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RESEARCH ARTICLE PHYSICAL SCIENCE

HIGHER ORDER IMPROVED SOLUTION OF SECOND ORDER OVER-DAMPED NON-LINEAR SYSTEMS

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Abstract: A second order nonlinear differential system modeling non-oscillatory processes by over damping is considered. Then second order approximate solution is found by means of an extension of the Krylov-Bogoliubov-Mitropolskii (KBM) method. The method is illustrated by an example. The solutions for different initial conditions show a good agreement with those obtained by numerical solution.

Key words: Perturbation, asymptotic solutions, over-damping, non-linear system

Introduction

The asymptotic method of Krylov and Bogoliubov (1947) is one of the widely used techniques to study weakly nonlinear systems. The method was originally developed for obtaining periodic solutions of second order ordinary differential equation with small nonlinearities. Then the method was amplified and justified by Bogoliubov and Mitropolskii (1961) and later extended by Popov (1965) to damped nonlinear system. It is noteworthy that because of the importance of physical processes involving damping, Popov's results have later been rediscovered by Bojadziev (1983) and Mendelson (1970). Murty *et al*. (1969) has studied second order over-damped nonlinear systems in the sense of Krylov and Bogoliubov's method. Murty (1971) has also presented a unified theory to study second order nonlinear oscillatory, damped oscillatory and non-oscillatory systems. Sattar (1986) has investigated an asymptotic solution of a second order critically damped nonlinear equation.

In all the papers (Krylov and Bogoliubov, 1947; Bogoliubov and Mitropolskii, 1961; Popov, 1965; Murty *et al*., 1969; Mendelson, 1970; Murty, 1971; Bojadziev, 1983; Sattar, 1986) there have been investigated only the first order of small nonlinearity (considered up to ε) improved solution. Second order of small nonlinearity improved solutions (considered up to ε^2) are not investigated. In this paper we have investigated second order of small

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nonlinearity improved solution i.e. considered both ε and ε^2 for second order over-damped nonlinear systems.

Materials and Methods

Consider a weakly nonlinear over-damped system is governed by the differential equation

$$\ddot{x} + k_1 \dot{x} + k_2 x = -\varepsilon\, f(x, \dot{x}) \tag{1}$$

where over-dots denote differentiation with respect to *t*, ε is a small parameter, *f* is the given nonlinear function and k_1, k_2 are constants defined by $k_1 = \lambda_1 + \lambda_2$ and $k_2 = \lambda_1 \lambda_2$ where $-\lambda_1$ and $-\lambda_2$ are the real negative eigen-values of the characteristic equation of the equation (1) for $\varepsilon = 0$. It is noted that equation (1) is like to the equation of Murty *et al.* (1969). The over damping force in the system is represented by these real negative eigen-values. When $\varepsilon = 0$, the solution of correspond linear equation is

$$x(t,0) = a_{1,0} e^{-\lambda_1 t} + a_{2,0}\, e^{-\lambda_2 t} \tag{2}$$

where $a_{1,0}$, $a_{2,0}$ are arbitrary constants.

When $\varepsilon \neq 0$, following Alam (2002) the solution of the nonlinear differential equation (1) is sought in the form

$$x(t,\varepsilon) = a_1\, e^{-\lambda_1 t} + a_2\, e^{-\lambda_2 t} + \varepsilon\, u_1(a_1, a_2, t) + \varepsilon^2\, u_2(a_1, a_2, t) + \ldots \tag{3}$$

where each a_1, a_2 satisfy the differential equation

$$\dot{a}_j(t) = \varepsilon\, A_j(a_1, a_2, t) + \varepsilon^2\, B_j(a_1, a_2, t) + \ldots \tag{4}$$

Confining only to the first few terms $1, 2, 3, \ldots, m$ in the series expansions of (3) and (4), we evaluate the functions $u_1, u_2, \ldots$ and A_j, B_j, $j = 1,2$ such that $a_j(t)$ appearing in (3) and (4) satisfy the given differential equation (1) with an accuracy of ε^{n+1}. Theoretically, the solution can be obtained up to the accuracy of any order of approximation. However, owing to the rapidly growing algebraic complexity for the derivation of the formulae, the solution is in general confined to a lower order, usually the first Murty (1971). In this paper, we have derived the formula for second order approximation.

