



## A NARRATIVE REVIEW OF THE USE OF FRACTAL GEOMETRY IN VARIOUS ASPECTS OF URBAN PLANNING

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### Abstract

Urban structure is one of the complex geometry due to its formation process and structural elements distribution over the space. Generally, its formation represents a degree of irregularity, spatial hierarchy, unevenness. But, at different observation scales, cities spatial arrangement represents the important characteristic of fractal, which is self-similarity (a small part of an object is exactly similar to the whole). Therefore, to characterize cities formation process and measure its structural complexity quantitatively, different researchers have introduced the concept of fractal geometry. Fractal geometry is used to explain the hierarchical arrangement of objects, structural self-similarity over different viewing scales, and heterogeneity within it. In order to plan a better city, a detailed study of its formation process is essential. The purpose of this study is to gather existing knowledge about how fractal geometry has been widely used in different domains of urban planning and synthesized existing knowledge. Most of the previous studies have carried out fractal dimension to measure fractal properties within objects. Box-counting method is one of the most common methods to calculate fractal dimension. Different studies concluded that if the fractal dimension of a city increases, the complexity of the city's physical arrangement also increases. Finally, this study will give future scholars with vital information regarding the use of fractal theory in urban planning and may provide profound insight in this area.

**Keywords:** Fractal geometry, fractal dimension, self-similarity, spatial hierarchy, power law

### Introduction

Traditionally, Euclidean geometry (e.g., line, circle, rectangle, angle, parabola) is widely used to measure the length, distance, and area of different objects found in nature. In 1967, mathematician Benoit Mandelbrot tried to measure the length of Britain's coastline. He found that the length of the coastline is not uniform; rather, the length changes with the measurement scale. If one changes the instrument length used to measure the coastline, the length of the coastline will also change (Bovill, 1996; B. Mandelbrot, 1967). He also explained fractal geometry by detailing the texture of the "Koch Curve". Mathematically, its texture is created in a recursive way by replicating a similar shape at different observation scales (Bovill, 1996). In a broad way, if we

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zoom into the texture of a Koch curve in more detail, we can see a similar shape/pattern is repeated over and over again to form the Koch curve. Therefore, a fractal is a complex geometric shape that is made from an infinite number of parts, and each of the parts is a small-scale, copied version of the whole (B. B. Mandelbrot, 1989). Hence, according to Benoit Mandelbrot, a fractal object has a single primary property: self-similarity. Self-similarity properties can be described as if a fractal object went through a series of transformations where the dimensions of the structure were all changed by the same factor. After changing, its size may be smaller, bigger, moved, and/or rotated, but its shape stays the same (Sala, 2003). Fractal dimension is one of the quantitative indexes that can summarize and express fractal characteristics as a number (Bovill, 1996). In addition, fractal dimension measures the space-filling capacity of an object. It provides information about the geometrical structure of an object at multiple scales. The value of the fractal dimension varies between 1 and 2 (Kim et al., 2003; Sreelekha et al., 2017).

As fractal geometry has the capability to explain complex natural feature formations (e.g., clouds, coastlines, plants) that show different degrees of irregularity, complexity, and self-similarity (Lorenz, 2002). Based on the capability of fractal, different researchers try to apply the concepts of fractal geometry in different arena of urban geography (Batty et al., 1993; Frankhauser, 1998a, 2008a; Shen, 2002). Results show that urban formation, spatial arrangement show spatial hierarchy, self-similarity in its formation (Shen, 2002; Lu & Tang, 2004). As fractals have the ability to summarize complexity, heterogeneity, spatial hierarchy in a single value: fractal dimension (Jevrić et al., 2014a). Hence, fractal geometry is an ideal choice for analyzing the dynamic urban growth, evolution of urban form, and spatial organization of physical features. Fractal theory is a relatively new scientific discipline in this field. It provides us a new way to visualize a city and analyze it holistically (Frankhauser, 1998a; Jevrić et al., 2014).

To plan for a better, healthier, and more sustainable city, it is necessary to consider every aspect of the urban area. The quality of urban life is directly linked to its formation process (Salinas, 2003). That's why it is essential to study the complex mechanism of a city formation and its evolution over the time. To make the whole system efficient, researchers argue that an urban system needs to be complex enough where all its components and elements must be interconnected (Aghili et al., 2022). Recently, fractal geometry has been widely used in different key components of urban area (e.g., Land use, street network, water distribution network, sprawl management, space management) (Kowalski et al., 2015; Terzi & Kaya, 2011a; Wang et al., 2011; Yu & Zhao, 2021).

