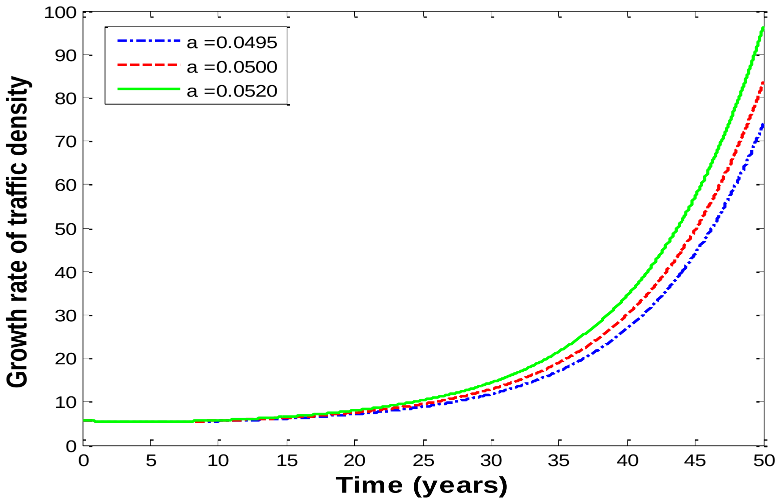

Figure 2.4. Growth rate of traffic density for different values of a.