Differentiating (3) twice with respect to *t*, substituting x and the derivatives $\dot{x}, \ddot{x}$ in the original equation (1), utilizing the relations in (4) and finally equating the coefficient of ε and ε^2, we obtain

$$e^{-\lambda_1 t}\left(\frac{\partial}{\partial t} + \lambda_2 - \lambda_1\right) A_1 + e^{-\lambda_2 t}\left(\frac{\partial}{\partial t} + \lambda_1 - \lambda_2\right) A_2 + \left(\frac{\partial}{\partial t} + \lambda_1\right)\left(\frac{\partial}{\partial t} + \lambda_2\right) u_1 = -f^{(0)}(a_1, a_2, t) \tag{5}$$

and

$$\begin{aligned} & e^{-\lambda_1 t}\left(\frac{\partial}{\partial t} + \lambda_2 - \lambda_1\right) B_1 + e^{-\lambda_2 t}\left(\frac{\partial}{\partial t} + \lambda_1 - \lambda_2\right) B_2 \\ & + \left(\frac{\partial}{\partial t} + \lambda_1\right)\left(\frac{\partial}{\partial t} + \lambda_2\right) u_2 = -\varepsilon\, f^{(1)}(a_1, a_2, t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} f^{(1)} &= u_1 f_x\left(a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t}, -\lambda_1 a_1 e^{-\lambda_1 t} - \lambda_2 a_2 e^{-\lambda_2 t}\right) \\ &+ \left(A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t} + \frac{\partial u_1}{\partial t}\right) f_{\dot{x}}\left(a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t}, -\lambda_1 a_1 e^{-\lambda_1 t} - \lambda_2 a_2 e^{-\lambda_2 t}\right) \\ &- \left[\left(A_1 \frac{\partial}{\partial a_1} + A_2 \frac{\partial}{\partial a_2}\right)\left(\frac{\partial}{\partial t} + k_1\right) + \frac{\partial}{\partial t}\left(A_1 \frac{\partial}{\partial a_1} + A_2 \frac{\partial}{\partial a_2}\right)\right] u_1 \\ &- \left(A_1 \frac{\partial}{\partial a_1} + A_2 \frac{\partial}{\partial a_2}\right)\left(A_1 e^{-\lambda_1 t} + A_2 e^{-\lambda_2 t}\right) \end{aligned}$$

Now $f^{(0)}$ and $f^{(1)}$ can be expressed in a Taylor series as

$$f^{(0)} = \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} F_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t}$$

and $$f^{(1)} = \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} G_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t}$$

Thus the equations (5) and (6) can be written as

$$\begin{aligned} & e^{-\lambda_1 t}\left(\frac{\partial}{\partial t} + \lambda_2 - \lambda_1\right) A_1 + e^{-\lambda_2 t}\left(\frac{\partial}{\partial t} + \lambda_1 - \lambda_2\right) A_2 + \left(\frac{\partial}{\partial t} + \lambda_1\right)\left(\frac{\partial}{\partial t} + \lambda_2\right) u_1 \\ & \quad = - \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} F_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t} \end{aligned} \tag{7}$$

$$\begin{aligned} & e^{-\lambda_1 t}\left(\frac{\partial}{\partial t} + \lambda_2 - \lambda_1\right) B_1 + e^{-\lambda_2 t}\left(\frac{\partial}{\partial t} + \lambda_1 - \lambda_2\right) B_2 \\ & + \left(\frac{\partial}{\partial t} + \lambda_1\right)\left(\frac{\partial}{\partial t} + \lambda_2\right) u_2 = - \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} G_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t} \end{aligned} \tag{8}$$