This paper mainly focuses on diverse usages of fractal in different domains of urban planning. Then, systematically organized thought process of different authors so that beginners in this field can better understand the diverse usages of fractal and how different authors make decision based on the inferences drawn from the research.

## **Materials and Methods**

As fractal geometry is newly introduced concept in urban planning. The concepts, terminology and wide ranges of usages of fractal are still unknown to many scholars. Additionally, as this concept of fractal was originated from typical mathematical discipline, therefore these terminologies may be tough to understand typically for the urban planning scholars. Basically, this limitations/ lack of understanding of the topic inspires me to write a review paper.

### ***Narrative review steps***

- Initially, searching and sorting the relevant literatures of fractal geometry on the basis of citation.
- Set out the specific terms for literature search (e.g., urban pattern design, land use, urban form and structure, road networks) in academic research database.
- Screening the relevant journal articles on the basis of above-mentioned area.

- Review Abstracts and conclusion.
- Structured overview of collected information and evaluates the sources.

Then the narrative review continues with:

- Identifying and summarizing the key concepts of fractal and how this concept has been used in above mentioned area of planning.
- Provide explanation of how fractal model fits in different area of urban planning.
- Identifies key research gaps by synthesizing exiting resources.
- Finally, explain the decision-making process based on the value of fractal dimension.

### ***Narrative review strategy***

As most of the journal articles was collected from renowned research paper database such as- Elsevier, JSTOR, Taylor and Francis group, SAGE, MDPI and from the international conference papers. Since this narrative review does not require inclusion or exclusion criteria, only pertinent literature was collected. Then, the titles of the studies were reviewed, and any papers that did not pertain to urban geography, urban planning, design, space management, and urban morphology were excluded from the collected sources. Then, exclude those research papers whose whole texts were unavailable were excluded. After that, the abstracts of the remaining research were examined and invalid ones were eliminated. The final selection was done by studying the full texts of selected research papers. In the following section, the whole selection process was highlighted by using a graphic.