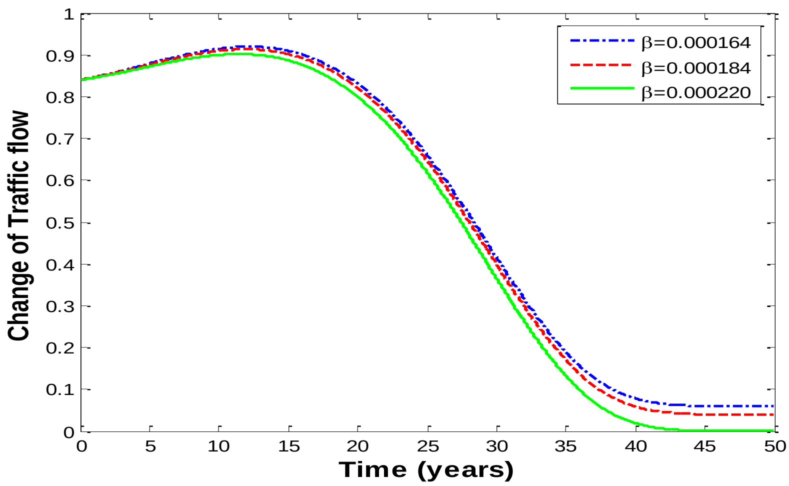

Figure 2.5. Change of traffic flow for different values of β

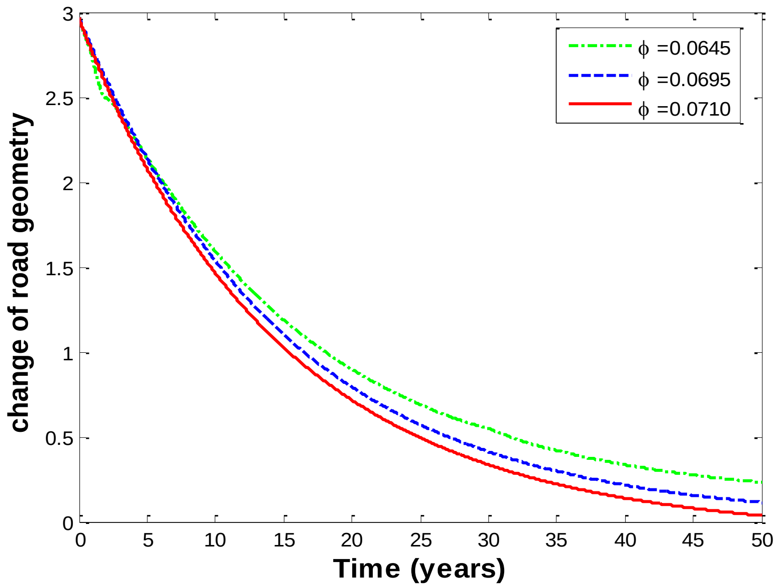

Figure 2.6. Change of Road Geometry for different values of φ

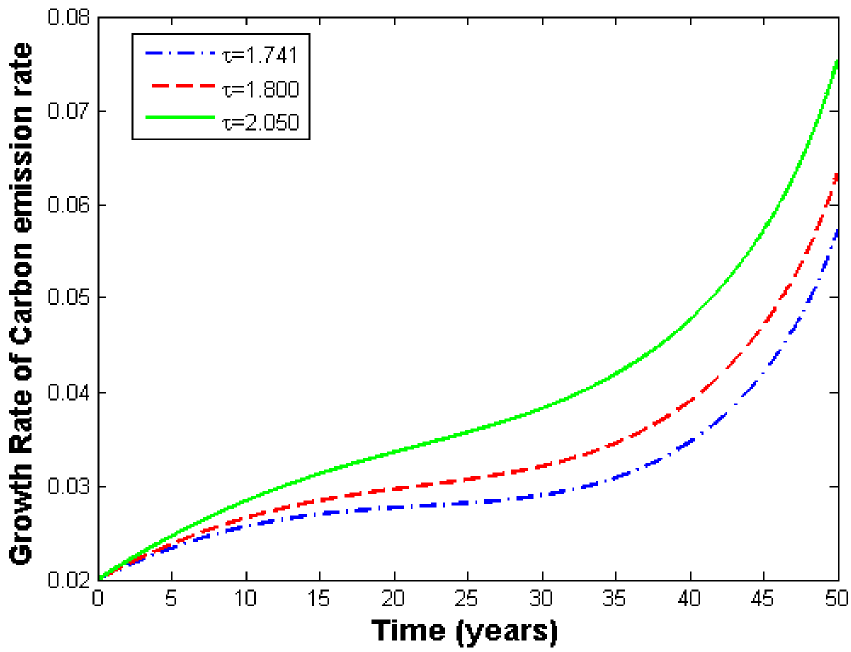

Figure 2.7. Growth Rate of carbon emission for different values of τ

We see that **Figures 2.4-2.7** shown the changing rate of four dynamic variables.

Results and Discussion

In this paper quantified the environmental impact of traffic jams in Bangladesh, with a focus on traffic flow, traffic density, road geometry and carbon emissions. Mathematical modeling is used to analyze the impact, incorporating stability theory of differential equations and equilibrium analysis to determine stable or unstable equilibriums. The model's analytical results were validated through numerical simulations, providing insight into the complex behaviors of the environmental impact of traffic congestion. Overall, the developed model serves as an important tool for policymakers and urban planners to understand how traffic flow, density, and road geometry contribute to carbon emissions in the specific context of Bangladesh.

Conclusion

In conclusion, a mathematical model encompassing four interconnected differential equations to capture the dynamics of traffic flow, density, road geometry, and carbon emissions are applied in the study to investigate environmental impact by traffic jam. Introduced some novel parameters such as decay constants and growth rates to account for the unique characteristics of traffic dynamics in the studied region. The model of four ordinary non-linear differential equations (2.16 − 2.19) sets out four equilibrium points E_1, E_2, E_3 and E_4. From the stability analysis, we see that all the equilibrium points will be stable under certain conditions and no equilibrium point is stable in general.

Provided insights into the intricate relationships between traffic parameters and environmental impact, emphasizing the role of road geometry, traffic flow, and density in carbon emissions. Analyzed the impact of congestion on carbon emissions, shedding light on the environmental consequences of high traffic density. Applied the developed model to the specific context of Bangladesh, considering the country's unique traffic patterns, road conditions, and environmental challenges. Addressed the implications of the findings for policymakers and urban planners in Bangladesh, offering a region-specific perspective on mitigating environmental impact. The proposed system of differential equations serves as a comprehensive framework for understanding how traffic flow, density, and road geometry contribute to carbon emissions. The case study in Bangladesh enriches the analysis by considering the region's specific challenges and conditions.

Conflict of Interest

The authors confirm that there is no conflict of interest with the publication of this article.

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