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AN ANALYTICAL MODEL OF SUBSTRATE CURRENT IN A SHORT CHANNEL MOSFET

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Abstract: The drain characteristics of a short channel MOSFET operating in the avalanche multiplication regime depends strongly on the flow of substrate current I_{sub}. In this work, the detailed physics and the corresponding mathematical analysis of the substrate current model of a short channel MOSFET in its avalanche multiplication regime has been presented, which can be used to find the different characteristics of the short channel MOSFET for a wide range of device parameters. It is shown that by using a multiplication model, the substrate current can be obtained for a particular drain source voltage V_{DS}. The model system shows excellent results, which were confirmed by the computer-aided simulation program (MATLAB).

Key words: Physical model, substrate current model, short channel, MOSFET, multiplication factor

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Introduction

In order to go on improving the speed and integrity of VLSI circuits comprised of MOSFETs and reducing its power consumption, at the same time breaking through technology and electrical characteristic limitations in the deep-submicrometer process, and providing technology support for simulating and designing VLSI quickly and accurately, it has been one of the hotspots to study the physical characteristics of the short channel MOSFETs and to develop its continuous and scaled-down analytical model (Yang *et al.*, 1999). The main applications of MOSFET are in VLSI/ULSI technology, which demands to reduce the device dimensions to improve the circuit performance. But, when the device dimensions are reduced by reducing the channel length, the behavior of the MOSFET departs from the long channel behavior, and the term ‘short channel’ comes into light for analyzing its effects in the device performance.

A number of analytical and numerical models on the breakdown of short channel MOSFET have been developed in the last few years (Kennedy and Phillips, 1973; Toyabe *et al.*, 1977; Sun *et al.*, 1978; Hsu *et al.*, 1982, 1983; Muller *et al.*, 1982; Hu *et al.*, 1982; Yang *et al.*, 1999; Skotnicki *et al.*, 2001). According to the physics of the short channel MOSFET device, the drain current in the avalanche multiplication regime consists of four different current components: channel current

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I_{ch}, substrate current I_{sub}, excess channel current ΔI_{ch} and emitter current I_e. But, excess channel current ΔI_{ch} and emitter current I_e are functions of substrate current I_{sub} (Tsividis, 2006). In this paper, the analytical model of substrate current I_{sub} for a short channel MOSFET has been developed, as it is the most influencing factor in determining the drain characteristics of a short channel MOSFET in the avalanche multiplication regime. A computer-aided simulation program using MATLAB is used to analyze the outputs of the substrate current model.

Materials and Methods

Physical model of breakdown in a short channel MOSFET: The breakdown in a short channel MOSFET is not a simple junction avalanche breakdown, rather, it is a breakdown induced by avalanche multiplication. Two consecutive phases of breakdown occur in such a small channel MOSFET (Skotnicki *et al.*, 2001) - first, the rise in drain current is caused by the enhanced body effect (EBE) and then the sharp rise in drain current is initiated by the parasitic bipolar action of the MOSFET.

The physical mechanism of EBE is that the holes, multiplied in the drain junction, bias the substrate spreading resistance when flowing towards the substrate terminal. This bias corresponds to a reduction in the channel region depletion width, and thus, releases a certain amount of bulk charge of ionized impurities, which was initially tied by the gate field. This reduction in bulk charge implies a threshold lowering of the MOSFET and enhancement of inversion layer charge, so as to preserve the overall charge neutrality. This EBE is quite different from the body effect resulting from the applied external positive biasing of the bulk (body), which also causes a reduction in threshold. But the difference between these two body effects results from the fact that the ordinary body effect does not involve any electric field in the bulk, whereas, the EBE requires a non-vanishing electric field in the bulk, necessary to drive the substrate current. In case of EBE, the variation in potential in the bulk no longer disappears along the depth from depletion edge boundary to the substrate terminal and the substrate current in the bulk flows due to drift of the holes rather than diffusion. The holes after multiplication spread towards the bulk and follow the doping distribution $(p = n)$ which makes the gradient of concentration along the depth of the MOSFET zero and leads to zero diffusion current in the bulk.

