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RESEARCH ARTICLE PHYSICAL SCIENCE

MAGNETOHYDRODYNAMIC FREE CONVECTION FLOW OF FLUID OF A VERTICAL PLANE WITH VISCOSITY AND THERMAL CONDUCTIVITY DEPENDING ON TEMPERATURE

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Abstract: A two-dimensional natural convection flow of a viscous incompressible and electrically conducting fluid with both the viscosity and the thermal conductivity depending on temperature past a vertical impermeable flat plate is considered in presence of a uniform transverse magnetic field. The governing equations are reduced to non-similar boundary layer equations by introducing coordinate transformations appropriate to the cases (i) near the leading edge (ii) in the region far away from the leading edge and (iii) for the entire regime from leading edge to down stream. The governing equations for the flow in the upstream regime are investigated by perturbation method for smaller values of ξ, the stream-wise distributed magnetic field parameter. The equations governing the flow for large ξ and for all ξ, have been investigated by employing the implicit finite difference method with Keller box scheme. The effects of the viscosity variation parameter, ε and the thermal conductivity variation parameter γ, on the skin friction as well as the rate of heat transfer for the fluid for low Prandtl number are shown graphically.

Key words: Grashof number, Prandtl number, Hartmann number, thermal diffusivity, stream function

Introduction

The studies of forced, free and mixed convection flow of a viscous incompressible fluid, in the absence of magnetic field, along a vertical surface have extensively been conducted by Sparrow *et al.* (1959) and Loyed and Sparrow (1970). Wilks and Hunt (1981) introduced a group of continuous transformations computation for the boundary layer equations between the similarity regimes for mixed convection flow. In the case of similarity regimes Wilks and Hunt (1984) recognized $\zeta(=H_x^{10}/Gr_x^{\ 2})^{1/2}$, where Gr_x is the local Grashof number and H_x is the local Hartmann number, a governing parameter for the flow from a vertical plate. Empirical patching of two perturbation solutions have also been carried out to provide a uniformly valid solution by Raju et al (1984) which covers the whole range of the values of ζ. They obtained a finite difference solution applying an algebraic transformation $Z=1/(1+\zeta^2)$. Many solutions have been developed by considering the free convection as a perturbation quantity. Tingwi *et al.* (1982) have also studied the effect of forced and free convection along a vertical flat plate with uniform heat flux by considering the buoyancy parameter ζ_p to be $Gr_x/Re_x^{5/2}$. The solutions were obtained for the small buoyancy parameter taking into account of the perturbation technique.

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Because of its application for magneto hydrodynamic natural convection flow in the nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors, such as Sparrow and Cess (1961), Riley (1964) and Kuiken (1970). Wilks (1976) recognized a parameter ξ, defined by $\xi = (\sigma H_0^2)^2 x/g\beta(T_0-T_\infty)$ to investigate the MHD free convection flow about a semi-infinite vertical plate in a strong cross magnetic field.

Materials and methods

The basic equations of steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with both the viscosity and thermal conductivity depending on temperature past a semi-infinite vertical impermeable flat plate in the presence of

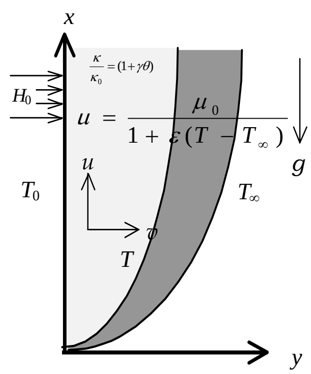

Fig.1. The flow configuration and the coordinates system.

a uniformly distributed transverse magnetic field of strength H_0 (the flow configuration and the coordinates system are shown in Fig.1)are as given below

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta(T-T_\infty) - \frac{\sigma H_0^{\ 2} u}{\rho_\infty} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) \tag{3}$$

