



MHD VISCO-ELESTIC BOUNDARY LAYER FLOW OF FLUID THROUGH POROUS MEDIUM

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Abstract: In the present paper the behavior of the free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through a porous medium over a continuously moving surface in the presence of uniform magnetic field with constant suction is studied. A uniform magnetic field is assumed to be applied perpendicularly to the moving surface. A similarity transformation is used which reduces the partial differential equation to ordinary differential equation. The velocity and temperature field is obtained. The effect of Reynolds number (Re), Hartmann number (H), Grashof number (Gr), viscoelastic parameter (K_0) and permeability parameter (K) on velocity field and temperature field are discussed with the help of graphs.

Keywords: Walter's "liquid B", Schmidt number, Newtonian fluid, porous media, visco-elastic

Introduction

The studies of the transport of heat, mass, momentum in the laminar boundary layer on moving inextensible or stretching flat surface are considered in electrochemistry and polymer processing. The steady boundary layer flow of a Newtonian fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the sheet has been extended to fluids obeying Non-Newtonian constitutive equations. While Siddappa and Abel (1985) studied the shear-driven flow of the viscoelastic Walter's 'liquid B', Anderson and Dandapat (1992) considered the boundary layer flows of micropolar and power law fluids. The heat and mass transfer problem associated with the Newtonian boundary flow past a stretching sheet was studied by Gupta and Gupta (1977). Unfortunately, the presence of a chemical reaction term in the mass diffusion equation generally destroys the possibilities of finding a similarity solution, except in the case of a stagnation point flow (Chambre and Young (1958)). However Anderson (1993) demonstrated that the mass transfer problem studied by Gupta and Gupta (1977) can be extended to diffusion of a chemically reactive species and still allow for similarity solutions.

From a technical point of view, the study of boundary layer flow on a continuously moving surface is always important. The applications of such flows in different areas such as aerodynamic, extrusion of plastic sheet, cooling of an infinite metallic plate in a cooling bath and boundary layer along a liquid in condensation process. In view of these applications Soundalgekar (1977) had studied free convection effects on the stokes problem for an infinite vertical plate. Abdelhafez (1985) discussed the skin friction and heat transfer on a continuous flat surface moving in a parallel free

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stream. Singh (1991) studied the effects of hydro magnetic convection flow past a porous plate. Choudhury and Das (2000) worked on magneto hydrodynamics boundary layer flow of non-Newtonian fluid past a flat plate. Recently Ahmad *et. al* (2004) have discussed the hall effects on the free convection flow of a non-Newtonian power law fluid at a stretching surface.

In this research work, the effect of free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through a porous medium over a continuously moving surface in the presence of uniform magnetic field is considered.

Materials and Methods

Consider the boundary layer flow of an electrically conducting incompressible visco-elastic fluid over a continuous moving flat surface through a porous medium. The incompressible prototype fluid, designated as liquid B by Walters (1962), is confined to the half-space $y>0$ above the surface. Let us denote the imposed uniform magnetic field perpendicular to the surface is B_0 and temperature of the fluid in the free stream by T under these assumptions the physical variables are functions of y only. The continuity, momentum and energy equations governing the visco-elastic boundary layer flow past a continuous moving surface thus becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$-K_0^* \left[u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right] - \frac{\sigma B_0^2}{\rho} u - \frac{\mu}{K} u$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are

$$u = U_0, v = -V_0 = \text{constant}, T = T_0 \text{ at } y = 0, \tag{4}$$

$$u \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty$$

where u and v are the velocity components of the fluid in the x and y directions respectively. Here ρ is the density, σ is the electric conductivity, ν is the kinematic viscosity, α is the thermal diffusivity, β is the coefficient of volume expansion, μ is the coefficient of viscosity, g is the acceleration due to gravity, V_0 is suction velocity, K_0^* is the visco-elastic parameter and \bar{K} is the permeability parameter

Making use of the assumption that the velocity field is independent of distance parallel to the surface, equations (1), (2), (3) and boundary conditions (4) can be written as

$$-V_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + K_0^* \frac{\partial^3 u}{\partial y^3} - \frac{\sigma B_0^2}{\rho} u - \frac{\mu}{K} u \tag{5}$$

$$-V_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{6}$$

The boundary conditions are

$$\begin{aligned} u = U_0, T = T_0 \text{ at } y = 0, \\ u \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \quad (7)$$