The mechanism of bipolar induced breakdown due to impact ionization in a short channel MOSFET has been studied by many researchers. Channel electrons that travel through the high-field avalanche region cause impact ionization. The electric field in the high-field region sweeps the electrons generated by impact ionization into the drain, and holes into the bulk. The holes created at the high field avalanche region first causes a reduction in the channel region depletion width, and thus lowers the threshold voltage V_T of the MOSFET, and then the ohmic drop caused by the hole flowing through the high resistivity substrate forward-biases the source junction to initiate the bipolar action of the MOSFET, and hence cause it to inject electrons. Part of these injected electrons travel to the drain junction and will further be multiplied by the drain field, causing positive feedback action. So, after the bipolar turn-on of the MOSFET, the drain current (I_d) consists of channel current (I_{ch}), excess channel current (ΔI_{ch}) due to threshold lowering of the MOSFET and the collector current (I_c) of the bipolar transistor. On the other hand, the hole current (I_m) due to avalanche multiplication,

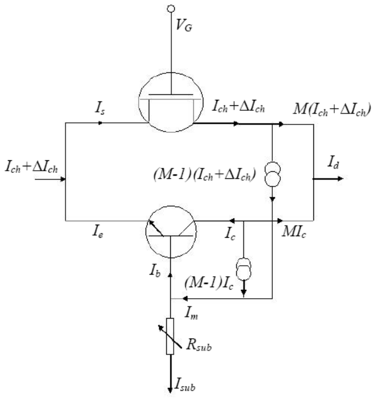

Fig. 1. Equivalent circuit of a short channel MOSFET after the bipolar turn-on.

consists of base current (I_b) and the substrate current (I_{sub}).

From the equivalent circuit (Fig. 1) of a MOSFET after the bipolar turn-on, the necessary equations relating to different current components can be represented as follows:

$$I_d = M(I_c + I_{ch} + \Delta I_{ch}) \tag{1}$$

$$I_m = I_{sub} + I_b = (M-1)(I_c + I_{ch} + \Delta I_{ch}) \tag{2}$$

where, M is the multiplication factor.

Now, manipulating Eqs. (1) and (2), we get,

$$I_m = (1 - \frac{1}{M}) I_d \tag{3}$$

But, the drain current is related to substrate current as follows (from Eqs. 1 and 2):

$$\begin{aligned} I_d &= M(I_c + I_{ch} + \Delta I_{ch}) \\ &= (M-1)(I_c + I_{ch} + \Delta I_{ch}) + (I_c + I_{ch} + \Delta I_{ch}) \\ &= I_m + (I_c + I_{ch} + \Delta I_{ch}) \\ &= I_{sub} + I_b + (I_c + I_{ch} + \Delta I_{ch}) \\ &= I_{sub} + I_e + I_{ch} + \Delta I_{ch} \end{aligned}$$

$$\Rightarrow I_d = I_{sub} + I_s \tag{4}$$

where,

$I_e (= I_b + I_c)$ is the emitter current, and $I_s (= I_e + I_{ch} + \Delta I_{ch})$ is the source current.

So, manipulating the Eqs. (2)-(4), the expression for substrate current can be given by the relation,

$$I_{sub} + I_b = I_m = (1 - \frac{1}{M}) I_d = (1 - \frac{1}{M})(I_{sub} + I_b)$$

$$\Rightarrow I_{sub} = (1 - \frac{1}{M})(I_{sub} + I_s) - I_b \tag{5}$$

If the multiplication factor M can be found, we shall be able to find the substrate current I_{sub}.

Thus, the substrate current is a function of multiplication factor M as shown in Eq. (5). But, the general expression of multiplication factor M (Hasler *et al.*, 1998) is given by

$$1 - \frac{1}{M} = \int_{x_n}^{x_p} \alpha_n \exp\left[- \int_x^{x_p} (\alpha_n - \alpha_p) dx'\right] dx = \int_0^{\Delta L} \alpha_{eff} dx \tag{6}$$

But, $\alpha_{eff} = A_i e^{-\frac{B_i}{g}}$ (7)

where, α_n and α_p are ionization rates for electrons and holes respectively, and, A_i and B_i are ionization coefficients.