With the boundary conditions

$$\begin{aligned} &u = v = 0, \quad T = T_0 \quad \text{at } y = 0 \\ &u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

Here u, v are the velocity components associated with the direction of increase of coordinates x and y measured along and normal to the vertical plate. T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β is the coefficient of thermal expansion, κ is the thermal conductivity, ρ_∞ is the density of the fluid, c_p is the specific heat at constant pressure and T_∞ is the temperature of the ambient fluid μ is the viscosity of the fluid depending on temperature defined in equation (5) and κ is the temperature depending on thermal conductivity defined in equation (6). The thermal conductivity increases with the increase of temperature. The temperature dependent viscosity μ and thermal conductivity κ can be of the form

$$\mu = \frac{\mu_0}{1 + \varepsilon (T - T_\infty)} \tag{5}$$

$$\frac{\kappa}{\kappa_0} = 1 + \gamma\theta \tag{6}$$

where $\theta = T - T_\infty$

Solution for entire regime

Now we introduce the following transformations to the equations (2) and (3)

$$\psi = c\nu x^{3/4}(1+\xi)^{-1/4} f(\xi,\eta), \quad \eta = \frac{cy(1+\xi)^{-1/4}}{x^{1/4}}$$
$$T - T_\infty = (T_0 - T_\infty)\theta(\xi,\eta), \quad c = \left[\frac{g\beta(T_0 - T_\infty)}{\nu^2}\right]^{1/4} \tag{7}$$

and we get the following equations

$$(1+\varepsilon\theta) f''' - \varepsilon\theta' f'' + (1+\varepsilon\theta)^2 \left\{ \frac{3+2\xi}{4(1+\xi)} ff'' - \frac{1}{2(1+\xi)} f'^2 + (1+\xi)\theta - \xi^{1/2}(1+\xi)^{1/2} f' \right\} = (1+\varepsilon\theta)^2 \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \tag{8}$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \Pr \frac{3+2\xi}{4(1+\xi)} f\theta' = \Pr \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \tag{9}$$

With the boundary conditions

$$\begin{aligned} & f = f' = 0; \quad \theta = 1 \quad \text{at} \ \eta = 0 \\ & f' \to 0; \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \\ & f = \theta = 0; \quad \text{at} \quad \xi = 0, \eta \rangle 0 \end{aligned} \tag{10}$$

Here the coefficient of skin-friction, τ_w, and the coefficient of the rate of heat transfer, Q are defined by the following equations.

$$\tau_w = \frac{H_0}{g\beta(T_0 - T_\infty)} \left(\frac{\sigma\nu}{\rho_\infty}\right)^{1/2} \left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi^{1/4}(1+\xi)^{-3/4} f''(\xi,0) \tag{11}$$

$$Q = -\left(\frac{\rho_\infty \nu}{\sigma H^2{}_0}\right)^{1/2} \frac{1}{T_0 - T_\infty} \left(\frac{\partial T}{\partial y}\right)_{y=0} = \xi^{-1/4}(1+\xi)^{-1/4} \theta'(\xi,0) \tag{12}$$

Solution near the leading edge

For small ξ from the equation (8) and (9) we get the following equations

$$(1+\varepsilon\theta) f''' - \varepsilon\theta' f'' + (1+\varepsilon\theta)^2 \left\{ \frac{3}{4} ff'' - \frac{1}{2} f'^2 + \theta - \xi^{1/2} f' \right\} = (1+\varepsilon\theta)^2 \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \tag{13}$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \frac{3}{4} \Pr f\theta' = \Pr \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \tag{14}$$

where the boundary conditions (10) remains the same
To get the solutions of the above equations (13) and (14) we use the 2nd and 3rd order series solution and the finite difference method.

