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MEASURING CHILD MORTALITY IMPACT BY COX PROPORTIONAL HAZARD MODEL WITH TWO TIME SCALES

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Abstract: In most demographic surveys, demographer measures child mortality on the basis of time scale (lifetime) but here we introduce dual time number (lifetime and calendar time) in survival analysis. The two-way Cox proportional hazard model is used to compare as well as relate them with a symmetric maximum likelihood that avoids necessitate of primary scale. Finally, the scrutiny indicates, the time point influence both the lifetime of the children as well as the period when birth occurs.

Keywords: Dual time number, proportional hazard model, symmetric MLF

Introduction

Regression methods have become vital concerns with investing the relationship between a response variable and one or more explanatory variables. Since 1960, biostatisticians have become interested in developing mathematical models for investigating the relationship between survival time and patient characteristics (possibly prognostic and risk variables). In this reason, early days statisticians tried to adapt multiple regression method for use with censored survival data. Glasser (1967) proposed a log linear regression model for censored survival data. There he introduced exponential model with covariance adjustments and identifying important prognostic and risk factors. Prentice (1973) investigated this model from the point of view of Fraser's (1968) structural inference and extended to the case of multiple covariates in a classical way by Breslow (1974). Lehman (1953) proposed regression models for survival distributions generally engage proportional hazard function. Finally, Cox (1972) introduced a general non-parametric model appropriate for the analysis of survival data with or without censoring named Cox proportional hazards model. Cox and Oakes (1984) and Kalbfleisch and Prentice (1980) provided the exhaustive introduction to proportional hazards model. Normally, a parametric model is used if the failure time follows a known parametric model but if the distribution is unknown, Cox model will give a reliable answer. Another reason for choosing the Cox model is that the model is a robust model, i.e., result of the model will approximately close for the correct parametric model. Since Cox proportional hazards model is expressed as an expression for the hazards model at a specific time for an individual with a given specification of a set of explanatory variables. An important advantage of Hazards model over other regression model is being able to accommodate censored observation and time varying covariates in analysis of duration or event history data.

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Ample studies show that infant mortality is high among the first born, but relatively less among second and the third order births and that it goes up later with increasing order of births(Chandrasekher,1972).Other factors which influence the child mortality are maternal height and weight, immunization status, delivery practice and infant weight. Mortality levels and trends are considered important indicators of the well being and health of the population. Generally, infant and child mortality rate are still high in Bangladesh. The study of child mortality in developing countries is an important issue in public health and development programs. Considering the importance of analyzing the child mortality in Bangladesh, lot of works have been done so far; but the scope of further works on this particular topic still persist. Some of the works concerning the child mortality are mentioned hereby, such as Cleland and Sathar (1984) described the effect of birth spacing on child mortality in Pakistan. Hobcraft *et al.* (1985) worked in demographic determinants of infant and early child mortality. Pollani and Tienda (1986) described the effect of breastfeeding and place of childbearing on mortality at early ages. Mosley and Chen (1984) acknowledge all social and economic determinants of child mortality necessarily operate common set of proximate determinants to exert an impact of mortality. Bairagi *et al.* (1999) briefly described levels, trends and determinants of child mortality Matlab in Bangladesh. Kabir et al. (2001) attempted to identify important factors influencing infant and child mortality. Their analysis was conducted by means of Cox's proportional hazards model which showed that socio- economic status of the parents was associated with child mortality. The aim of this paper is to analyze child mortality using Cox proportional hazard model with two time points- lifetime and the calendar time and find the impact on both lifetime and calendar time on child mortality which may help the policy maker to hasten the mortality decline.