In a short channel MOSFET, the substrate current is composed of holes generated by impact ionization in the drain regime due to high lateral electric field.

So, embedding Eq. (7) into Eq. (6), and also considering applied lateral electric field E in y direction, we obtain,

$$1-\frac{1}{M}=\int_0^{\Delta L}\alpha_{eff}\,dy=\int_0^{\Delta L}A_i e^{-\frac{B_i}{g}}\,dy=\int_0^{\Delta L}A_i e^{-\frac{B_i}{g}}\frac{dy}{d\mathrm{E}}d\mathrm{E} \tag{8}$$

Since the lateral electric field E in the channel increases sharply toward the drain and substrate current is very strongly dependent on the field, Eq. (8) reduces approximately to

$$1-\frac{1}{M}\approx\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}A_i\int_{\mathrm{E}_{SAT}}^{\mathrm{E}_{SD}}e^{-\frac{B_i}{g}}\,d\mathrm{E} \tag{9}$$

$$\begin{aligned}
&\approx A_i\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}\left[\mathrm{E}^2+e^{-\frac{B_i}{g}}\left(1-\frac{2}{B_i}\mathrm{E}+6\frac{\mathrm{E}^2}{B_i^2}\right)\right]_{\mathrm{E}_{SAT}}^{\mathrm{E}_{SD}}\\
&\approx\frac{A_i}{B_i}\mathrm{E}_{SD}^2\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}\left(1-2\frac{\mathrm{E}_{SD}}{B_i}+6\frac{\mathrm{E}_{SD}^2}{B_i^2}\right)e^{-\frac{B_i}{g_{SD}}}\\
&-\frac{A_i}{B_i}\mathrm{E}_{SAT}^2\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}\left(1-2\frac{\mathrm{E}_{SAT}}{B_i}+6\frac{\mathrm{E}_{SAT}^2}{B_i^2}\right)e^{-\frac{B_i}{g_{SAT}}}\\
&=(C_1-C_2)
\end{aligned} \tag{10}$$

i.e., $1-\dfrac{1}{M}=C_1-C_2$ (11)

where,

$$C_1=\frac{A_i}{B_i}\mathrm{E}_{SD}^2\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}\left(1-2\frac{\mathrm{E}_{SD}}{B_i}+6\frac{\mathrm{E}_{SD}^2}{B_i^2}\right)e^{-\frac{B_i}{g_{SD}}} \tag{12}$$

and

$$C_2=\frac{A_i}{B_i}\mathrm{E}_{SAT}^2\left|\frac{dy}{d\mathrm{E}}\right|_{\mathrm{E}_{SD}}\left(1-2\frac{\mathrm{E}_{SAT}}{B_i}+6\frac{\mathrm{E}_{SAT}^2}{B_i^2}\right)e^{-\frac{B_i}{g_{SAT}}} \tag{13}$$

Now, the substrate current I_{sub} can be represented as follows by using the Eqs. (5) and (11):

$$I_{sub} = (C_1 - C_2)(I_{sub} + I_s) - I_b \tag{14}$$

However, the lateral electric field E can be related to drain source voltage (Tam *et al.*, 1992) as,

$$\mathrm{E} = \sqrt{A^2 (V - V_{DS}^{'})^2 + \mathrm{E}_{SAT}^2} \tag{15}$$

where,

$$A = \sqrt{\frac{\frac{\varepsilon_{ox}}{\varepsilon_o}}{t_{ox}\left\{d_j + 0.01(V - V_{DS}^{'})(d_l - d_j)\right\}}} \tag{16}$$

and

$$d_l = \sqrt{(\frac{2\varepsilon_o}{qN_{sub}})(V_{DS}^{'} + 2\varphi_f)}\ ;$$

$$\frac{d\mathrm{E}}{dy} = \frac{dV}{dy}\frac{d\mathrm{E}}{dV} = \mathrm{E}\frac{d\mathrm{E}}{dV} \tag{17}$$

(V is the lateral potential along the y direction and $\frac{dV}{dy} = \mathrm{E}$).