Series solution methods

For series solution we consider the following series

$$f(\xi,\eta) = f_0 + \xi^{1/2} f_1 + \xi f_2 + \cdots\cdots \tag{15}$$

$$\theta(\xi,\eta) = \theta_0 + \xi^{1/2}\theta_1 + \xi\theta_2 + \cdots\cdots \tag{16}$$

Where f_0, θ_0 are the well known free convection similarity solutions for flow around a constant temperature semi-infinite vertical plate and f_1, θ_1 are effectively the first order correction and f_2, θ_2 are effectively second order correction to the flow due to the presence of magnetic field.

Using equations (15) and (16) in (13) and (14) we get a set of equations with boundary conditions. Then we get the solutions for skin friction and rate of heat transfer which are shown graphically.

Solution far away from the leading edge

For large ξ from the equations (8) and (9) we get

$$(1+\varepsilon\theta) f''' - \varepsilon\theta' f'' + (1+\varepsilon\theta)^2 \left\{ \frac{1}{2} ff'' + \xi(\theta - f') \right\} = (1+\varepsilon\theta)^2 \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \tag{17}$$

$$(1+\gamma\theta)\theta'' + \gamma\theta'^2 + \frac{1}{2} \Pr f\theta' = \Pr \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \tag{18}$$

with the boundary conditions

$$\begin{aligned} & f = f' = 0; \quad \theta = 1 \quad \text{at}\ \eta = 0 \\ & f' \to 0; \quad \theta \to 0 \quad \text{as}\ \eta \to \infty \\ & f = \theta = 0; \quad \text{at}\ \xi = 0,\ \eta \rangle 0 \end{aligned} \tag{19}$$

Here we calculate the skin friction and the rate of heat transfer.

Results

The solutions of the governing non-similar equations are obtained by the implicit finite difference method or simply Keller box method for all ξ. The solutions of the present problem with wide range of parameter, ξ are obtained by the finite difference method.

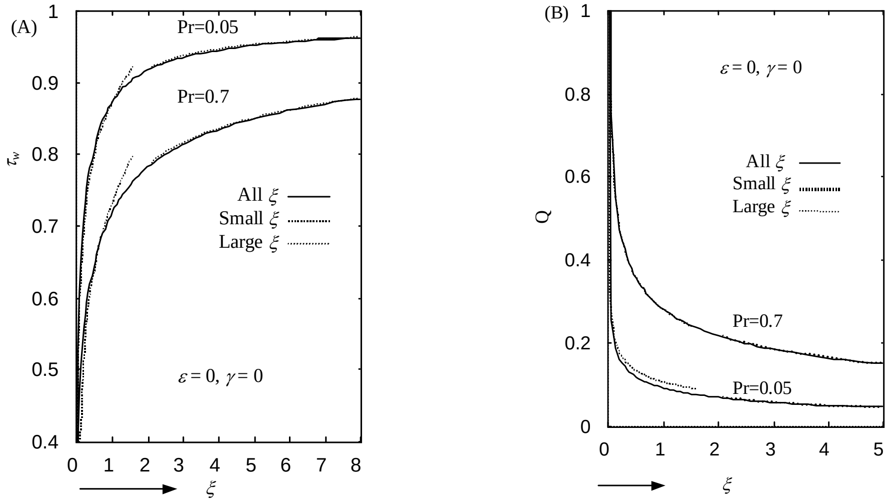

Fig.2. (A) The skin-friction coefficient and (B) the rate of heat transfer against ξ for different values of Pr with $\varepsilon = 0.0$ and $\gamma = 0.0$