Materials and methods

Study Area: The child survival data from Bangladesh Demographic and Heath Survey (BDHS, 2007) is analyzed in this study. This survey was conducted under the authority of the national Institute of Population Research and Training of the ministry of Health and Family Welfare. The survey was implemented by Mitra and Associates, a Bangladeshi research farm located in Dhaka. ORC Macro of Calverton, Maryland, USA provided technical assistance to the project as a part of its International Demographic and Health Surveys program and financial assistance was provided by US Agency for International Development (US-Aid), Bangladesh. Data collection took place over a five-month period from 24 March to 11 August 2007. This survey included ever-married women aged 10-49 and ever-married men aged 15-54. However, the number of ever-married women aged 10-14 was very low and thus this group had to be excluded from the analysis. The survey was designed to obtain 11,485 completed interviews with ever-married women aged 10-49. According to the sample design 4,360 interviews were allocated to urban areas and 7,125 to rural areas. All ever-married women aged 10-49 in selected households were eligible respondents for the women's questionnaire. In addition, ever-married men aged 15-54 in every second were eligible to be interviewed. In the BDHS, clusters or enumeration areas (EAs), which were created for the 2001 census based on a convenient number of dwelling units, were used as primary sampling unit (PSUs). In each division, the list of clusters constituted the corresponding sample frame. A multistage sampling procedure was used to select 361 clusters from the total of 550 clusters.

The covariates considered in our analysis can be divided into two categories as- child's variable and the mother's variable. The variable lifetime U_i, used in the study is the children age at death in month and the variable V_i, indicates the calendar time which is the elapsed time of death from the beginning of the study. Though the child survival data are available from 1964 but for some limitations we consider here the cases from 1985. The children enter into the study with the birth and the date of birth of children obtained from the data as denoted by $b_i = b_i - 1020$, the study data starts from 1985 which is 1020 month later since Janaury'1900 according to CMC calculated. Then the calendar date is considered as $v_i = u_i + b_i$. Some of the most important covariates such as breast feeding is ignored as data is not found for all fifteen children. So, these variables consist a standard set of proximate determinants of children mortality. The summary statistics of the variables (Proximate determinant of child mortality) is presented in the following table (See Appendix, Table 1)

In Table 1, we see that percentage of male and female among the selected children is almost same. The mean age at death is 12.59 months in last 20 years. Mean age of mothers at their first birth is 17.19 years and 32.18 is the mean age of mothers at the interview period. The result added in the table also indicates that a large portion of the respondents come from rural area, which is around 69% and largest portion of the respondents come from Dhaka division.

Survival analysis usually analyses data using statistical tools, considering random variable of interest is 'time until an event occurs'. More generally, survival analysis involves the modeling of time to event data, in this study, death is considered an event.

Hazard and Survival Function

Let *T* be the time of death and *t* be some specific time period of death, '*Pr*' be stands for probability and *S(t)* be the survival function. If *T* is a non-negative random variable denoting the lifetime of the individuals, with $T = \min(T_e, T_c)$, T_e is the failure or death time and T_c is the censored time, $\delta = I\{T_e < T_c\}$ the indicator has a value of 1 if the event observed or 0 if it censored. Then the generic survival data is presented in the form (T, δ). Most often, we are also interested in including covariates in the data. Then the survival data becomes (T, δ, Z), where $Z = (Z_1, Z_2, \ldots\ldots, Z_n)^{/}$, is a *P*-dimensional vector of covariates, *T* is a non-negative random variable that denotes the time until the occurrence of some specified event and can be continuous or discrete. Considering the continuous case, there are many functions that describe the distribution of *t*, the cumulative distribution function $F(t) = P(T \leq t)$ and the density function $f(t) = \frac{dF(t)}{dt}$ are the usual functions, characterizing functions of a random variable. The survivor function is

$$S(t) = 1 - F(t) = I(T \geq t) \tag{1}$$

means the probability of the duration time (e.g., lifetime) being longer than t and the hazard function is

$$\lambda(t) = \lim_{\Delta \to 0} \frac{P(t \leq T \leq t + \Delta t)}{\Delta t} \tag{2}$$

i.e., the probability of getting an event (e.g., death) within a short interval, conditional upon survival to time t. Applying conditional probability and the relations between $F(t)$, $f(t)$ and $S(t)$, the relation between $\lambda(t)$ and $S(t)$ can be derived as

$$\lambda(t) = \frac{dF(t)}{dt}\frac{1}{S(t)} = \frac{f(t)}{s(t)} \tag{3}$$

It also follows that $\lambda(t) = -\frac{d}{dt}\log S(t)$ and $S(t) = -\exp(\Lambda(t)),\ \Lambda(t)\int_0^t \lambda(u)du$ is the integrated or cumulative hazard function. Fleming and Lin (2000) observed (T,δ) rather than T_e and net hazard $\lambda_{nct} = \lim_{\Delta t\to 0} \frac{P(t \le T \le t+\Delta t \mid T_e > t)}{\Delta t}$.So, the equality of crude hazard and net hazard is an important assumption in survival analysis.

