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RESEARCH ARTICLE PHYSICAL SCIENCE

VISCOUS AND JOULE HEATING EFFECTS ON MHD FREE CONVECTION AND MASS TRANSFER FLOW WITH THERMAL DIFFUSION AND LARGE SUCTION

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Abstract: MHD free convection and mass transfer flow of an incompressible viscous fluid past a continuously moving infinite vertical porous plate is made in the presence of joule heating and thermal diffusion. The corresponding momentum, energy and concentration equations are made similar by introducing the usual similarity transformations. These similarity equations are then solved analytically by employing the perturbation technique. The solutions are obtained for the case of large suction. The effects of the various parameters entering in to the problem on the velocity field are shown graphically.

Key words: Free convection, Joule heating, large suction, thermal diffusion

Introduction

Free convection flow is often encountered in cooling of nuclear reactors or in the study of the structure of stars and planets. Along with the free convection flow the phenomenon of mass transfer is also very common in the theories of stellar structure. The science of magnetohydrodynamics (MHD) was concerned with geophysical and astrophysical problems for a number of years. In recent years the possible use of MHD is to affect a flow steam of an electrically conducting fluid for the purpose of thermal protection, braking, propulsion and control. From the point of applications, model studies on the effect of magnetic field on free convection flows have been made by several investigators. Some of them are Georgantopoulos (1979), Nanousios *et al.* (1980) and Raptis and Singh (1983). Along with the effects of magnetic field, the effect of transpiration parameter, being an effective method of controlling the boundary layer has been considered by Kafousias *et al.* (1979) and Singh (1982).

Along with these studies, the effect of thermal diffusion on MHD free convection and mass transfer flows have also been considered by many investigators due to its important role particularly in isotopic separation and in mixtures between gases with very light molecular weight (H_2, H_6) and medium molecular weight (N_2, air) (Eckert and Drake, 1972). Considering these aspects, model studies were carried out by many investigators of whom the names of Kafousias (1992) and Sattar and Alam (1994), are worth mentioning.

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In the above studies, the effect of Joule heating was neglected due to low fluid velocity. The effect of joule heating was, however, considered by Hossain (1990) for MHD forced and free convection flow. Thus considering the importance of Joule heating along with viscous dissipation and thermal diffusion, our aim is to study the effects of viscous dissipation as well as joule heating on steady MHD free convection heat and mass transfer flow of an electrically conducting viscous incompressible fluid past an impulsively started vertical porous plate with variable concentration. The similarity solutions of the governing equations are obtained by employing the perturbation technique based on large suction as has been demonstrated by Singh and Dikshit (1988) and Bestman (1990).

Governing equations of the flow

Consider a steady MHD free convective heat and mass transfer flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate ($y = 0$) in presence of transverse magnetic field with the effects of viscous dissipation and joule heating. The flow is assumed to be in the x-direction which is taken along the plate in the upward direction and y-axis is normal to it. A uniform magnetic field $\mathbf{B_0}$ is taken to be acting along the y-axis. The plate is impulsively started in its own plane with a constant velocity U_0 and the plate temperature and concentration quickly raised from T and C to T_∞ and C_∞ respectively. The uniform magnetic field is assumed to be negative so that $\mathbf{B} = (0, B_0, 0)$. The equation of conservation of electric charge $\nabla . \mathbf{J} = 0$ gives $J_y =$ constant, where $\mathbf{J} = (J_x, J_y, J_z)$. Since the plate is electrically non-conducting, this constant is zero and $J_y = 0$ everywhere in the flow.

It is assumed that the plate is infinite in extent and hence all physical quantities depend on x and y. Thus accordance with the above assumptions and Boussineq's approximation, the basic equations relevant to the problem are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g_0\beta(T - T_\infty) + g_0\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2 u^2}{\rho C_p}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_T\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where, (u, v) are the velocity components along x, y directions respectively; T and C are respectively the temperature and concentration of the fluid, ν is the kinematic viscosity of the fluid, ρ is the density of the fluid, κ is the thermal conductivity, C_p is the specific heat at a constant pressure, β is the volumetric co-efficient of thermal expansion, β^* is the volumetric co-efficient of expansion for concentration, g_0 is the acceleration due to gravity, σ is the electrical conductivity, D is the molecular diffusivity and D_T is the thermal diffusivity.

The boundary conditions for the problem are:

$$\left.\begin{array}{l} u = U_0, \quad v = v_0(x), \quad T = T_w, \quad C = C(x) \quad at \quad y = 0, \\ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty, \end{array}\right\} \tag{5}$$

where U_0 is the constant velocity, $v_0(x)$ the velocity of suction and $C(x)$ is a variable concentration at the plate.

Mathematical formulations

We now introduce the usual similarity technique to reduce the governing equations (1)-(5). For the reasons of similarity, the suction velocity at the plate and the plate concentration are assumed to be $v_0(x) = -\sqrt{\frac{\nu U_0}{2x}} f_w$ and $C(x) = C_\infty + (C_0 - C_\infty)\frac{U_0 x}{\nu}$ where f_w is the dimensionless suction velocity and C_0 is considered to be the mean concentration. We now introduce the following similarity variables:

$$\left.\begin{array}{l} \eta = y\sqrt{\frac{U_0}{2\nu x}}, \\ u = U_0 f^{/}(\eta), \\ T = T_\infty + (T_w - T_\infty)\theta(\eta), \\ C(x) = C_\infty + (C_0 - C_\infty)\frac{U_0 x}{\nu}\phi(\eta). \end{array}\right\} \tag{6}$$

Thus introducing (6) into equation (1), we obtain:

$$v = \sqrt{\frac{\nu U_0}{2x}}(\eta f^{/} - f)\,. \tag{7}$$

Further, introducing the relations (6) and (7) in equations (2)- (4) we respectively have the following nondimensional equations:

$$f^{///} + f f^{//} + G_r\theta + G_m\phi - Mf^{/} = 0\,, \tag{8}$$

$$\theta^{//} + P_r f\theta^{/} + J_h P_r ({f^{//}}^2 + M{f'}^2) = 0\,, \tag{9}$$

$$\phi^{//} - 2S_c f^{/}\phi + S_c f\phi^{/} + S_0 S_c \theta^{//} = 0\,, \tag{10}$$

where $G_r\left(= \frac{g_0\beta(T_w - T_\infty)2x}{U_0^2}\right)$ the Grashof number, $G_m\left(= \frac{g_0\beta^*(C_0 - C_\infty)2x}{U_0^2}\right)$ the modified Grashof number, $P_r\left(= \frac{\nu\rho C_p}{\kappa}\right)$ the Prandtl number, $S_c\left(= \frac{\nu}{D}\right)$ the Schmidt number,

$S_0\left(=\dfrac{(T_w-T_\infty)}{(C_0-C_\infty)}\dfrac{D_T}{\nu}\right)$ the Soret number, $M\left(=\dfrac{2\sigma B_0^2 x}{\rho U_0}\right)$ the magnetic parameter and $J_h\left(=\dfrac{U_0^2}{C_p(T_w-T_\infty)}\right)$ the Joule heating parameter.