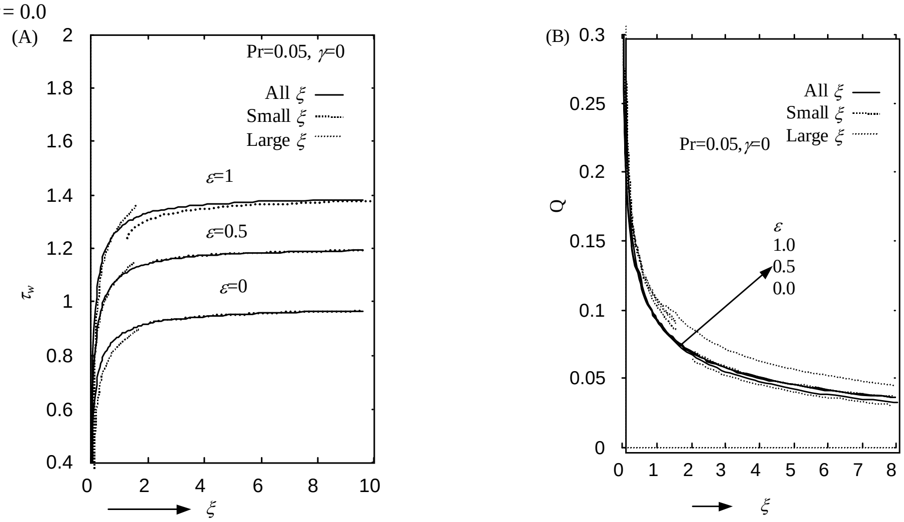

Fig.3. (A) The skin-friction coefficient and (B) the rate of heat transfer against ξ for different values of ε with Pr = 0.05 and $\gamma = 0.0$

The effect of the viscosity variation parameter ε and the thermal conductivity variation parameter γ, on the local skin-friction τ, as well as the local rate of heat transfer coefficient Q, against the magnetic field parameter ξ, for the fluids with Pr = 0.05 and 0.7 (Pr = 0.05 for lithium) are displayed in Fig 2A-B, Fig 3A-B, Fig 4A-B and Fig 5A-B.

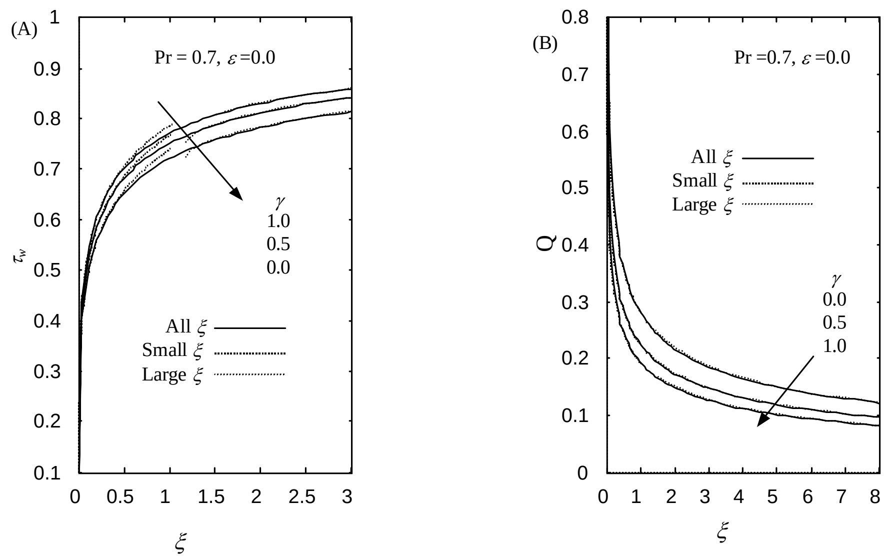

Fig.4. (A) the skin-friction and (B) the rate of heat transfer against ξ for different values of γ with Pr = 0.7 and ε =0.0