The Cox's Proportional Hazards Model

Cox proportional hazards regression model (Cox, 1972) is the most frequently used regression model in survival analysis. In Cox's model, it is assumed that survival times are available along with measurements on one or more covariates and hazard function is taken to be a function of the explanatory variables as well as unknown regression coefficients multiplied by an arbitrary unknown function of time. Then, if $\lambda_1(t;x_1)$ and $\lambda_2(t;x_2)$ are the hazard function for two individual with covariate vectors x_1 and x_2 respectively, then the ratio of those two hazard functions does not vary with time *t*. This implies that the hazard function of *T* given *x* can be (Kalbfleisch and Prentice, 1980) in the following form

$$\lambda(t;x) = \lambda_0(t)c(x) \tag{4}$$

where, $\lambda_0(t)$ is an arbitrary hazard function for an individual and $c(x)$ be a known function of *x* that contains parameters of interest. Under this model $\frac{\lambda_1(t;x_1)}{\lambda_2(t;x_2)} = \frac{\lambda_0(t)c(x_1)}{\lambda_0(t)c(x_2)} = \frac{c(x_1)}{c(x_2)}$, is independent of time t. If Cox's regression model may be considered as a non-parametric model then let $\lambda(t,z)$ represents the hazard function at time *t* for an individual with covariates $Z = (Z_1, Z_2, \ldots\ldots, Z_n)'$ and proportional hazards model (Cox, 1972) specifies that

$$\lambda(t,z) = \lambda_0(t)\exp(Z'\beta) \tag{5}$$

where, $\lambda_0(t)$ is called the baseline hazard function. The second quality is the exponential expression to the linear sum of $\beta_i Z_i$, here the sum is over the p explanatory *Z* variables. It is important to know the baseline hazard is a function of *t*, but doesn't involve *Z*. The value of any hazard function must range between 0 and ∞.

Stratified Proportional Hazards Model

The proportional hazards model requires that any two covariates, say x_1 and x_2 , the hazard functions are proportional which is independent of time. If the assumption is violated then the ratio will depend upon the time *t*. For such case stratum analysis is necessary. Let *X* be a covariate with a strata and the subject in the j^{th} stratum have an arbitrary base line hazard function are presented by a proportional hazards model in that stratum as

$$h_j(t \mid x) = h_{0j(t)}\exp(X'\beta)\quad j = 1,2,\ldots\ldots\ldots,s \tag{6}$$

The assumption is that all other covariates except the variable on which stratification is made act similarly in each stratum. In such case the parameters are estimated by maximizing a partial likelihood function defined as

$$L(\beta) = \sum_{j=1}^{s} L_j(\beta) \tag{7}$$

where, $L_j(\beta)$ is the likelihood function of the j^{th} stratum.

Two way model

The two-way proportional hazards model is a symmetric statement of the dual timescale situation. Let u_i be the lifetime for the i^{th} individual and also let b_i be the event occurs for the i^{th} individual. Then the calendar date for the i^{th} individual is denoted by v_i and defined as $v_i = b_i + u_i$ and x_i be the covariate for the i^{th} individual. The hazard rate for the i^{th} individual is defined as

$$h_i(u,v) = r_0(u)s_0(v)\exp(\alpha' x_i) \tag{8}$$

where, $h_i(u,v)$ is the hazard rate for patient *i* at lifetime *u* and calendar date *v*, x_i is the vector of covariates for the i^{th} individual. The sobriquet 'two-way' for model (8) contrasts it with familiar one-way model proportional hazards models that include only a single base-line hazard rate $r_0(u)$ if the modeling is done on the lifetime scale or $s_0(v)$ if the hazard rates are modeled on the calendar date scale. If the model is considered as discrete, calling the time unit 'day' or 'month', hazard rate models consist of a formula for the conditional probability that an individual does not survive one day or month further along his/her history line

$$h_i = \Pr ob(Patient\, i\, dies\, on\, (u,v) \,|\, patient\, i\, at\, risk) \tag{9}$$