According to our assumption u_1, u_2 do not contain the fundamental terms, therefore equation (7) can be separated into three equations for unknown functions u_1, A_1 and A_2 (see (2002) also for details).

$$e^{-\lambda_1 t}\left(\frac{\partial}{\partial t} + \lambda_2 - \lambda_1\right) A_1 = - \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} F_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t}; \qquad m_1 = m_2 + 1 \tag{9}$$

$$e^{-\lambda_2 t}\left(\frac{\partial}{\partial t} + \lambda_1 - \lambda_2\right) A_2 = - \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty} F_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t}; \qquad m_1 = m_2 - 1 \tag{10}$$

and

$$\left(\frac{\partial}{\partial t} + \lambda_1\right)\left(\frac{\partial}{\partial t} + \lambda_2\right) u_1 = - \sum_{m_1=-\infty,\, m_2=-\infty}^{\infty,\infty}{}' F_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1 \lambda_1 + m_2 \lambda_2) t} \tag{11}$$

Again since u_1, u_2 do not contain the fundamental terms, so the equation (8) can also be separated into three equations for unknown functions u_2, B_1 and B_2.

$$e^{-\lambda_1 t}\left(\frac{\partial}{\partial t}+\lambda_2-\lambda_1\right)B_1=-\sum_{m_1=-\infty,\,m_2=-\infty}^{\infty,\infty} G_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1\lambda_1+m_2\lambda_2)t}; \qquad m_1=m_2+1 \tag{12}$$

$$e^{-\lambda_2 t}\left(\frac{\partial}{\partial t}+\lambda_1-\lambda_2\right)B_2=-\sum_{m_1=-\infty,\,m_2=-\infty}^{\infty,\infty} G_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1\lambda_1+m_2\lambda_2)t}; \qquad m_1=m_2-1 \tag{13}$$

and

$$\left(\frac{\partial}{\partial t}+\lambda_1\right)\left(\frac{\partial}{\partial t}+\lambda_2\right)u_2=-{\sum_{m_1=-\infty,\,m_2=-\infty}^{\infty,\infty}}' G_{m_1,m_2}\, a_1^{m_1}\, a_2^{m_2}\, e^{-(m_1\lambda_1+m_2\lambda_2)t} \tag{14}$$

where $\sum'$ exclude those terms for $m_1=m_2\pm1$.

The particular solution of equations (9)-(14) give the values of A_1, A_2, B_1, B_1, u_1 and u_2. Substituting the values of A_1, A_2, B_1, B_1 into the equation (4) and integrating we shall get the value of a_1 and a_2. Finally substituting the values of a_1, a_2, u_1 and u_2 into equation (3) we shall get the value of x.

Thus the determination of the second order approximate solution is complete.

Example

As an example of the above procedure, we have considered the Duffing's equation

$$\ddot{x}+k_1\dot{x}+k_2x=-\varepsilon x^3 \tag{15}$$

For equation (15), the equation (5) and (6) become

$$\begin{aligned}e^{-\lambda_1 t}\left(\frac{\partial}{\partial t}+\lambda_2-\lambda_1\right)A_1+e^{-\lambda_2 t}\left(\frac{\partial}{\partial t}+\lambda_1-\lambda_2\right)A_2+\left(\frac{\partial}{\partial t}+\lambda_1\right)\left(\frac{\partial}{\partial t}+\lambda_2\right)u_1\\ =-\left\{a_1^3e^{-3\lambda_1 t}+3a_1^2a_2e^{-(2\lambda_1+\lambda_2)t}+3a_1a_2^2e^{-(\lambda_1+2\lambda_2)t}+a_2^3e^{-3\lambda_2 t}\right\}\end{aligned} \tag{16}$$