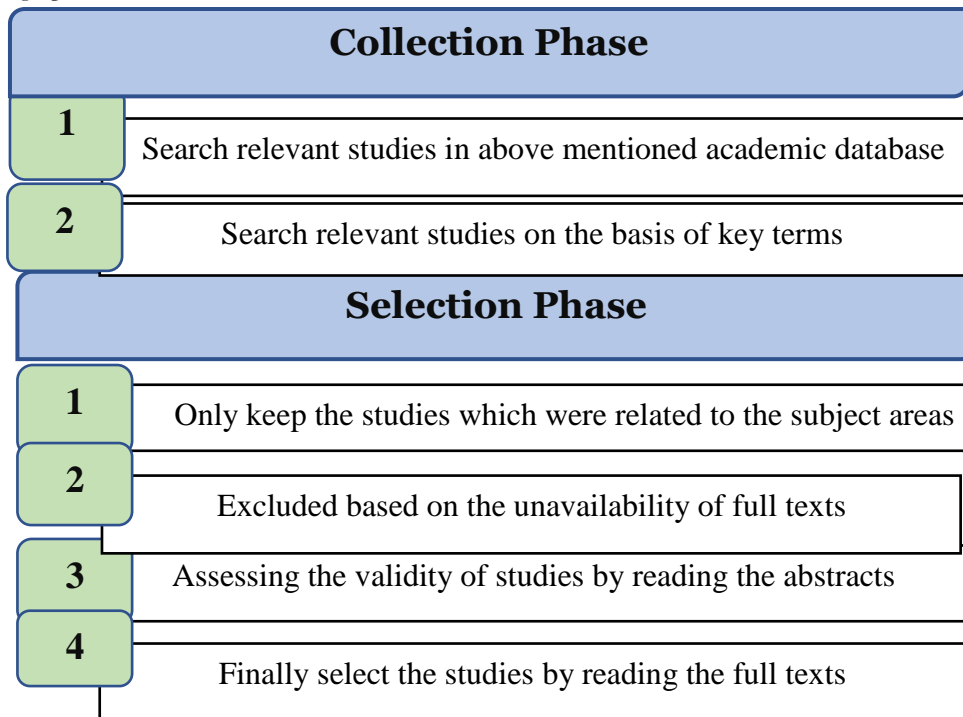


Figure 1. Narrative review strategy and Journal article Selection process

## Different Usages of Fractal Geometry in Urban Planning

### *Fractal geometry in urban water distribution network*

The demand for water is increasing with the growing population. To fulfill the need, it requires more pumping stations, reservoirs, and supply networks. As the network extends over a large urban space, it becomes more complex to manage as well as increases the operational difficulty. It not only increases the difficulties but also increases the economic cost. Thus, to manage its efficiently, the planned division of distribution points is crucial (Qi et al., 2014). To plan the water distribution network properly, Qi et al. (2014) introduce the fractal growth theory of self-organizing theory. However, to run the fractal growth of self-organization model in network, it is required to have the information of water demand points, required flow, and elevation. Fractal growth of self-organization model simply postulates an organizational arrangement of objects where it follows hierarchical order and self-similarity in different scales. Later on, different researchers found that the water supply network follows fractal properties (Diao et al., 2017; Kowalski et al., 2014, 2015; Qi et al., 2014). Kowalski et al. (2014) examined the characteristics of fractal in urban water distribution networks by answering one question. Question: Does it represent self-similar characteristics? He examines the geometrical structure of water supply network to answer the question. Research findings show that the geometrical structure of the water distribution network looks like a tree-shaped (dendritic) structure. It seems like the organization of network structure is created by copying, moving, and rotating a basic segment at different levels of observation. Thus, it is obvious that a scale hierarchy is present in its geometrical structure.

### *Fractal geometry in urban morphology*

Urban morphology is the study of urban formation, its creation, evolution and transformation over the time (F. Chen, 2014). This evolution occurs as a result of natural process and climatic determinants. Another reason for urban transformation is the interactions of various social, economic, and technological agents in urban spaces over time (Mcadams, 2007). As fractal dimension can explain the space-filling process, it could give information about urban evolution over time (Y.-G. Chen, 2018). Fractal geometry has been considered one of the innovative methods to study urban morphology. However, as a beginner in this field, two questions may arise after learning about the applications of fractals in urban morphology. First, what does mean by fractal and usages of fractal to describe urban objects? A fractal is a complex geometric shape, which looks self-similar at different scales. An object can be considered as fractal if it follows two principles- 1) it must follow power-law distribution 2) scale-free distribution (J. S. Kim et al., 2007). Power-law degree distribution simply means when one variable is raised to the power of another variable (e.g.,  $y = x^n$ ). Power law indicates that the system is scale-free/self-similar. Scale-free distribution means that at different scales of observation, the objects look the same. For example, if we take a small component of an object and magnify it to an infinite level, the shape of the object remains consistent across scales (J. S. Kim et al., 2007; Z. Lu et al., 2016). In conventional mathematics, characteristic lengths (e.g., length, density, area, volume) are used to measure an object or compare among objects. Nevertheless, an urban area is a gigantic entity that comprises buildings and road networks, among others. Definitely, urban areas cannot be effectively measure with such characteristic measurement scale because of its hidden morphological complexity (Y. Chen, 2020). However, urban area belongs to scale-free phenomenon, which means measurement of physical features depends on relative measurement scale. As it has no characteristics length, thus it can be considered as scale-free distributions (Y. Chen et al., 2017; Y. Chen, 2018, 2020). Fractal geometry is a powerful tool for the scaling analysis of cities (Fankhauser, 1998). In this case, urban morphology can be well characterized with fractal parameters.

Nevertheless, different empirical studies have shown that cities are not really a true fractal, rather pre-fractal. A true fractal can be repeated at infinite levels, whereas a pre-fractal has limited hierarchy (Y. Chen, 2020). A number of empirical studies have shown that a city presents fractal properties within a limited scaling range. To explain urban form and describe urban growth using fractal, we need to carry out fractal dimension. Different researchers have used different types of fractal dimensions, but the most commonly used methods to

compute fractal dimension are box counting method and match dimension. The box dimension is mainly suitable for explain the city spatial structure and systems, while the others are carried out to explain scale-free distribution. Generally, fractal dimension value ranges between 0 to 2 (Ariza-Villaverde et al., 2013; F. Chen, 2014; Man & Chen, 2020; Mcadams, 2007).