'at risk' meaning that patient i neither died nor was lost to follow-up before day (u,v) or days is redundant, since given the 'birth' date or entry date b_i we know, $v = b_i + u,\ \ u = v - b_i$. The logarithm of the two-way model is

$$\log h_i(u,v) = \log\{r_0(u)\} + \log s_0(v) + \alpha' x_i \tag{10}$$

If the calendar date axis *v* is considered, then the Risk set $S(v)$ is defined as

$$S(v) = \{i : b_i \le v \text{ and } v_i \ge v\} \tag{11}$$

and the patient-specific hazard rate can be re-expressed as in the date notation as

$$s_i = \Pr ob\{patient\, i\, dies\, on\, date\, v \,|\, i \in (v)\}$$

so,

$$s_i(v) = h_i(v - b_i, v) \tag{12}$$

The log-additive from (12) makes Poisson modeling particularly convenient. Let $I_i(v)$ denote patient i's survival indicators,

$$I_i = \begin{Bmatrix} 0 \;\; for \quad v = b_i, b_i + 1, \ldots, v_i - 1 \\ c_i \;\; for \quad v = v_i \end{Bmatrix}$$

where c_i, be the censoring indicator and takes value 1 for event and 0 for censored. Then $E\{I_i(v) \,|\, i \in S(v) = s_i(v)\}$ the equation (12) and the Poisson assumption can be stated as

$$I_i(v) \,|\, i \in S(v) \xrightarrow{ind} P_0\{s_i(v)\} \tag{13}$$

The independence applies overall v and all $i \in S(v)$. To model survival data realistically, it is required that $s_i << 1$ in expression (13) to avoid multiple deaths or events, especially in discrete situation. This is no problem in a continuous context, where $s_i(v) \to 0$ as the time is divided more finely. Through Bernoulli process can be substituted for expression (13) as in Efron (1977), but the Poisson model is more tractable. Poisson assumptions are especially attractive for modeling repeated events as in Prentice et al. (1981). First, the usual survival situation is considered. Suppose that $s_i(v)$ has the log-additive form,

$$\log s_i(v) = \eta + \alpha' x_i + \beta' y(u) + \gamma' z(v) \quad (u = v - b_i) \tag{14}$$

where, β and γ are unknown parameter vectors whereas $y(u)$ and $z(v)$ are known vector functions of u and v respectively; η is a single unknown intercept parameter. In this paper the functions are considered to be the cubic polynomials:

$$\beta' y(u) = \beta_1 u + \beta_2 u^2 + \beta_3 u^3$$
$$\gamma' z(v) = \gamma_1 v + \gamma_2 v^2 + \lambda_3 v^3 \tag{15}$$

The equation (12)-(15) jointly describes a standard Poisson generalized linear model. Compare equation (10) with (15) the base-line hazards can be modeled as,

$$r_0 = \exp\{\beta' y(u)\} \quad s_0 = \exp\{\gamma' z(v)\}$$

so, $s_0 = \exp\{\gamma' z(v)\}$ is the effect of calendar date on survival and the estimated date effect is, $\hat{s}_0 = \exp\{\hat{\gamma}' z(v)\}$, obtained from the maximum likelihood estimate.

Parameter Estimation

Conditional Likelihood: Cox (1975) suggested partial likelihood concept for estimating the parameters of the proportional hazards model. Consider the set $R(t_{(i)})$ of individuals at risk at $t_{(i)} - 0$. The conditional probability that item (i) fails at time $t_{(i)}$ given that the items $R(t_{(i)})$ are at risk and that exactly one failure occurs at $t_{(i)}$ is

$$\frac{\lambda(t_{(i)}, Z_{(i)})}{\sum_{i \in s(v)} \exp \lambda(t_{(i)}, Z'_{(i)}} = \frac{(\exp(Z'_{(i)}, \beta)}{\sum_{I \in R(t_{(i)}} (\exp(Z'_{(i)}, \beta)} \tag{16}$$

The partial likelihood for β is now formed as follows:

$$L(\beta) = \prod_{i=1}^{k} \frac{(\exp(Z'_{(i)}, \beta)}{\sum_{I \in R(t_{i)})} (\exp(Z'_{(i)}, \beta)} \tag{17}$$