The corresponding boundary conditions are:

$$\left.\begin{array}{l} f=f_w,\quad f^{/}=1,\quad \theta=1,\quad \phi=1 \quad at \quad \eta=0, \\ f^{/}=0,\quad \theta=0,\quad \phi=0 \quad as \quad \eta\to\infty, \end{array}\right\} \tag{11}$$

where $f_w=-v_0(x)\sqrt{\dfrac{2x}{\nu U_0}}$ is the transpiration parameter and is obtained from equation (7). The primes denote derivatives with respect to η. Here $f_w<0$ indicates the suction and $f_w>0$ the injection. The solutions of the equations (8) - (10) subjected to the boundary conditions (11) are now sought by using perturbation technique and are presented in the following section.

Solutions

Since the solutions are sought for large suction, further transformations are made as: (for making LDE)

$$\varsigma=\eta f_w,\quad f(\eta)=f_w F(\varsigma),\quad \theta(\eta)=f_w^2 H(\varsigma),\quad \phi(\eta)=f_w^2 P(\varsigma)\,. \tag{12}$$

Substituting (12) in equations (8)-(10), we have:

$$F^{///}+FF^{//}+\in\left[G_r G+G_m H-MF^{/}\right]=0\,, \tag{13}$$

$$G^{//}+P_r FG^{/}+J_h P_r\left(\frac{1}{\in}F^{//2}+MF^{/2}\right)=0\,, \tag{14}$$

$$H^{//}-2S_c F^{/}H+S_c FH^{/}+S_0 S_c G^{//}=0\,, \tag{15}$$

with the boundary conditions:

$$\left.\begin{array}{l} F=1,\quad F^{/}=\in,\quad G=\in,\quad H=\in \quad at \quad \varsigma=0, \\ F^{/}=0,\quad G=0,\quad H=0 \quad as \quad \varsigma\to\infty, \end{array}\right\} \tag{16}$$

where, $$\in=\frac{1}{f_w^2}\,. \tag{17}$$

Now for large suction $f_w>1$, so that $\in$ is very small. Therefore, F, G and H can be expanded interms of small perturbation quantity $\in$ as:

$$F(\varsigma)=1+\in F_1(\varsigma)+\in^2 F_2(\varsigma)+\in^3 F_3(\varsigma)+\ldots.. \tag{18}$$

$$G(\varsigma)=\in G_1(\varsigma)+\in^2 G_2(\varsigma)+\in^3 G_3(\varsigma)+\ldots.. \tag{19}$$

$$H(\varsigma)=\in H_1(\varsigma)+\in^2 H_2(\varsigma)+\in^3 H_3(\varsigma)+\ldots.. \tag{20}$$

Then substituting $F(\varsigma)$, $G(\varsigma)$ and $H(\varsigma)$ from (18)-(20) in equations (13)-(15), we get the following set of ordinary differential equations and the boundary conditions for $F_i(\varsigma), G_i(\varsigma)$ and $H_i(\varsigma)$, $(i=1,2,3,....)$;

First order $o(\in)$ **:**

$$F_1^{///} + F_1^{//} = 0\,, \tag{21}$$

$$G_1^{//} + P_r G_1^{/} + J_h P_r F_1^{//2} = 0\,, \tag{22}$$

$$H_1^{//} + S_c H_1^{/} + S_0 S_c G_1^{//} = 0\,, \tag{23}$$

$$\left.\begin{array}{l} F_1^{/} = 1, \quad F_1 = 0, \quad G_1 = 0, \quad H_1 = 1, \qquad at \quad \varsigma = 0, \\ F_1^{/} \to 0, \quad G_1 \to 0, \quad H_1 \to 0, \qquad as \quad \varsigma \to \infty \end{array}\right\} \tag{24}$$

Second order $o(\in^2)$ **:**

$$F_2^{///} + F_2^{//} + F_1 F_1^{//} + G_r G_1 + G_m H_1 - M F_1^{/} = 0\,, \tag{25}$$

$$G_2^{//} + P_r (F_1 G_1^{/} + G_2^{/}) + J_h P_r (2F_1^{//} F_2^{//} + M F_1^{/2}) = 0\,, \tag{26}$$

$$H_2^{//} - 2S_c F_1^{/} H_1 + S_c (F_1 H_1^{/} + H_2^{/}) + S_0 S_c G_2^{//} = 0\,, \tag{27}$$

$$\left.\begin{array}{l} F_2^{/} = 0, \quad F_2 = 0, \quad G_2 = 0, \quad H_2 = 0, \qquad at \quad \varsigma = 0, \\ F_2^{/} \to 0, \quad G_2 \to 0, \quad H_2 \to 0, \qquad as \quad \varsigma \to \infty \end{array}\right\} \tag{28}$$

Third order $o(\in^3)$ **:**

$$F_3^{///} + F_3^{//} + F_2 F_1^{//} + F_1 F_2^{//} + G_r G_2 + G_m H_2 + 4RG_2 - M F_2^{/} = 0\,, \tag{29}$$

$$G_3^{//} + P_r G_3^{/} + P_r (F_2 G_1^{/} + F_1 G_2^{/}) + J_h P_r (F_2^{//2} + 2F_1^{//} F_3^{//} + 2MF_1^{/} F_2^{/}) = 0, \tag{30}$$

$$H_3^{//} + S_c H_3^{/} + S_c (F_1 H_2^{/} - 2F_1^{/} H_2) + S_c (F_2 H_1^{/} - 2F_2^{/} H_1) + S_0 S_c G_3^{//} = 0\,, \tag{31}$$

$$\left.\begin{array}{l} F_3^{/} = 0, \quad F_3 = 0, \quad G_3 = 0, \quad H_3 = 0, \qquad at \quad \varsigma = 0, \\ F_3^{/} \to 0, \quad G_3 \to 0, \quad H_3 \to 0, \qquad as \quad \varsigma \to \infty, \end{array}\right\} \tag{32}$$

etc.