$$\begin{aligned}e^{-\lambda_1 t}\left(\frac{\partial}{\partial t}+\lambda_2-\lambda_1\right)B_1+e^{-\lambda_2 t}\left(\frac{\partial}{\partial t}+\lambda_1-\lambda_2\right)B_2+\left(\frac{\partial}{\partial t}+\lambda_1\right)\left(\frac{\partial}{\partial t}+\lambda_2\right)u_2\\ =-\left(3a_1^2e^{-2\lambda_1 t}+6a_1a_2e^{-(\lambda_1+\lambda_2)t}+3a_2^2e^{-2\lambda_2}\right)u_1\\ -\left[\left(A_1\frac{\partial}{\partial a_1}+A_2\frac{\partial}{\partial a_2}\right)\left(\frac{\partial}{\partial t}+k_1\right)+\frac{\partial}{\partial t}\left(A_1\frac{\partial}{\partial a_1}+A_2\frac{\partial}{\partial a_2}\right)\right]u_1\\ -\left(A_1\frac{\partial}{\partial a_1}+A_2\frac{\partial}{\partial a_2}\right)\left(A_1e^{-\lambda_1 t}+A_2e^{-\lambda_2 t}\right)\end{aligned} \tag{17}$$

Therefore, the equation (9)-(11) respectively become

$$e^{-\lambda_1 t}\left(\frac{\partial}{\partial t}+\lambda_2-\lambda_1\right)A_1=-3a_1^2a_2e^{-(2\lambda_1+\lambda_2)t} \tag{18}$$

$$e^{-\lambda_2 t}\left(\frac{\partial}{\partial t}+\lambda_1-\lambda_2\right)A_2=-3a_1a_2^2e^{-(\lambda_1+2\lambda_2)t} \tag{19}$$

$$\left(\frac{\partial}{\partial t}+\lambda_1\right)\left(\frac{\partial}{\partial t}+\lambda_2\right)u_1=-a_1^3e^{-3\lambda_1 t}-a_2^3e^{-3\lambda_2 t} \tag{20}$$

Now solving equations (18)-(20) and substituting $\lambda_1=k-\omega$ and $\lambda_2=k+\omega$, we obtain

$$A_1=\frac{3a_1^2a_2e^{-2kt}}{2(k-\omega)} \tag{21}$$

$$A_2=\frac{3a_1a_2^2e^{-2kt}}{2(k+\omega)} \tag{22}$$

$$u_1=-\frac{a_1^3e^{-3(k-\omega)t}}{4(k-\omega)(k-2\omega)}-\frac{a_2^3e^{-3(k+\omega)t}}{4(k+\omega)(k+2\omega)} \tag{23}$$

Substituting the value of u_1, A_1 and A_2 from equations (21)-(23) into the equation (17) and solving for B_1, B_2, u_2 and finally substituting $\lambda_1=k-\omega$ and $\lambda_2=k+\omega$, we obtain

$$B_1=\frac{3a_1^3a_2^2e^{-4kt}}{8(k-\omega)\left\{9(k-\omega)^2-(k+\omega)^2\right\}} \tag{24}$$

$$B_2=\frac{a_1^2a_2^3e^{-4kt}}{8(k+\omega)\left\{9(k+\omega)^2-(k-\omega)^2\right\}} \tag{25}$$

$$\begin{aligned}u_2=&\frac{3a_1^5e^{-5(k-\omega)t}}{8(k-\omega)^2(k-2\omega)(2k-3\omega)}+\frac{3a_1^4a_2e^{-(5k-3\omega)t}}{4(k+\omega)^2(k+2\omega)(2k+3\omega)}\\&+\frac{3a_1a_2^4e^{-(3\omega+5k)t}}{4(k-\omega)^2(k-2\omega)(2k-\omega)}+\frac{3a_2^5e^{-5(k+\omega)t}}{8(k+\omega)^2(k+2\omega)(2k+\omega)}\end{aligned} \tag{26}$$

Substituting the values of A_1, A_2, B_1 and B_2 from equations (21), (22), (24) and (25) into equation (4), we obtain

$$\begin{aligned}\dot{a}_1&=\varepsilon\frac{3a_1^2a_2e^{-2kt}}{2(k-\omega)}+\varepsilon^2\frac{3a_1^3a_2^2e^{-4kt}}{8(k-\omega)\left\{9(k-\omega)^2-(k+\omega)^2\right\}}\\\dot{a}_2&=\varepsilon\frac{3a_1a_2^2e^{-2kt}}{2(k+\omega)}+\varepsilon^2\frac{a_1^2a_2^3e^{-4kt}}{8(k+\omega)\left\{9(k+\omega)^2-(k-\omega)^2\right\}}\end{aligned} \tag{27}$$