Estimation of parameters from Two way model: The estimates of parameters are now obtained using two way proportional hazard models. The two-way proportional hazard model for i^{th} patient is given as (Efron'2002)

$$h_i(u,v) = r_0(u) s_0(v) \exp(\alpha' x_i) \tag{18}$$

where, $h_i(u,v)$ is the hazard rate for patient i at lifetime u and calendar date v, $r_0(u)$ is baseline hazard function at u, $s_0(v)$ is baseline hazard function at v, x_i the vector of covariates for the i^{th} individual. The likelihood function for the date scale is,

$$L = \prod_{v} \prod_{i \in S(v)} \exp\{-s_i(v)\} s_i(v)^{I_i(v_i)}$$

Let define,

$$S_i(v) = \exp(\eta + \alpha'' x_i + \beta' y(u) + \gamma' z(v)) \tag{18}$$

$$S_+(v) = \sum_{i \in S(v)}, \; s_i(v) = \sum \exp(\eta + \alpha' x_i + \beta' y(u) + \gamma' z(v)), \; S_i(v) = \frac{s_i(v)}{s_+(v)}$$

Then the likelihood decomposition on date scale can be expressed as,

$$I = A + \sum_{v} [n(v) \log B_1 - B_1] \tag{19}$$

where,

$$A = \sum_i c_i \log[\frac{\exp\{\eta + \alpha' x_i + \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \gamma_1 v + \gamma_2 v^2 + \gamma_3 v^3\}}{\sum_{i \in s(v)} \exp\{\eta + \alpha' x_i + \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \gamma_1 v + \gamma_2 v^2 + \gamma_3 v^3\}}]$$

$$B_1 = \sum_{i \in s(v)} \exp(\exp\{\eta + \alpha' x_i + \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \gamma_1 v + \gamma_2 v^2 + \gamma_3 v^3)$$

After calculating derivatives and using this, the score vector and information matrix is produced to estimate the parameters by Newton-Raphson iteration method.

The score vector is,

$$U(\eta, \alpha, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) = \left[\frac{\delta l}{\delta \eta}, \frac{\delta l}{\delta \alpha}, \frac{\delta l}{\delta \beta_1}, \frac{\delta l}{\delta \beta_2}, \frac{\delta l}{\delta \beta_3}, \frac{\delta l}{\delta \gamma_1}, \frac{\delta l}{\delta \gamma_2}, \frac{\delta l}{\delta \gamma_3} \right]$$

The information matrix can be obtained from the following:

$I(\eta, \alpha, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) =$

$$\begin{bmatrix}
-\frac{\delta^2 l}{\delta\eta^2} & -\frac{\delta^2 l}{\delta\eta\delta\alpha} & -\frac{\delta^2 l}{\delta\eta\delta\beta_1} & -\frac{\delta^2 l}{\delta\eta\delta\beta_2} & -\frac{\delta^2 l}{\delta\eta\delta\beta_3} & -\frac{\delta^2 l}{\delta\eta\delta\gamma_1} & -\frac{\delta^2 l}{\delta\eta\delta\gamma_2} & -\frac{\delta^2 l}{\delta\eta\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\alpha\delta\eta} & -\frac{\delta^2 l}{\delta\alpha^2} & -\frac{\delta^2 l}{\delta\alpha\delta\beta_1} & -\frac{\delta^2 l}{\delta\alpha\delta\beta_2} & -\frac{\delta^2 l}{\delta\alpha\delta\beta_3} & -\frac{\delta^2 l}{\delta\alpha\delta\gamma_1} & -\frac{\delta^2 l}{\delta\alpha\delta\gamma_2} & -\frac{\delta^2 l}{\delta\alpha\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\beta_1\delta\eta} & -\frac{\delta^2 l}{\delta\beta_1\delta\alpha} & -\frac{\delta^2 l}{\delta\beta_1^2} & -\frac{\delta^2 l}{\delta\beta_1\delta\beta_2} & -\frac{\delta^2 l}{\delta\beta_1\delta\beta_3} & -\frac{\delta^2 l}{\delta\beta_1\delta\gamma_1} & -\frac{\delta^2 l}{\delta\beta_1\delta\gamma_2} & -\frac{\delta^2 l}{\delta\beta_1\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\beta_2\delta\eta} & -\frac{\delta^2 l}{\delta\beta_2\delta\alpha} & -\frac{\delta^2 l}{\delta\beta_2\delta\beta_1} & -\frac{\delta^2 l}{\delta\beta_2^2} & -\frac{\delta^2 l}{\delta\beta_2\delta\beta_3} & -\frac{\delta^2 l}{\delta\beta_2\delta\gamma_1} & -\frac{\delta^2 l}{\delta\beta_2\delta\gamma_2} & -\frac{\delta^2 l}{\delta\beta_2\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\beta_3\delta\eta} & -\frac{\delta^2 l}{\delta\beta_3\delta\alpha} & -\frac{\delta^2 l}{\delta\beta_3\delta\beta_1} & -\frac{\delta^2 l}{\delta\eta\delta\beta_2} & -\frac{\delta^2 l}{\delta\beta_3^2} & -\frac{\delta^2 l}{\delta\beta_3\delta\gamma_1} & -\frac{\delta^2 l}{\delta\beta_3\delta\gamma_2} & -\frac{\delta^2 l}{\delta\beta_3\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\gamma_1\delta\eta} & -\frac{\delta^2 l}{\delta\gamma_1\delta\alpha} & -\frac{\delta^2 l}{\delta\gamma_1\delta\beta_1} & -\frac{\delta^2 l}{\delta\gamma_1\delta\beta_2} & -\frac{\delta^2 l}{\delta\gamma_1\delta\beta_3} & -\frac{\delta^2 l}{\delta\gamma_1^2} & -\frac{\delta^2 l}{\delta\gamma_1\delta\gamma_2} & -\frac{\delta^2 l}{\delta\gamma_1\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\gamma_2\delta\eta} & -\frac{\delta^2 l}{\delta\gamma_2\delta\alpha} & -\frac{\delta^2 l}{\delta\gamma_2\delta\beta_1} & -\frac{\delta^2 l}{\delta\gamma_2\delta\beta_2} & -\frac{\delta^2 l}{\delta\gamma_2\delta\beta_3} & -\frac{\delta^2 l}{\delta\gamma_2\delta\gamma_1} & -\frac{\delta^2 l}{\delta\gamma_2^2} & -\frac{\delta^2 l}{\delta\gamma_2\delta\gamma_3} \\
-\frac{\delta^2 l}{\delta\gamma_3\delta\eta} & -\frac{\delta^2 l}{\delta\gamma_3\delta\alpha} & -\frac{\delta^2 l}{\delta\gamma_3\delta\beta_1} & -\frac{\delta^2 l}{\delta\gamma_3\delta\beta_2} & -\frac{\delta^2 l}{\delta\gamma_3\delta\beta_3} & -\frac{\delta^2 l}{\delta\gamma_3\delta\gamma_1} & -\frac{\delta^2 l}{\delta\gamma_3\delta\gamma_2} & -\frac{\delta^2 l}{\delta\gamma_3^2}
\end{bmatrix}$$

Discussion

Proportional hazards model and Poisson regression model are considered for the data in this study. In both cases, we consider two time points- lifetime (u_i) and calendar time (u_i).The models considered in this study are:

Model I: $\lambda(u_i \mid Z_i) = \lambda_0(u_i)\exp(Z_i'\beta)$

Model II: $\lambda(v_i \mid Z_i) = \lambda_0(v_i)\exp(Z_i'\beta)$

Model III: $\lambda(u_i, v_i) = r_0(u)s_0(v)\exp(Z_i'\beta)$

Model IV: $\lambda_g(u_i \mid Z_i) = \lambda_{0g}(u_i)\exp(Z_i'\beta) \quad g = 1,2,3, \quad \ldots \quad 12$

Model V: $\lambda_g(v_i \mid Z_i) = \lambda_{0g}(v_i)\exp(Z_i'\beta) \quad g = 1,2,3, \quad \ldots \quad 12$

Model VI: $\lambda_g(u_i \mid Z_i) = \lambda_{0g}(u_i)\exp(Z_i'\beta) \quad g = 1,2$

Model VII: $\lambda_g(v_i \mid Z_i) = \lambda_{0g}(v_i)\exp(Z_i'\beta) \quad g = 1,2$

$\lambda_g(v_i|Z_i) = \lambda_{0g}(v_i)\exp(Z_i'\beta) \quad g = 1,2$

The values in the Appendix Table -2 shows both of the calendar time have significant effect on the lifetime. Since calendar date is the most important variable, each additional month in calendar date associated with 1.8% increase in the lifetime. Again the covariate lifetime has effect on the calendar time and each month increase in the lifetime increase the calendar on average 1.2%. In both model the covariate sex has no significant on lifetime as well as calendar time.