The solutions of the equations up to order 3 under the prescribed boundary conditions are obtained in a straightforward manner and are

$$F_1 = 1 - e^{-\varsigma}\,, \tag{33}$$

$$G_1 = A_1 e^{-2\varsigma} + A_2 e^{-P_r \varsigma}\,, \tag{34}$$

$$H_1 = B_6 e^{-S_c \varsigma} - B_3 e^{-2\varsigma} - B_5 e^{-P_r \varsigma}\,, \tag{35}$$

$$F_2 = D_9 + D_{10} e^{-\varsigma} - D_1 \varsigma e^{-\varsigma} + \frac{1}{4} D_2 e^{-2\varsigma} + D_7 e^{-P_r \varsigma} + D_8 e^{-S_c \varsigma}\,, \tag{36}$$

$$G_2 = B_{14}e^{-P_r\varsigma} - B_8e^{-3\varsigma} - B_{12}\varsigma e^{-2\varsigma} - B_{13}e^{-2\varsigma} - B_{10}e^{-(P_r+1)\varsigma} + B_{11}e^{-(S_c+1)\varsigma}, \tag{37}$$

$$H_2 = G_{10}e^{-S_c\varsigma} + G_9e^{-2\varsigma} + G_2e^{-3\varsigma} - G_3\varsigma e^{-S_c\varsigma} + G_5\varsigma e^{-\varsigma} - G_4e^{-P_r\varsigma} + G_7e^{-(P_r+1)\varsigma} - G_8e^{-(S_c+1)\varsigma}, \tag{38}$$

$$F_3 = T_1 + T_2e^{-\varsigma} + T_3\varsigma e^{-\varsigma} + T_4e^{-2\varsigma} - \frac{1}{18}O_3e^{-3\varsigma} - \frac{1}{2}O_4\varsigma^2e^{-\varsigma} + \frac{1}{4}O_5\varsigma e^{-2\varsigma} + T_5e^{-S_c\varsigma} + T_6e^{-P_r\varsigma} - O_8O_{16}\varsigma e^{-S_c\varsigma} - O_9O_{23}e^{-(S_c+1)\varsigma} - O_{10}O_{24}e^{-(P_r+1)\varsigma}, \tag{39}$$

$$G_3 = V_{24}e^{-2\varsigma} - V_{25}e^{-3\varsigma} - V_3e^{-4\varsigma} - V_{26}\varsigma e^{-2\varsigma} - V_8\varsigma^2e^{-2\varsigma} - V_6\varsigma e^{-3\varsigma} + V_0e^{-P_r\varsigma} + V_{27}\varsigma e^{-P_r\varsigma} - V_{13}e^{-2P_r\varsigma} - V_{14}e^{-2S_c\varsigma} - V_{28}e^{-(P_r+1)\varsigma} - V_{17}\varsigma e^{-(P_r+1)\varsigma} - V_{16}e^{-(P_r+2)\varsigma} - V_{29}e^{-(S_c+1)\varsigma} - V_{20}\varsigma e^{-(S_c+1)\varsigma} - V_{22}e^{-(S_c+2)\varsigma} - V_{23}e^{-(S_c+P_r)\varsigma}, \tag{40}$$

$$H_3 = Z_{43}e^{-2\varsigma} + Z_{44}e^{-3\varsigma} + Z_3Z_{23}e^{-4\varsigma} + Z_{51}e^{-S_c\varsigma} + Z_{12}Z_{33}e^{-2S_c\varsigma} + Z_{45}e^{-P_r\varsigma} + Z_{13}Z_{34}e^{-2P_r\varsigma} + Z_{46}\varsigma e^{-2\varsigma} + Z_5Z_{22}\varsigma e^{-3\varsigma} + Z_6Z_{21}\varsigma^2e^{-2\varsigma} - Z_{47}\varsigma e^{-S_c\varsigma} - \frac{1}{2}Z_8Z_{42}\varsigma^2e^{-S_c\varsigma} + Z_{48}\varsigma e^{-P_r\varsigma} + Z_{49}e^{-(S_c+1)\varsigma} + Z_{50}e^{-(P_r+1)\varsigma} + Z_{17}Z_{36}\varsigma e^{-(S_c+1)\varsigma} + Z_{16}Z_{35}\varsigma e^{-(P_r+1)\varsigma} + Z_{18}Z_{39}e^{-(S_c+2)\varsigma} + Z_{19}Z_{40}e^{-(P_r+2)\varsigma} + Z_{20}Z_{41}e^{-(S_c+P_r)\varsigma}, \tag{41}$$

where, the constants A_j, B_j, D_j, E_j, G_j, O_j, T_j, W_j, V_j, and Z_j, are shown in Appendix 1. The velocity, the temperature and the concentration fields can be calculated from the combinations of (6), (12) and (18)-(20) as:

$$\frac{u}{U_0} = f^{/}(\eta) = F_1^{/} + \in F_2^{/} + \in^2 F_3^{/}, \tag{42}$$

$$\theta(\eta) = G_1 + \in G_2 + \in^2 G_3, \tag{43}$$

$$\phi(\eta) = H_1 + \in H_2 + \in^2 H_3. \tag{44}$$

Thus with the help of the solutions (33)-(41), the velocity, temperature and concentration distributions can be calculated out from (42)-(44). However, the results of the velocity distributions, temperature and concentration distributions are put forward graphically in Figs. 1- 9.