By observing the above equations, we find that the multiplication factor M is a strong function of lateral electric field near the drain junction, and thus is a function of drain source voltage V_{DS}, oxide thickness t_{ox}, junction depth d_j and substrate doping N_{sub}. The substrate current can be found from Eq. (14) by knowing the parameters C_1 and C_2. The essential variable required to evaluate these parameters are the electric field E_{SD} and the gradient of electric field $\left|\frac{d\mathrm{E}}{dy}\right|_{\mathrm{E}_{SD}}$ at the drain end of the channel which can be found in the following way (differentiating Eq. (15)):

$$\frac{d\mathrm{E}}{dV} = \frac{d}{dV}\left\{A^2\left(V - V_{DS}^{'}\right)^2 + \mathrm{E}_{SAT}^2\right\}^{1/2} = \frac{\left(V - V_{DS}^{'}\right)}{\mathrm{E}}A^2\left[1 + \left(V - V_{DS}^{'}\right)\frac{1}{A}\frac{dA}{dV}\right]$$

$$\Rightarrow \mathrm{E}\frac{d\mathrm{E}}{dV} = \left(V - V_{DS}^{'}\right)A^2\left[1 + \left(V - V_{DS}^{'}\right)\frac{1}{A}\frac{dA}{dV}\right] \tag{18}$$

Putting $x_1 = \frac{\varepsilon_{ox}}{\varepsilon_o}$ and $y_1 = t_{ox}\left\{d_j + 0.01\left(V - V_{DS}^{'}\right)\left(d_l - d_j\right)\right\}$ in Eq. (16), and then differentiating and manipulating them,

$$\frac{dA}{dV} = -\frac{x_1^{1/2}}{y_1}\frac{d}{dV}\left(y_1^{1/2}\right) = -\frac{\left(x_1^{1/2} / y_1^{1/2}\right)}{2y_1}\frac{d}{dV}\left[t_{ox}\left\{d_j + 0.01\left(V - V_{DS}^{'}\right)\left(d_l - d_j\right)\right\}\right]$$

$$= -\frac{A}{2y_1}t_{ox}\left\{0.01\left(d_l - d_j\right)\right\}$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dV} = -\frac{1}{2y_1}t_{ox}\left\{0.01\left(d_l - d_j\right)\right\} = -bA^2 \qquad (19)$$

where, $b = \frac{1}{2}0.01\left(d_l - d_j\right)\varepsilon_o\frac{t_{ox}}{\varepsilon_{ox}}$

From Eqs. (17)-(19), we get,

$$\frac{d\mathrm{E}}{dy} = \mathrm{E}\frac{d\mathrm{E}}{dV} = \left(V - V_{DS}^{'}\right)A^2\left[1 + \left(V - V_{DS}^{'}\right)\frac{1}{A}\frac{dA}{dV}\right] = \left(V - V_{DS}^{'}\right)A^2\left[1 - bA^2\left(V - V_{DS}^{'}\right)\right]$$

$$\Rightarrow \frac{d\mathrm{E}}{dy} = A^2\left(V - V_{DS}^{'}\right)\left[1 - bA^2\left(V - V_{DS}^{'}\right)\right] \qquad (20)$$

The lateral electric field E_{SD} and the gradient $\left|\frac{d\mathrm{E}}{dy}\right|_{\mathrm{E}_{SD}}$ at the drain end can be found by replacing V in Eqs. (15) and (20) by drain source voltage V_{DS}.

Results

The substrate current I_{sub} can be calculated from the Eq. (14). However, the substrate current I_{sub} depends on the lateral electric field E_{SD}, which can be calculated or modeled from the Eqs. (15) and (20).

The effect of drain source voltage V_{DS} on lateral electric field E_{SD} has been depicted in Fig. 2, which was obtained by a computer-aided simulation program in the MATLAB platform following the concepts of the Eqs. (15)-(20).

On the other hand, Figs. 3 and 4 show the substrate current (I_{sub}) vs. drain source voltage (V_{DS}) characteristics for V_{GS} = 3V and V_{GS} = 4V, respectively, which were obtained from the Eqs. (14) and (15) by using MATLAB software.