Appendix Table-3 presents different results using model IV, which indicates that the covariates lifetime of children, total number of children, number of child under five year, birth order number and birth in last five years have significant effect on the hazard of calendar time. The hazard ratio for the lifetime is 0.985 which indicates hazard rate decreases with the increase of lifetime by 1.5%. Mother's age at first birth has hazard ratio 0.995which indicates hazard decreases with the increase of the age. The hazard ratio for the covariate total children is 2.414 which indicates hazard of the calendar time increases with the increase of the total children and the hazard ratio for the covariate children under five is 0.869. The covariate sex of the children has no significant effect on the hazard of the calendar time.

The results shown in table-4 obtained by using the model V which indicates that the covariate sex of the children and the age of the mothers at the first birth have no significant effect on the hazard of lifetime but the covariate calendar time, total number of children ever born, numbers of children under age five, birth order number and the birth in last five years have significant effect on the hazard of the lifetime.

Now we discuss and compare the parameters and corresponding p-values of the model VI and VII for different divisions of Bangladesh. Model VI indicates that the covariates of lifetime, total children, children under five, birth order number and birth in last five years are found to be significant effect on the hazard of the calendar time for all division. On the other hand, the covariate sex of the children and the age of mother at first birth have no significant on the hazard of calendar time. The hazard ratio for the covariate lifetime is 0.990(Dhaka division), which indicates that each unit (month) increase in the lifetime associated with 0.01 percent decrease in hazard of calendar time and similarly we say for all other division. Again, the result using the model VII, gives almost the same but the difference is the covariate sex has significant effect on the lifetime for Chittagong division and covariate age of mother at the first birth has significant effect for Khulna division.

Conclusion

Most demographic surveys in the developing world collect child survival data that are considered only the time scale lifetime but here we apply dual time scale both lifetime as well as calendar time in two-way proportional hazard model. In the analysis birth order number, number of children under 5 was found to have significant on time effect on time data, lifetime and the calendar time. Finally, the study shows that the time point (calendar time) is also in important time scale which indicates that child mortality not only depend on the lifetime of the children but also depend on the period when the birth occur.

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Appendix

Table 1: Summary statistics of variables (proximate determinant of Child Mortality).

Variable	Sex of Child		Current Age	Age at death	Age of Mother	Age of Mother at the first birth	Region						TNB's	TND's
	M	F					Dhaka	Khulna	Chitta-gong	Sylhet	Raj-shahi	Bari-shal		
Mean/ %	50.7	49.3	8.60 (m's)	12.59	32.18	17.19	22.1	13.1	19.8	12.06	20.9	12.5	2,55,883	2,947

Table: 2 Estimates of parameters, hazards ratio, standard error, *z*-value and *p*-value

Variables	Coefficient	Exp (coefficient)	Se (coefficient)	z	*p*-value
Calendar time	0.017	1.018	0.000	141.320	0.000
Child's Sex(Male & Female)	0.023	1.023	0.013	1.740	0.080
Lifetime	0.0116	1.012	0.000	42.120	0.000
Child's Sex(Male & Female)	-0.014	0.986	0.013	-1.090	0.280

Table: 3 Estimates of parameters and *p*-values using model IV

Variables	Coefficient	Exp(coefficient)	Se(coefficient)	z	*p*-value
Lifetime, (u_i)	-0.014	0.985	0.000	-33190	0.000
Child Sex	-0.009	0.991	0.013	-0.680	0.500
Mother's Age at First Birth	-0.004	0.995	0.002	-1.990	0.047
Total Children	0.881	2.414	0.006	130.360	0.000
Children Under 5	-0.140	0.869	0.009	-14.19	0.000
Birth Order Number	-0.851	0.427	0.007	-118.47	0.000
Birth in Last 5 Years	-1.224	0.294	0.013	-90.580	0.000