Substituting $a_1=\frac{a}{2}e^{\varphi_1}$ and $a_2=\frac{a}{2}e^{-\varphi_1}$ into equations (23), (26) and (27) and then simplifying, we obtain

$$\begin{aligned}\dot{a}&=\varepsilon l_1a^3e^{-2kt}+\varepsilon^2l_2a^5e^{-4kt}\\\dot{\varphi}&=\varepsilon m_1a^2e^{-2kt}+\varepsilon^2m_2a^4e^{-4kt}\end{aligned} \tag{28}$$

$$u_1 = a^3 e^{-3kt}\{p_1 \cosh 3(\omega t+\varphi) + p_2 \sinh 3(\omega t+\varphi)\} \tag{29}$$

and

$$u_2 = 3a^5 e^{5k_1 t}\begin{Bmatrix} n_1 \cosh 3(\omega t+\varphi) + n_2 \sinh 3(\omega t+\varphi) \\ + n_3 \cosh 5(\omega t+\varphi) + n_4 \sinh 5(\omega t+\varphi) \end{Bmatrix} \tag{30}$$

where

$$l_1 = \frac{3k}{8(k^2-\omega^2)}, \qquad l_2 = \frac{3\left(16k^5 - 59k^3\omega^2 - 65k\omega^4\right)}{128(k^2-\omega^2)(k+\omega)^2(4k^2-\omega^2)(k^2-4\omega^2)}$$

$$m_1 = \frac{3\omega}{8(k^2-\omega^2)}, \qquad m_2 = \frac{3(26k^4\omega - 124k^2\omega^3 - 10\omega^5)}{128(k^2-\omega^2)(k+\omega)^2(4k^2-\omega^2)(k^2-4\omega^2)}$$

$$p_1 = \frac{k^2+2\omega^2}{16(k^2-\omega^2)\,(k^2-4\omega^2)}, \qquad p_2 = \frac{3kw}{16(k^2-\omega^2)\,(k^2-4\omega^2)}$$

$$n_1 = \frac{-\left(20k^6 + 153k^4\omega^2 - 51k^2\omega^4 - 14\omega^6\right)}{512(k^2-\omega^2)^2\,(k^2-4\omega^2)(k^2-4\omega^2)(4k^2-\omega^2)}$$

$$n_2 = \frac{-\left(96k^5 w + 69k^3\omega^3 - 57k\omega^5\right)}{512(k^2-\omega^2)^2\,(k^2-4\omega^2)(k^2-4\omega^2)(4k^2-\omega^2)}$$

$$n_3 = \frac{2\left(2k^4 + 11k^2\omega^2 + 3\omega^4\right)}{512(k^2-\omega^2)^2\,(k^2-4\omega^2)(4k^2-9\omega^2)}$$

$$n_4 = \frac{2k^3\omega + 19k\omega^3}{512(k^2-\omega^2)^2\,(k^2-4\omega^2)(4k^2-9\omega^2)}$$

Equation (28) has no exact solution. Since $\dot{a}$ and $\dot{\varphi}$ are proportional to small parameter ε, therefore $\dot{a}$ and $\dot{\varphi}$ are slowly varying functions of time *t*, with a period T. So, we may consider them as constants in the right hand side of the equation (28) (see also (1969) for details)

$$\begin{aligned} a &= a_0 + \varepsilon \frac{l_1 a_0^3\left(1-e^{-2kt}\right)}{2k} + \varepsilon^2 \frac{l_2 a_0^5\left(1-e^{-4kt}\right)}{4k} \\ \varphi &= \varphi_0 + \varepsilon \frac{m_1 a_0^2\left(1-e^{-2kt}\right)}{2k} + \varepsilon^2 \frac{m_2 a_0^4\left(1-e^{-4kt}\right)}{4k} \end{aligned} \tag{31}$$

Therefore, we obtain the second order approximate solution of the equation (15) as

$$x(t,\varepsilon) = a e^{-kt}\cosh(\omega t+\varphi) + \varepsilon\, u_1 + \varepsilon^2\, u_2 \tag{32}$$

where a and φ are given by the equation (31), u_1 and u_2 given by (29) and (30) respectively.