Solution of the problem: Let us now introduce the following dimensionless variables in to the Eqs. (5) to (7)

$$Y = \frac{y}{L}, U = \frac{u}{U_0}, \theta = \frac{T - T_\infty}{T_0 - T_\infty}, k = \frac{\bar{K}U_0^2}{\nu^2} \quad (8)$$

where L is the characteristic length (between the slit and wind up roll)
The non dimensional equations take the following form Eqs. (9) to (11)

$$K_0 \frac{\partial^3 U}{\partial Y^3} + \frac{1}{\text{Re}} \frac{\partial^2 U}{\partial Y^2} + \frac{\partial U}{\partial Y} - M_1 U = -Gr\theta \quad (9)$$

$$\frac{\partial^2 \theta}{\partial Y^2} + \text{Re Pr} \frac{\partial \theta}{\partial Y} = 0 \quad (10)$$

$$U = 1, \theta = 1 \text{ at } Y = 0, \quad (11)$$

$$U \rightarrow 0, \theta \rightarrow 0 \text{ as } Y \rightarrow \infty$$

where

$$\text{Re} = \frac{V_0 L}{\nu}, H = \frac{\sigma B_0^2}{\rho V_0}, Gr = \frac{g\beta(T_0 - T_\infty)}{V_0 U_0}, K_0 = \frac{K_0^*}{L^2}, K = \frac{\rho U_0^2 L}{k\nu}, \quad (12)$$

$$\text{Pr} = \frac{\nu}{\alpha}, M_1 = H + K$$

are the Reynolds number, Hartmann number, Grashof number, visco-elastic parameter, permeability parameter and Prandtl number respectively.

Solving equations (9) and (10) with the boundary conditions (11), we get

$$U = \exp(-mY) + Gr(\exp(-\text{Re Pr} Y) - \exp(-mY)) \quad (13)$$

$$\theta = \exp(-\text{Re Pr} Y) \quad (14)$$

Where m is the cubic root of the equation

$$K_0 m^3 + \frac{1}{\text{Re}} m^2 + m - M_1 = 0 \quad (15)$$

$$\begin{aligned} m = & -\frac{1}{3K_0 \text{Re}} + \\ & \frac{2^{1/3}(-1 + 3K_0 \text{Re}^2)}{3K_0 \text{Re}(2 - 9K_0 \text{Re}^2 - 27K_0^2 \text{Re}^3 M_1 +} \\ & \sqrt{4(-1 + 3K_0 \text{Re}^2)^3 + (2 - 9K_0 \text{Re}^2 - 27K_0^2 \text{Re}^3 M_1)^2}^{1/3} \\ & (2 - 9K_0 \text{Re}^2 - 27K_0^2 \text{Re}^3 M_1 + \\ & \sqrt{4(-1 + 3K_0 \text{Re}^2)^3 + (2 - 9K_0 \text{Re}^2 - 27K_0^2 \text{Re}^3 M_1)^2}^{1/3}) \\ & \frac{3.2^{1/3} K_0 \text{Re}}{251} \end{aligned}$$

$$\text{Skin friction } \tau = \left(\frac{\partial U}{\partial Y} \right)_{Y=0} = Gr(m - Pr Re) - m \quad (16)$$

$$\text{Rate of heat transfer} = - \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} = Pr Re \quad (17)$$

Results

The free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through a porous medium over a continuously moving surface in the presence of uniform magnetic field with constant suction is studied. Here skin friction and rate of heat transfer are calculated in Eq. (16) and Eq. (17) respectively. The velocity profiles and temperature profiles are shown graphically in the figures from Fig. 1 to Fig. 6.

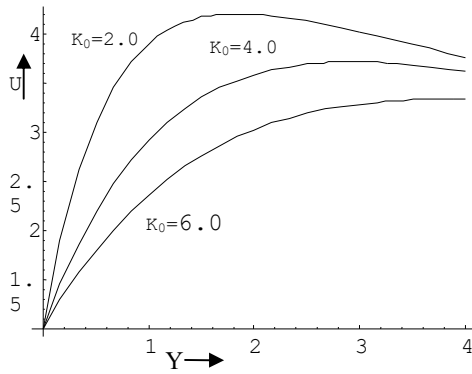


Fig.1. Velocity Profiles for different values of K_0 with $Re = 0.1$, $Pr = 0.71$, $K = 0.3$ and $H = 1.2$

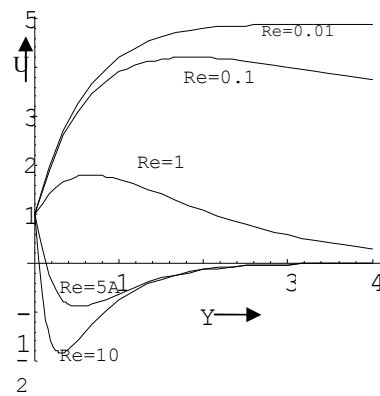


Fig.2. Velocity Profiles for different values of Re with $K_0 = 2$, $Pr = 0.71$, $K = 0.3$ and $H = 1.2$