Skin-friction coefficients, Nusselt number and Sherwood number

The quantities of chief physical interest are the skin friction coefficients, Nusselt number and the Sherwood number. The equation defining the wall skin frictions is:

$$\tau_x = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \text{ hence from (42) we have, } \tau_x \propto f^{//}(0)$$

where

$$f^{//}(0) = -1 + \in[D_{11} + D_2 + D_7P_r^2 + D_8S_c^2] + \in^2[-T_7 - T_8 + 2T_9 - 1/2O_5 - 1/2O_3 + T_{10}S_c + T_{11}P_r + T_{12} + T_{13} - T_{14}(S_c+1) - T_{15}(P_r+1)]. \tag{45}$$

The local Nusselt number, denoted by N_u, is proportional to $-\left(\frac{\partial T}{\partial y}\right)_{y=0}$, hence from (43) we have, $N_u \propto -\theta^{/}(0)$, where,

$$\begin{aligned}\theta^{/}(0) = & -2A_1 - A_2P_r + \in [-B_{14}P_r + 3B_8 - B_{12} + 2B_{13} + B_{10}(P_r+1) - B_{11}(S_c+1)] \\ & + \in^2 [-2V_{24} + 3V_{25} + 4V_3 - V_{26} - V_6 - V_0P_r + V_{27} + 2V_{13}P_r + 2V_{14}S_c + V_{28}(P_r+1) \\ & - V_{17} + V_{16}(P_r+2) + V_{29}(S_c+1) - V_{20} + V_{22}(S_c+2) + V_{23}(S_c+P_r)]. \end{aligned} \quad (46)$$

The local Sherwood number denoted by S_h is proportional to $-\left(\frac{\partial C}{\partial y}\right)_{y=0}$. Therefore from (44) we have, $S_h \propto -\phi^{/}(0)$ where,

$$\begin{aligned}\phi^{/}(0) = & -B_6S_c + 2B_3 + B_5P_r + \in [-G_{11}S_c - 2G_9 - 3G_2 - G_3 + G_5 + G_{10}P_r \\ & - G_7(P_r+1) + G_8(S_c+1)] + \in^2 [-2Z_{43} - 3Z_{44} - 4Z_3Z_{23} - Z_{51}S_c \\ & - 2Z_{12}Z_{33}S_c - Z_{45}P_r - 2Z_{13}Z_{34}P_r + Z_{46} + Z_5Z_{22} - Z_{47} + Z_{48} - Z_{49}(S_c+1) \\ & - Z_{50}(P_r+1) + Z_{17}Z_{36} + Z_{16}Z_{35} - Z_{18}Z_{39}(S_c+2) - Z_{19}Z_{40}(P_r+2) \\ & - Z_{20}Z_{41}(S_c+P_r)]. \end{aligned} \quad (47)$$

Thus the values proportional to the skin friction coefficients, the Nusselt number and the Sherwood number are respectively obtained from (45)- (47). These values are not shown for brevity.

Results and Discussion

The velocity, temperature and concentration profiles, obtained in the form of dimensionless velocity ($f^{/}$), temperature ($\theta(\eta)$) and concentration ($\phi(\eta)$), are presented in Figs. 1-9 for different values of M, f_w, S_c, P_r, S_0 and J_h. The values of M and G_r are taken to be large, since these values respectively correspond to a strong magnetic field and to a cooling problem that is generally considered in nuclear engineering in connection with the cooling reactors. Negative values of G_r and G_m, which indicates the heating of the plate, are also taken into account. Three values of Prandtle number (P_r) are considered such as 0.71, 1.01 and 7.0. Here P_r = 0.71 represents air at 20^0 C, P_r = 1.01 corresponds to electrolyte solutions (such as salt water) and P_r = 7.0 represents water. Three values of Schmidt number (S_c), 0.60, 0.22 and 0.75 are considered where the values 0.60, 0.22 and 0.75 represent for water vapour, carbondyoxide (Co_2) and oxygen (O_2) respectively. The values of G_m, M, f_w, and J_h are however chosen arbitrarily.

With the above flow parameters, it is thus observed from Fig. 1 that the velocity profiles increase with the increase of Soret number (S_0) for cooling plate while decreases with the increase of

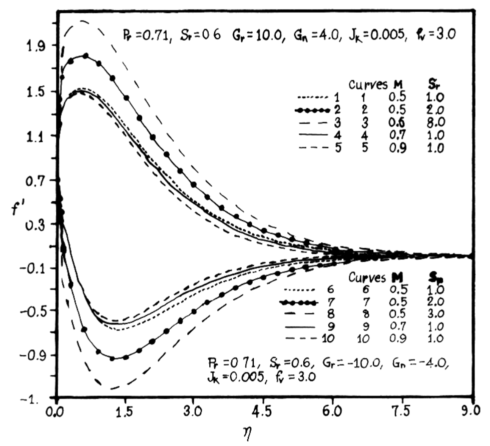

Fig. 1. Velocity profiles (cooling and heating plate) for different values of M and Sc.

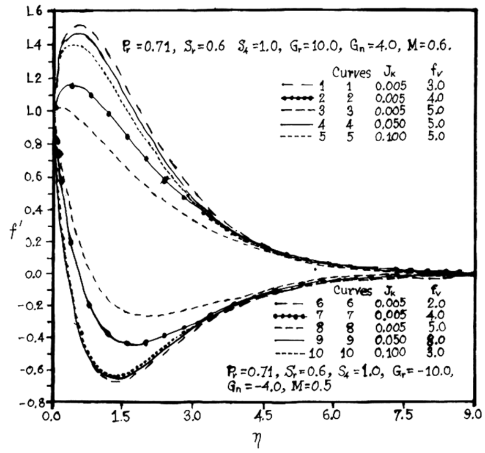

Fig. 1. Velocity profiles (cooling and heating plate) for different values of J_h and f_w.

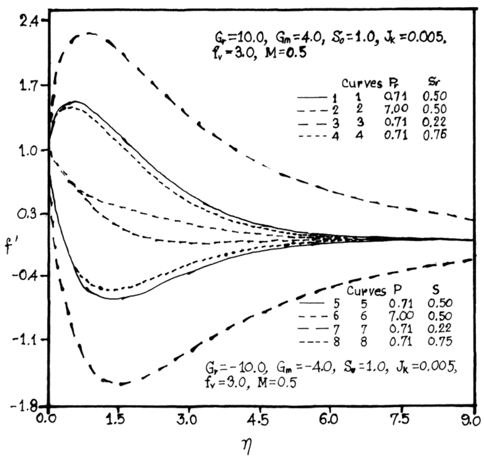

Fig. 3. Velocity profiles (cooling and heating plate) for different values of P_r and S_c.

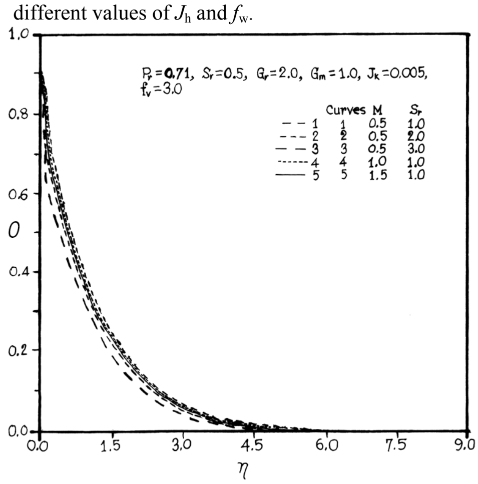

Fig. 3. Temperature profiles for different values of M and S_c.