### ***Fractality in urban geography***

In the beginning, different researchers used Euclidean geometry in urban space to explain its complex nature, heterogeneity, and irregularity. However, it did not work well in urban form. Because cities are not a simple geometric shape, but rather a complex organization of different physical features. Mandelbrot (1997) introduced the concept of fractals in urban geography while measuring the coastline. Mandelbrot observed that the length of coastline increases with corresponding measurement scales as well as on different viewing scales, every individual component seems exactly similar to the whole (Mandelbrot, 1967). After that, different researchers have been used fractal theory to explain complex arrangement of built-up area (Fankhauser, 1998, 2008). If we examine the city's structure more closely, it may appear that each city began in a central location and then gradually expanded outward. While spreading from the center, the city becomes more fragmented and irregular. Due to the asymmetric interaction between center and periphery, different urban patterns may observe. It seems to be a continuous process of change between vacant and occupied spaces on different geographical scales (self-similarity) (Arlinghaus, 1985; Shen, 2002). Therefore, the distribution of urban mass on space is not uniform, neither dense nor thin. In general, urban element density decreases as distance from the city center increases. It can be concluded that analysis of urban areas is compatible with the principles of fractal geometry. Because urban areas show hierarchy (e.g., local, regional, and city), irregularity, and self-similarity through their land-use patterns, population distribution, service concentration, open spaces, and so on (Tannier and Pumain, 2005).

Recent research on urban form and different cities suggests that its follow multifractals characteristics (Y. Chen & Wang, 2013; Man & Chen, 2020). Multifractal characteristics indicate different subareas of urban region show different fractal dimension values at the same time. And in general, the value of fractal dimension decays from the center to the edge, which gives us a clear indication of urban sprawl (Man & Chen, 2020).

### ***Fractal in space management:***

Cities are the major source for various type of nuisance (such as- Air, Noise, sound and water pollution). Nuisance occurs due to human interactions with natural phenomenon and environment. It not only creates adverse impact on human health but also gives bad experience in their everyday life. Therefore, it gives us indication that most of the people in big cities want to live nearby park area/ open space/ greenery area. Thus, the issue is directly related to spatial planning and space management. Cities are considered as a complex organism. The reason behind its complexity lies on its process of formation. Every city is formed through the gradual interaction of diverse local factors, which causes different urban shapes or pattern. Fractal is widely used to analyze complexities and irregularities of cities. Fractal has an unique ability to summarize complexity, compactness, spatial heterogeneity in a single value, which is called 'Fractal Dimension' (Jevric & Romanovich, 2016; Man & Chen, 2020; Tucek & Janoska, 2013). Generally, the value of fractal dimension varies with the availability of open spaces in urban area. If an urban area offers more open space, the value of the fractal dimension will be lower. Alternatively, the value will be higher for compact urban areas (Shen, 2002; Wang et al., 2011).

Along with urban areas, the fractal dimension value of urban borders is also important for space management. From the perspective of urban planning, urban areas should be compacted with better public utility services, open spaces, and discouraged peripheral development. In this instance, the fractal dimension value of the urban boundary and urban area can be utilized as a parameter for spatial development decision-

making. For good spatial planning, the value of fractal dimension for urban areas needs to be high and alternatively, for urban boundary needs to be low. An increase of metropolitan area FD and decrease of urban boundary FD indicate more dense, homogeneous development with the tendency to offer more free spaces (Jevrić, Knežević, Kalezić, et al., 2014). The opposite trend indicates the process of urban sprawl. (Y. Chen, 2011; Jevric & Romanovich, 2016).