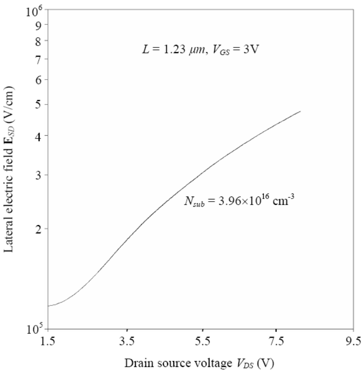

Fig. 2. Effect of drain source voltage on lateral electric field.

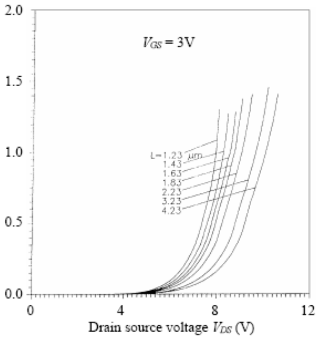

Fig. 3. Substrate current vs. drain source voltage characteristics for V_{GS} = 3V.

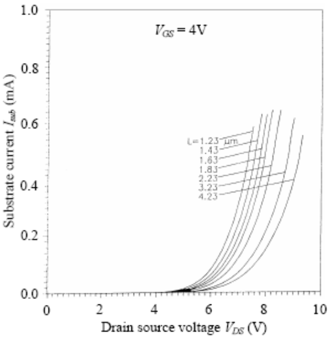

Fig. 4. Substrate current vs. drain source voltage characteristics for V_{GS} = 4V

Discussion

Lateral electric field in a short channel MOSFET is a strong function of drain source voltage V_{DS}. The electric field E_{SD} near drain end of the channel at pinch-off or saturation point is given by E_{SAT} and must increase with V_{DS} beyond pinch-off. This can be explained from Eq. (15). For a fixed channel length (L) and substrate doping N_{sub}, V_{DS}' (typically 1-2V below the drain source voltage V_{DS}, so that the substrate current can be detected) is fixed. Although the parameter A decreases with V_{DS} but the increase of the term ($V_{DS} - V_{DS}'$) with V_{DS} overcompensates the decrease and ultimately the product of the two increases with V_{DS}. So, the electric field E_{SD} increases with V_{DS} and the plot shown in Fig. 2 agrees well with the fact.

Figs. 3 and 4 show the variation of substrate current I_{sub} with V_{DS} for two different values of V_{GS} (3V and 4V, respectively) for a series of MOSFET differing only in channel lengths (L). These plots show that the substrate current increases with V_{DS} which is quite obvious and can be explained from Eqs. (14) and (15). The substrate current is a strong function of multiplication factor M as shown from Eqs. 5 and (14). The multiplication factor M on the other hand increases with V_{DS} due to the increase of E_{SD} with V_{DS} as given by Eqs. (15) and (20). These facts justify the increase of substrate current with V_{DS}. The effect of V_{GS} on substrate current can be explained by considering two conflicting factors. The substrate current I_{sub} depends on both ionization rate α_{eff} and channel current I_{ch} (Sze, 2006). When V_{GS} increases I_{ch} increases, but α_{eff} decreases due to the decrease of electric field E_{SD}. The decrease in E_{SD} is due to the increase of V_{DS}' with V_{GS}. The increase in channel current tends to increase the substrate current I_{sub}, whereas, the decrease in α_{eff} tends to decrease the substrate current I_{sub}. Ultimately, the substrate current for a fixed drain source voltage is determined by the factor of the two (i.e., α_{eff} and I_{ch}) which dominates the other.

Conclusion

Based on basic physical effects of short channel MOSFETs, an accurate analytical physical substrate current model has been developed. The results based on the model show that the multiplication induced substrate current is the main reason that leads to the breakdown of the short channel MOSFET. The major components of the drain current that contribute to these breakdown phenomena are excess channel current and emitter current, both of which are determined by the substrate current. But, the substrate current again depends on the source current and the multiplication factor M , which is a strong function of electric field E_{SD} at the drain end. Knowing the substrate current I_{sub} , the excess channel current ΔI_{ch} and the emitter current I_e and finally, the drain characteristics can be obtained. Being analytic model, it can be calculated faster and used in VLSI circuit and device simulation. Furthermore, the model being physically based, it can also be scaled down and predict characteristics of diminished devices as well.

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