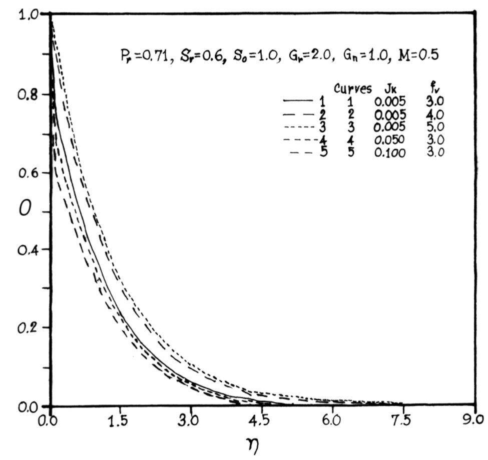

Fig. 5. Temperature profiles for different values of J_h and f_w.

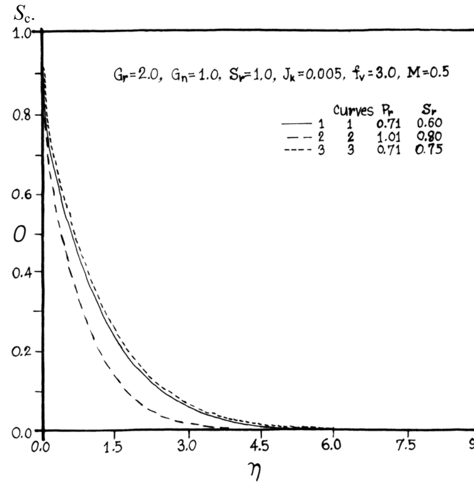

Fig. 6. Temperature profiles for different values of P_r and S_c.

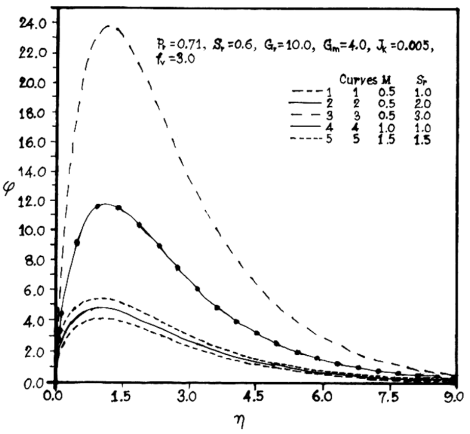

Fig. 7. Concentration profiles for different values of M and Sc.

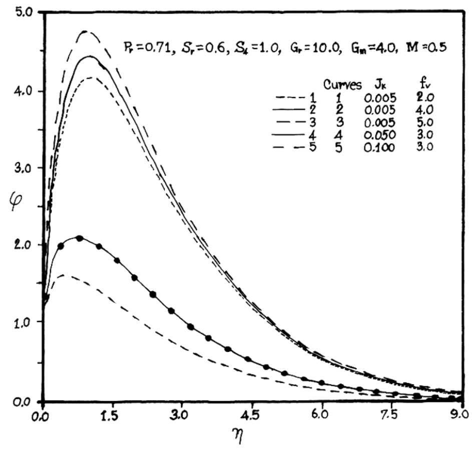

Fig. 8. Concentration profiles for different values of J_h and f_w.

magnetic parameter (M) keeping P_r, S_c, G_r, G_m, f_W, and J_h constant. While incase of heating of the plate the velocity is just reverse. These effects indicate that for cooling of the plate, when thermal diffusivity is introduced into the flow field, it entrances into the velocity field which decreases the velocity incase of heating of the plate. From Fig. 2, the usual effects of the suction of the boundary layer control are observed. It is also observed from this figure that the joule heating parameter (J_h) reduces the velocity at a particular y-position of the boundary layer while incase of heating of the plate, the velocity is just opposite. From Fig. 3 for cooling of the plate, we observed that the velocity profiles decrease for P_r =7.0 with compared to P_r = 0.71 but it increases with the increase of Schmidt number (S_c). From Fig. 4, we observe that temperature profile decreases with the increase of S_0 and M. From Fig. 5, we see that with the increase of joule heating parameter (J_h), the temperature profiles increase. On the contrary, opposite effect is observed for suction parameter. From Fig. 6, we observe that temperature profile decreases with the increase of Prandtl number (P_r) with compared to air and water. Also minor increase is observed with the increase of Schmidt number (S_c). From Fig. 7, it is seen that the concentration profiles increase with increasing the Soret number (S_0) and decrease with increasing the magnetic parameter (M) keeping other parameters fixed. From Fig. 8, it is observed that the concentration profiles increase with the increasing of joule heating (J_h) and decrease with the increase of suction parameter (f_W). In Fig. 9, minor effect is observed for P_r = 0.71 and 7.0 where S_c = 0.60. On the contrary, a drastic change is observed for S_c = 0.75 (in the case of oxygen) where P_r = 0.71.

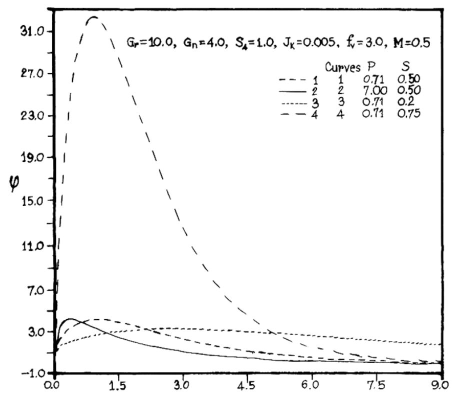

Fig. 8. Concentration profiles for different values of P_r and S_c.

References

Bestman, A.R. 1990. *Astrophysics and Space Science*, 173: 93.

Eckert, E.R.G. and Drake, R.M. 1972. *Analysis of Heat and Mass Transfer*. McGraw-Hill Book Co., New York.

Georgantopoulos, G.A. 1979. *Astrophysics and Space Science,* 65(2): 433.

Hossain, M.A. 1990. *ICTP,* International Print No. IC/9/265.

Kafousias, N.G.; Nanousis, N.D. and Geograntopoulas, G.A. 1979. *Astrophysics and Space Science*, 64: 391.