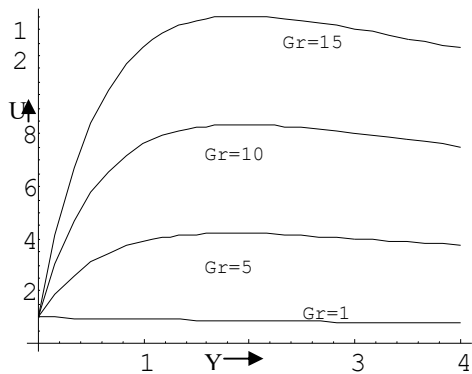


Fig.3. Velocity Profiles for different values of Gr with $K_0 = 2$, $Re = 0.1$, $Pr = 0.71$, $K = 0.3$ and $H = 1.2$

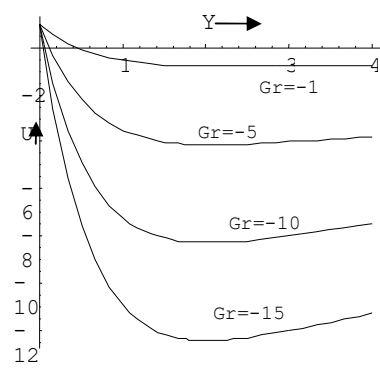


Fig.4. Velocity Profiles for different values of Gr with $K_0 = 2$, $Re = 0.1$, $Pr = 0.71$, $K = 0.3$ and $H = 1.2$

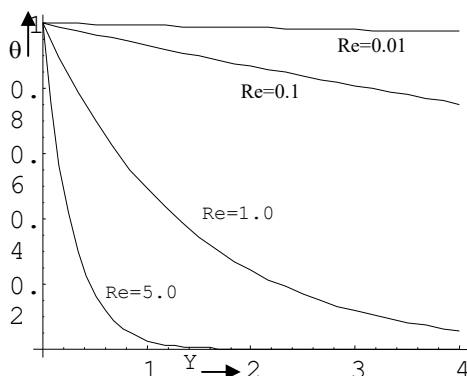


Fig.5. Temperature Profiles for different values of Re with $Pr = 0.71$

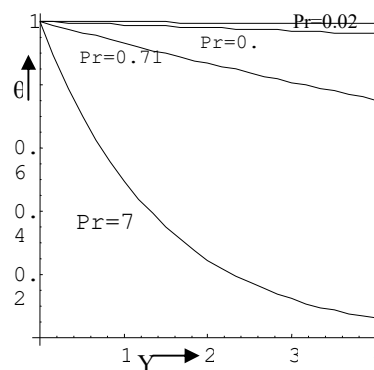


Fig.6. Temperature Profiles for different values of Pr with $Re = 0.1$

Discussion

The velocity profiles of the boundary layer flow are plotted in figures from Fig.1 to Fig.4 for different values of visco-elastic parameter (K_0), Reynolds number (Re), Grashof number (Gr), Hartmann number (H) and permeability parameter (K). In the Fig.1 we see that the increasing values of K_0 from 2.0 to 4.0 and 4.0 to 6.0 keeping Re , H , Gr and K unchanged, the fluid velocity decreasing but not same ratio. All the velocity profiles go to a limiting point for large values of Y . In the Fig.2 we observe that the graphs for different values of Re are plotted with a fixed values of K_0 , H , Gr , and K . We see in Fig.2 that increasing values of Reynolds number (Re) the velocity profiles decreasing and for $Re = 5.0$ and 10.0 the graphs are of negative side of U . In the Fig.3 and Fig.4 it is shown that the velocity profiles are plotted for positive and negative values of Gr respectively considering K_0 , Re , Pr , H and K are fixed. Here increasing values of Grashof number (Gr) the velocity profiles increasing.

The temperature profiles are plotted in the Fig. 5 and Fig. 6 for different values of Re and Pr . In the Fig. 5, the graphs for $Re = 0.01, 0.1, 1.0$ and 5.0 with for $Pr = 0.72$ (air) is plotted. Here the temperature profiles for increasing values of Re decreases, for fixed values of Prandtl number. In the Fig. 6 it is clear that for $Re = 0.1$ for $Pr = 0.02$ (mercury), 0.1 (lithium at low temperature), 0.71 (air) and 7.0 (water) are plotted and we observe that increasing values of Prandtl number the temperature profiles also decreasing.

The skin friction that was obtained in the Eq.(16) is depended on Re , Pr , K , H , Gr and K_0 . But the rate of heat transfer in the Eq.(17) only depended on the Pr and Re .

Conclusion

From the study of free convective boundary layer flow of an electrically conducting visco-elastic, incompressible fluid through a porous medium over a continuously moving surface in the presence of uniform magnetic field with constant suction the concluding remark may be as follows:

The velocity profiles is increasing for decreasing values of viscoelastic parameter (K_0) and Reynolds number (Re) while for increasing values of Grashof number (Gr).

For increasing values of Reynolds number (Re) and Prandtl number (Pr) the temperature profiles decreasing.

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