#### *Fractal in Urban sprawl management:*

In every perspective, urban dynamics are always a complex and unpredictable system. These complexities are more than usual, when urban sprawl is added with it. Different scholars have defined urban sprawl in different ways, but most of the definitions share common facts like outward extension of development, low density, fragmentation of land use, and discontinuity (Frenkel & Ashkenazi, 2008; Jiang et al., 2007; Terzi & Kaya, 2008a). Over time researchers have been used different methods to calculate urban sprawl. Among them the most used methods are: Sprawl index, density gradient (Banai & DePriest, 2010; Frenkel & Ashkenazi, 2008; Torrens & Alberti, 2000). In recent times, different researchers have proposed fractal theory to measure urban sprawl (Terzi & Kaya, 2008, 2011). Generally, fractal theory is used to describe how urban objects fill space. An urban area is considered fractal due to its geometric configuration. When urban sprawl occurs, physical features tend to fill the space in different directions, with varying degrees of discontinuity. As a result, different types of development patterns may be found (Batsuuri et al., 2020; Terzi & Kaya, 2011). Results show that the extension of urban development gives a lower fractal dimension value (Terzi & Kaya, 2011). The value of fractal dimension varies with urban expansion. Generally, the value of fractal dimension is higher for concentrated development. Hence, it is evident that fractal dimension could be an efficient way for measuring sprawl (Terzi & Kaya, 2008, 2011).

#### *Fractal in Urban pattern design:*

Most cities in the world contain planned and unplanned parts, and usually, its forms are neither regular nor strictly geometrical. Irregularity occurs due to the uneven distribution of its elements (e.g., buildings, roads, and open spaces) in urban spaces (Jevrić et al., 2014). Although the structural arrangement of objects may seem irregular, the spatial distribution of urban elements obeys power-law distribution. Fractal Dimension quantifies how objects occupy space at various scales. The value of the fractal dimension varies with the distribution pattern of urban objects. The value is close to 2, when urban features are uniformly distributed. And it would be 0, if the urban mass is concentrated in one point such as city center (K. S. Kim et al., 2003). The big challenges for new urban pattern design are limited space and urbanization. Most residents require immediate access to green spaces. Therefore, it is evident that a uniform distribution of green spaces in urban areas is necessary. In this case, fractal geometry helps us to design a landscape that offers a uniform distribution of green spaces at different levels of observations. In the previous section, it is mentioned that rise in metropolitan area FD and a fall in urban boundary FD indicate more dense, homogenous development with a tendency to provide more open space. The opposite trend indicates urban sprawl, which is characterized by dispersed development. Therefore, this idea can be helpful for better urban planning. At each level of planning, several relevant factors need to be considered, which gives us different choices about urban patterns (Frankhauser, 2004, 2008; Jevrić et al., 2014).

According to recent research, diverse city forms result from the gradual interaction of local factors, which eventually generates a highly organized global pattern. Due to the interaction of local factors, it demonstrates a certain hierarchical order at various scales of observation. Different studies have found that a small change in local factors may lead to an unexpected change in urban pattern. Therefore, to make the urban pattern fractal, its composition must follow spatial hierarchical order (e.g., city center, sub-center, rural area) and uniform distribution of its elements over different observation scales (Frankhauser, 2015).