Kafoussias, N.G. 1992. *Astrophysics and Space Science,* 192: 11.

Nanousis, N.; Georgantopoulos, G.A. and Papaioannou, A. 1980. *Astrophysics and Space Science*, 70: 377.

Raptis, A.A. and Singh, A.K. 1983. *Int. Comm. Heat and Mass Transfer*, 10(4): 313.

Sattar, M.A. and Mahmud Alam, M. 1994. *Indian Journal of Pure and Appied Mathematics*, 25(6): 679.

Singh, A.K. 1982. *Astrophysics and Space Science*, 87: 455.

Singh, A.K. and Dikshit, C.K. 1988. *Astrophysics and Space Science*, 148: 249.

Appendix 1

$A_1 = -\dfrac{J_h P_r}{4-2P_r}$, $A_2 = 1 - A_1$, $B_1 = 1$, $B_2 = 4A_1 S_0 S_c$, $B_3 = \dfrac{B_2}{4-2S_c}$, $B_4 = S_0 S_c A_2 P_r^2$,

$B_5 = \dfrac{B_4}{P_r(P_r - S_c)}$, $B_6 = B_1 + B_3 + B_5$, $D_1 = -1$, $D_2 = 1 + G_r A_1 - G_m B_3$, $D_3 = G_r A_2 - G_m B_5$,

$D_4 = G_m B_6$, $D_5 = \dfrac{D_3 P_r}{P_r^2 (P_r - 1)}$, $D_6 = \dfrac{D_4 S_c}{S_c^2 (S_c - 1)}$, $D_7 = \dfrac{D_3}{P_r^2 (P_r - 1)}$, $D_8 = \dfrac{D_4}{S_c^2 (S_c - 1)}$,

$D_9 = D_1 + \dfrac{1}{4} D_2 + D_5 + D_6 - D_7 - D_8$, $D_{10} = -D_1 - \dfrac{1}{2} D_2 - D_5 - D_6$, $D_{11} = 2D_1 + D_{10}$,

$D_{12} = -2A_1 P_r - 2J_h P_r D_{11})$, $D_{13} = 2A_1 P_r - 2D_2 J_h P_r$, $D_{14} = 2D_1 J_h P_r$, $D_{15} = -2D_7 J_h P_r^3$,

$D_{16} = 2D_8 J_h P_r S_c^2$, $B_7 = \dfrac{D_{12}}{4-2P_r}$, $B_8 = \dfrac{D_{13}}{9-3P_r}$, $B_9 = \dfrac{D_{14}(4-2P_r)}{(4-2P_r)^2}$, $B_{10} = \dfrac{D_{15}}{P_r + 1}$,

$B_{11} = \dfrac{D_{16}}{(S_c + 1)(1 + S_c - P_r)}$, $B_{12} = \dfrac{D_{14}}{4-2P_r}$, $B_{13} = B_7 + B_9$, $B_{14} = B_7 + B_8 + B_9 + B_{10} - B_{11}$,

$E_1 = B_{14} P_r^2$, $E_2 = 9B_8$, $E_3 = 4B_{12}$, $E_4 = 4B_{12} - 4B_{13}$, $E_5 = B_{10}(P_r + 1)^2$, $E_6 = B_{11}(S_c + 1)^2$,

$E_7 = 2B_3 S_c + E_4 S_0 S_c$, $E_8 = E_2 S_0 S_c$, $E_9 = B_6 S_c^2$, $E_{10} = B_5 S_c P_r + E_1 S_0 S_c$,

$E_{11} = E_3 S_0 S_c$, $E_{12} = B_5 S_c P_r - 2B_5 S_c + E_5 S_0 S_c$, $E_{13} = B_6 S_c^2 + E_6 S_0 S_c - 2B_6 S_c$

$G_1 = \dfrac{E_7}{4-2S_c}$, $G_2 = \dfrac{E_8}{9-3S_c}$, $G_3 = \dfrac{E_9}{S_c}$, $G_4 = \dfrac{E_{10}}{P_r^2 - S_c P_r}$, $G_5 = \dfrac{E_{11}}{4-2S_c}$, $G_6 = \dfrac{E_{11}(4-S_c)}{(4-2S_c)^2}$,

$G_7 = \dfrac{E_{12}}{(P_r+1)(1+P_r-S_c)}$, $G_8 = \dfrac{E_{13}}{S_c+1}$, $G_9 = G_6 - G_1$, $G_{10} = -G_9 - G_2 + G_4 - G_7 + G_8$,

$O_1 = -D_9 + D_{11}$, $O_2 = -D_{10} - D_{11} + D_2 - B_{13} G_r + G_9 G_m$,

$O_3 = \dfrac{1}{4} D_2 + D_2 + B_8 G_r - G_2 G_m$, $O_4 = -D_1$, $O_5 = -2D_1 - B_{12} G_r + G_5 G_m$, $O_6 = D_8 S_c^2 + G_{10} G_m$,

$O_7 = D_7 P_r^2 + G_r B_{14} - G_4 G_m$, $O_8 = -G_3 G_m$, $O_9 = D_8 + D_8 S_c^2 - B_{11} G_r + G_8 G_m$,

$O_{10} = D_7 + D_7 P_r^2 + B_{10} G_r - G_9 G_m$, $O_{11} = \dfrac{1}{S_c^2}$, $O_{12} = \dfrac{1}{P_r^2}$, $O_{13} = \dfrac{1}{S_c(1-S_c)}$, $O_{14} = \dfrac{1}{P_r(1-P_r)}$,

$O_{15} = \dfrac{O_{11}}{(1-S_c)}$, $O_{16} = \dfrac{O_{12}}{(1-P_r)}$, $O_{17} = \dfrac{1}{(S_c+1)^2}$, $O_{18} = \dfrac{1}{(P_r+1)^2}$, $O_{19} = \dfrac{O_{11}(3S_c-2)}{(1-S_c)^2}$,

$O_{20} = \dfrac{O_{12}(3P_r-2)}{(1-P_r)^2}$, $O_{21} = \dfrac{1}{S_c(S_c+1)}$, $O_{22} = \dfrac{1}{P_r(P_r+1)}$, $O_{23} = \dfrac{1}{S_c(S_c+1)^2}$,