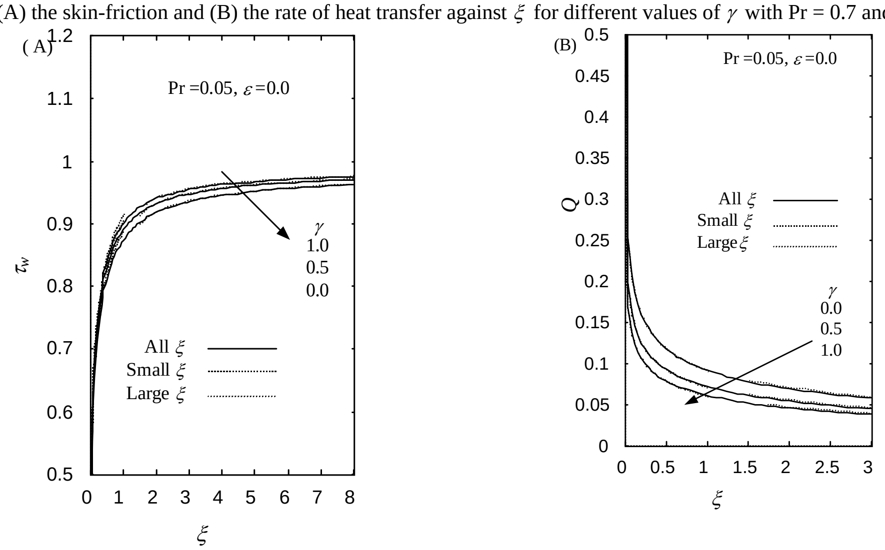

Fig.5. (A) the skin-friction and (B) the rate of heat transfer against ξ for different values of γ with Pr = 0.05 and ε =0.0

These figures show that the skin-friction decreases where as the rate of heat transfer increases due to decrease in thermal conductivity parameter γ. It is further observe that for the increasing values of the magnetic field parameter ξ, the values of the skin-friction increase and the rate of heat transfer decrease. For the increasing values of the viscosity variation parameter ε, both the skin-friction and the rate of heat transfer increases.

Discussion

In this section we discuss the results obtained from the solution of the equations governing the MHD free convection flow of a viscous incompressible and electrically conducting fluid with both the viscosity and thermal conductivity depending on temperature, in the presence of uniform transverse magnetic field along an impermeable vertical flat plate.

Fig.2 suggests that the series solution for small ξ has good agreement with the solution from finite difference method. Also the asymptotic solution for large ξ has an excellent agreement with the solution from the finite solution method for both the skin friction and the rate of heat transfer. The skin friction and the the rate of heat transfer (defined in the equation (11) and (12)) for the liquid metal of low Prandtl number value (i.e. Pr = 0.05 for lithium) are depicted graphically in the Figs. 2, 3 and 5, (A) and (B) for ε= 0.0, 0.5 and 1.0 also in Figs 2 and 4 (A) and (B) for γ = 0, 0.5 and 1.0 for Pr = 0.7.

From the Fig.2, we observe that the skin friction increase and the rate of heat transfer decrease due to the increasing values of ξ. The effect of the parameter ξ is not significant close to the surface of the plate. For increasing values of Pr the skin friction decrease while the rate of heat transfer increase. From Fig.3, we also observe that for the increasing values of ε both the skin friction and the rate of heat transfer increase. From Fig.5, we see that for increasing values of γ the skin friction increase and the rate of heat transfer decrease.

In all the figures 2, 3, 4 and 5, we see that the increasing values of magnetic field parameterξ, the skin friction increase while the rate of heat transfer decrease and this effect is significant for large values of it.

Conclusion

In this paper, the problem of Magnetohydrodynamic free convection flow along a vertical flat plate with variable thermal conductivity is investigated. The numerical computations were carried out only for the case of assisting flow for the fluids having low Prandtl number appropriate for liquid metals (Pr = 0.05 lithium) and air (Pr = 0.7).

The skin friction and the rate of heat transfer coefficient are presented in graphical form in this paper and it has been observed that for increasing values of Prandtl number, the local skin friction decreases monotonically and the rate of heat transfer increases.

The skin friction increase and the rate of heat transfer decrease for increasing values of the magnetic field parameter ξ and as the increasing values of the thermal conductivity parameter γ, the skin friction increases and the rate of heat transfer decreases.

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