Results

It is usual to compare the perturbation solution to the numerical solution to test the accuracy of the approximate solution. For $k = 5.0$, $\omega = 1.0$, and $\varepsilon = 0.1$, we have computed $x(t, \varepsilon)$ by (32) in which a, φ are computed by (31) with initial conditions $x(0) = 1.015764$, $\dot{x}(0) = -4.898804$ [or $a_0 = 1.0$, $\varphi(0) = 0.17453$]. The solutions for various values of *t* are presented in the Table 1.

Again, for $k = 5.0$, $\omega = 1.0$ and $\varepsilon = 0.1$, we have computed $x(t, \varepsilon)$ by (32) in which a, φ are computed by (31) with initial conditions $x(0) = 1.255536$ $\dot{x}(0) = -6.158840$ or [or $a_0 = 1.25$, $\varphi(0) = 0.08726$]. The solutions for various values of *t* are presented in the Table 2.

Table 1. Comparison between perturbation and numerical results when $a_0 = 1.0$, $\varphi(0) = 0.17453$.

t	*x*	x^*	*E%*
0.0	1.015764	1.015764	0.00000
0.5	0.101571	0.101301	0.26665
1.0	0.011957	0.011900	0.47899
1.5	0.001529	0.001520	0.59210
2.0	0.000202	0.000201	0.49751
2.5	0.000027	0.000027	0.00000
3.0	0.000004	0.000004	0.00000
3.5	0.000000	0.000000	0.00000
4.0	0.000000	0.000000	0.00000
4.5	0.000000	0.000000	0.00000
5.0	0.000000	0.000000	0.00000

x computed by (32); *x** is computed by *Runge-Kutta* method.

Table 2. Comparison between perturbation and numerical results when $a_0 = 1.25$, $\varphi(0) = 0.08726$.

t	*x*	x^*	*E%*
0.0	1.255536	1.255536	0.00000
0.5	0.120980	0.120546	0.36002
1.0	0.013931	0.013841	0.65024
1.5	0.001764	0.001750	0.80000
2.0	0.000233	0.000231	0.86580
2.5	0.000031	0.000031	0.00000
3.0	0.000004	0.000004	0.00000
3.5	0.000001	0.000001	0.00000
4.0	0.000000	0.000000	0.00000
4.5	0.000000	0.000000	0.00000
5.0	0.000000	0.000000	0.00000

x computed by (32); *x** is computed by *Runge-Kutta* method.

Discussion

From the Table 1, by using the initial conditions $a_0 = 1.0$, $\varphi(0) = 0.17453$, we see that our asymptotic solutions are near to numerical solutions. Again, from Table 2, by using another set of initial conditions $a_0 = 1.0$, $\varphi(0) = 0.08726$, we observed that our asymptotic solutions also show good coincidence with corresponding numerical solutions. The results depend on initial phase angle φ. To clarify this fact, in table 1, we have taken $a_0 = 1.0$, $\varphi(0) = 0.17453$ and for table 2 $a_0 = 1.25$, $\varphi(0) = 0.08726$. From table 1, we see that the errors are smaller than the errors of table 2.

Conclusion

An asymptotic method, based on the theory of KBM, is developed in this paper for transient response of a nonlinear system governed by a second order ordinary differential equation, when two characteristic roots of the corresponding linear equation are all real and negative. The solutions obtained by this method are very near to the numerical solutions *i.e.* the solutions obtained by this method show good coincidence with corresponding numerical values.

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