$O_{24} = \dfrac{1}{P_r(P_r+1)^2}$,

$T_1 = O_1 + \dfrac{1}{4} O_2 - \dfrac{1}{9} O_3 + 2O_4 + \dfrac{1}{4} O_5 + O_6 O_{11} + O_7 O_{12} + O_8 O_{15} - O_9 O_{17} - O_{10} O_{18}$,

$$T_2 = -O_1 - \frac{1}{2} O_2 + \frac{1}{6} O_3 - 2O_4 + \frac{1}{5} O_5 - O_5 + O_6 O_{13} + O_7 O_{14} - O_8 O_{15} - O_8 O_{19} + O_9 O_{21} + O_{10} O_{22},$$

$T_3 = -O_1 - 2O_4$, $T_4 = \dfrac{1}{4} O_2 + \dfrac{1}{2} O_5$, $T_5 = -O_6 O_{15} + O_8 O_{19}$, $T_6 = -O_7 O_{16}$, $T_7 = T_3 - T_2$,

$T_8 = T_3 + O_4$, $T_9 = 2T_4 - \dfrac{1}{4} O_5$, $T_{10} = T_5 S_c + O_8 O_{15}$, $T_{11} = T_6 P_r$, $T_{12} = O_8 O_{15} S_c$,

$W_1 = -2A_1 D_9 P_r - B_{12} P_r + 2B_{13} P_r + D_{11}^2 J_h P_r + 2T_7 J_h P_r + 2T_8 J_h P_r$,

$W_2 = -2A_1 D_{10} P_r + 3B_8 P_r + B_{12} P_r - 2B_{13} P_r + 2D_2 D_{11} J_h P_r - 4T_9 J_h P_r + O_5 J_h P_r$,

$W_3 = -\dfrac{1}{2} A_1 D_2 P_r - 3B_8 P_r + D_2^2 J_h P_r + O_3 J_h P_r$, $W_4 = 2B_{12} P_r - 2D_1 D_{11} J_h P_r - 2T_8 J_h P_r - 2O_4 J_h P_r$,

$W_5 = 2A_1 D_1 P_r - 2B_{12} P_r - 2D_1 D_2 J_h P_r - 2O_5 J_h P_r$, $W_6 = D_1^2 J_h P_r + O_4 J_h P_r$,

$W_7 = -A_2 P_r^2 D_9 - B_{14} P_r^2$, $W_8 = -A_2 D_7 P_r^2 + D_7^2 J_h P_r^5$, $W_9 = D_8^2 J_h P_r S_c^4$,

$W_{10} = -A_2 D_{10} P_r^2 + B_{10} P_r (P_r + 1) + B_{14} P_r^2 + 2D_7 D_{11} J_h P_r^3 - 2T_{11} J_h P_r^2 - 2T_{13} J_h P_r$,

$W_{11} = -2A_1 D_7 P_r - \frac{1}{4} A_2 D_2 P_r^2 - B_{10} P_r (P_r + 1) + 2D_2 D_7 J_h P_r^3 + 2T_{15} J_h P_r (P_r + 1)$,

$W_{12} = A_2 D_1 P_r^2 - 2D_1 D_7 J_h P_r^3 + 2T_{13} J_h P_r^2$,

$W_{13} = -B_{11} P_r (S_c + 1) + 2D_8 D_{11} J_h P_r S_c^2 - 2T_{10} J_h P_r S_c - 2T_{12} J_h P_r$,

$W_{14} = -2D_1 D_8 J_h P_r S_c^2 + 2T_{12} J_h P_r S_c$,

$W_{15} = -2A_1 D_8 P_r + B_{11} P_r (S_c + 1) + 2D_2 D_8 J_h P_r S_c^2 + 2T_{14} J_h P_r (S_c + 1)$,

$W_{16} = -A_2 D_8 P_r^2 + 2D_7 D_8 J_h S_c^2 P_r^3$,

$V_0 = V_1 + V_2 + V_3 + V_5 + V_7 + V_9 + V_{11} + V_{13} + V_{14} + V_{15} + V_{16} + V_{18} + V_{19} + V_{21} + V_{22} + V_{23}$,

$V_1 = \frac{W_1}{4 - 2P_r}$, $V_2 = \frac{W_2}{9 - 3P_r}$, $V_3 = \frac{W_3}{16 - 4P_r}$, $V_4 = \frac{W_4}{4 - 2P_r}$, $V_5 = \frac{W_4 (4 - P_r)}{(4 - 2P_r)^2}$,

$V_6 = \frac{W_5}{9 - 3P_r}$, $V_7 = \frac{W_6 (9 - P_r)}{(9 - 3P_r)^2}$, $V_8 = \frac{W_6}{4 - 2P_r}$, $V_9 = \frac{W_6}{(4 - 2P_r)^2}$, $V_{10} = -\frac{W_6 (4 - P_r)}{(4 - 2P_r)^2}$,

$V_{11} = \frac{W_6 (P_r - 4)^2}{(4 - 2P_r)^2}$, $V_{12} = \frac{W_7}{P_r}$, $V_{13} = \frac{W_8}{2P_r^2}$, $V_{14} = \frac{W_9}{2S_c (2S_c - P_r)}$, $V_{15} = \frac{W_{10}}{P_r + 1}$,

$V_{16} = \frac{W_{11}}{2(P_r + 2)}$, $V_{17} = \frac{W_{12}}{P_r + 1}$, $V_{18} = \frac{W_{12} (P_r + 2)}{(P_r + 1)^2}$, $V_{19} = \frac{W_{13}}{(S_c + 1)(S_c - P_r + 1)}$,

$V_{20} = \frac{W_{14}}{(S_c + 1)(S_c - P_r + 1)}$, $V_{21} = \frac{W_{45} (2S_c - P_r + 2)}{(S_c + 1)^2 (S_c - P_r + 1)^2}$, $V_{22} = \frac{W_{15}}{(S_c + 2)(S_c - P_r + 2)}$,

$V_{23} = \frac{W_{16}}{S_c (S_c + P_r)}$, $V_{24} = -(V_1 + V_5 - 2V_9 + V_{11})$, $V_{25} = V_2 + V_7$, $V_{26} = V_4 - 2V_{10}$, $V_{27} = V_{12} + V_{15}$,

$V_{28} = V_{15} + V_{18}$, $V_{31} = V_{21} + V_{23}$,

$Z_1 = -2G_{11} S_c + G_5 S_c + 2B_3 D_9 S_c + 4V_{26} S_0 S_c + 4V_{28} S_0 S_c - 2V_8 S_0 S_c$,

$Z_2 = -3G_2 S_c + G_5 S_c - 2B_3 S_c D_1 - 9V_{25} S_0 S_c + 6V_6 S_0 S_c$,

$Z_3 = 3G_2 S_c - 2G_5 S_c - \frac{1}{2} B_3 S_c D_2 - 16V_3 S_0 S_c$,

$Z_4 = -2G_5 S_c - 4V_{26} S_0 S_c + 8V_8 S_0 S_c$, $Z_5 = -4G_5 S_c - 9V_6 S_0 S_c$, $Z_6 = -4V_8 S_0 S_c$,

$Z_7 = -G_{10} S_c - G_3 S_c - B_6 D_9 S_c^2$, $Z_8 = G_3 S_c^2$,

$Z_9 = +G_4 P_r S_c + B_5 D_9 S_c P_r + V_0 S_0 S_c P_r^2 - 2V_{27} S_0 S_c P_r$, $Z_{10} = V_{27} S_0 S_c P_r^2$,

$$Z_{12} = B_6 D_8 S_c^2 - 4V_{14} S_0 S_c^3, \; Z_{13} = -B_5 D_7 P_r S_c - 4V_{13} S_0 S_c P_r^2,$$

$$Z_{14} = -G_7 S_c (P_r + 1) + G_4 S_c P_r + 2G_4 S_c + B_5 D_{10} S_c P_r - 2B_5 S_c (D_1 + D_{10}) - V_{28} S_0 S_c (P_r + 1)^2 + 2V_{17} S_0 S_c (P_r + 1),$$

$$Z_{15} = G_8 S_c (S_c + 1) + G_{10} S_c^2 + G_3 S_c - 2G_{10} S_c - B_6 D_{10} S_c^2 + 2B_6 S_c (D_1 + D_{10}) - V_{24} S_0 S_c (S_c + 1)^2 + 2V_{20} S_0 S_c (S_c + 1),$$

$$Z_{16} = -B_5 D_1 S_c P_r + 2B_5 D_1 S_c - V_{17} S_0 S_c (P_r + 1)^2,$$

$$Z_{17} = G_3 S_c^2 + 2G_3 S_c + B_6 D_1 S_c^2 - 2B_6 D_1 S_c - V_{20} S_0 S_c (S_c + 1)^2,$$

$$Z_{18} = G_8 S_c (S_c + 1) + 2G_8 S_c - B_6 D_2 S_c (\frac{1}{4} S_c - 1) - 2B_3 D_8 S_c (S_c + 1) - V_{22} S_0 S_c (S_c + 2)^2,$$

$$Z_{19} = -G_7 S_c (P_r + 1) - 2G_7 S_c + 2B_3 D_7 S_c + B_5 D_2 S_c (\frac{1}{4} P_r - 1) - 2B_3 D_7 S_c P_r - V_{16} S_0 S_c (P_r + 2)^2,$$

$$Z_{20} = -B_6 D_7 S_c^2 + B_5 D_8 S_c P_r + 2B_6 D_7 S_c P_r - 2B_5 D_8 S_c^2 - V_{23} S_0 S_c (S_c + P_r)^2,$$

$$Z_{21} = \frac{1}{4 - 2S_c}, \; Z_{22} = \frac{1}{9 - 3S_c}, \; Z_{23} = \frac{1}{16 - 4S_c}, \; Z_{24} = \frac{4 - S_c}{(4 - 2S_c)^2},$$

$$Z_{25} = \frac{6 - S_c}{(9 - 3S_c)^2}, \; Z_{26} = \frac{1}{(4 - 2S_c)^2}, \; Z_{27} = \frac{4 - S_c}{(4 - 2S_c)^2}, \; Z_{28} = \frac{(4 - S_c)^2}{(4 - 2S_c)^3}, \; Z_{29} = \frac{1}{P_r (P_r - S_c)},$$

$$Z_{30} = \frac{S_c - 2P_r}{P_r^2 (P_r - S_c)^2}, \; Z_{31} = \frac{1}{P_r^2 (P_r - S_c)^2}, \; Z_{32} = \frac{(S_c - 2P_r)^2}{P_r^3 (P_r - S_c)^3},$$

$$Z_{33} = \frac{1}{2S_c^2}, \; Z_{34} = \frac{1}{2P_r (2P_r - S_c)}, \; Z_{35} = \frac{1}{(P_r + 1)(1 + P_r - S_c)}, \; Z_{36} = \frac{1}{(S_c + 1)},$$

$$Z_{37} = \frac{(2P_r - S_c + 2)}{(P_r + 1)^2 (P_r - S_c + 1)^2}, \; Z_{38} = \frac{S_c + 2}{(S_c + 1)^2}, \; Z_{39} = \frac{1}{2(S_c + 2)}, \; Z_{40} = \frac{1}{(P_r + 2)(P_r - S_c + 2)},$$

$$Z_{41} = \frac{1}{P_r (P_r + S_c)}, \; Z_{42} = \frac{1}{S_c}, \; Z_{43} = Z_1 Z_{21} + Z_4 Z_{24} - 2Z_6 Z_{26} + 2Z_6 Z_{28},$$

$$Z_{44} = Z_2 Z_{22} + Z_5 Z_{25}, \; Z_{45} = Z_9 Z_{29} - Z_{10} Z_{30}, \; Z_{46} = Z_4 Z_{21} + 2Z_6 Z_{27},$$

$$Z_{47} = Z_7 Z_{42} + 2Z_8 Z_{33}, \; Z_{48} = Z_{10} Z_{29}, \; Z_{49} = Z_{15} Z_{36} + Z_{17} Z_{38}, \; Z_{50} = Z_{14} Z_{35} + Z_{16} Z_{37},$$

$$Z_{51} = -Z_1 Z_{21} - Z_2 Z_{22} - Z_3 Z_{23} - Z_4 Z_{24} - Z_5 Z_{25} + 2Z_6 Z_{26} - 2Z_6 Z_{27} - 2Z_6 Z_{28} - Z_9 Z_{29} + Z_{10} Z_{30} - Z_{12} Z_{33} - Z_{13} Z_{34} - Z_{14} Z_{35} - Z_{15} Z_{36} - Z_{16} Z_{37} - Z_{17} Z_{38} - Z_{18} Z_{39} - Z_{19} Z_{40} - Z_{20} Z_{41}$$

Alam, M.M.; Islam, M.R. and Mondal, R.N. 2006. Viscous and joule heating effects on MHD free convection and mass transfer flow with thermal diffusion and large suction. *Khulna University Studies*, 7(2): 119-132.