*Fractal in street network:*

A series of study have already proved that street network follows fractal properties (Benguigui, 1992, 1995; Rodin & Rodina, 2000; Kim et al., 2003; Y. Lu & Tang, 2004; Sun et al., 2007; Xu et al., 2010; Zhang & Li, 2012; Mo et al., 2015; Z. Lu et al., 2016; Sreelekha et al., 2017; Karpinski et al., 2020; Zhang et al., 2021). In 1992, Benguigui first introduced the concept of fractal in railway network. He discovered that the spatial structure of railway networks follows a common organizational pattern. Since then, different researchers have been used fractal to analyze the spatial structure of road network. Rondon (2000) study suggests that the fractal dimension of the road network appears only if the road density is high and the road network expands over limited space. Kim et al. (2003) study shows that the value of the fractal dimension rises when new roads are added to the existing network. Lu & Tang (2004) study investigates the relationship between city size and complexity of road network. Study suggests that the road network becomes more complex, when a city gets larger. As a consequence, urban spaces are filled up by road networks with varying densities, which causes different fractal dimensions. Sun et al. (2007) treat a city as a distributive continuous space, where other researchers treat a city as a whole (Rodin & Rodina, 2000; Kim et al., 2003; Y. Lu & Tang, 2004). Fractal dimension is a crucial metric for determining the spatial distribution of objects over a given region which is completely different than simple road density (Sun et al., 2007; Sreelekha et al., 2016). Sun et al. (2007) clarifies that fractal dimension can be same in different scales of observation but density cannot. According to Xu et al. (2010) research, fractal dimension allows us to analyze comprehensively the spatial characteristics of transportation networks includes of network density, topological structure, connectivity, and function. Generally, Street network presents two different characteristics; 1) Structural and 2) Geometrical (Sun et al., 2012; Zhang & Li, 2012; Z. Lu et al., 2016; Molinero et al., 2017). The structural structure determines the hierarchical arrangement and connectivity of roads, on the other hand geometrical properties assesses the distribution of road network over space and determines the influence of each street segment on the system (Molinero et al., 2017; Zhang & Li, 2012). According to the study of Sun et al. (2012), topological characteristics (e.g., density pattern, accessibility pattern) of a road network cannot explain its complexity alone; rather, it should be evaluated from both geometric and structural (e.g., intersection, edge) perspectives. In addition, he emphasizes that a region with a high road density does not necessarily have a high fractal dimension. Z. Lu et al. (2016) study suggests that a larger geometric fractal dimension ( $D_g$ ) indicates a more uniform distribution of roads throughout the urban area, which facilitates access to suburban areas and encourages low-density development. A larger structural fractal dimension ( $D_s$ ) implies small world phenomenon where every local road tends to connect with higher order (e.g., Collector, arterial) roads. As a result, most roads can be reached by small number of neighboring roads. They concluded that higher  $D_g$  and  $D_s$  of the road network leads to higher per capita carbon emissions of transport, and lower quality of life. The similar type of result comes from Ma et al. (2020) study where the regression equation shows a negative relationship between fractal indices of the road network and CO<sub>2</sub> emission. Sreelekha et al. (2016) further explores the relationship between street network properties with fractal dimension. Their study show that fractal dimension of the road network varies directly with network connectivity and coverage over the area (Sreelekha et al., 2016, 2017). Sreelekha et al. (2016) stated that network density can be different for a same fractal dimension value. Fractal dimension varies with spatial arrangement of road network not with network density. But network density helps to predict the network fractal dimension for a given area. Wang et al. (2017) used two different fractal dimensions (Length-radius dimension and Branch-radius dimension) to characterize the transit network and road network of Strasbourg, France. The findings of the research suggest that new road construction efforts should concentrate on areas where fractal dimensions are generally smaller than the city average. To plan a better road layout, a linear relationship between Length dimension and Branch dimension should be maintained. For the first time, Sreelekha et al. (2017) evaluates the road network of a developing country based on fractal dimension. This study attempts to determine the variation of fractal

dimension within the city and compares the fractal dimension between different parts of the city to the whole city. The outcome indicates that the minimum value corresponds to the outer suburban zone, while the maximum value corresponds to the central business district zone. One of the objectives of Karpinski et al. (2020) research is similar to the Sreelekha et al. (2017) study. In addition, he tries to compare the transport development indicators between parts of the city. Dasari and Gupta's (2020) study suggests that fractal analysis can help us to rationally identify the impact of augmented road networks on development patterns. It can quickly assess the inequality of road supply within the urban area. Zhang, Gao, et al. (2021) focus on structural fractality of urban road network. Structural fractality is related to node, edge of the road network. Researchers didn't find a specific threshold value for structural fractal dimension, like the geometric fractal dimension of the road network (i.e., 1 to 2)(Sreelekha et al., 2017). Empirical study shows that structural fractal dimension increases with the connectivity between the nodes (Zhang et al., 2021).

A series of recent studies has found that urban street network follows multifractal characteristics (Long and Chen, 2021; Murcio et al., 2015; Pavón-Domínguez et al., 2018). Because, single fractal dimension cannot effectively address the local density variation of a street network, which is one of the primary reasons for using multifractal analysis (Long and Chen, 2021; Murcio et al., 2015). Hence, a question may come up "what is multifractal". Murcio et al. (2015) defines multifractal as an object that exhibits different fractal properties at different scales and regions. According to Pavón-Domínguez et al. (2018) research, the multifractal characteristics of a road network are not related to the density of the roads. Rather it depends on how road networks are distributed across a territory and how (national or regional) subnetworks connect one another. Long and Chen (2021) research confirmed that road network bears multifractal characteristics rather than single fractal characteristics. However, multifractal characteristics (e.g., global vs local fractal dimension) are only limited in certain scaling ranges rather than in an extensive scale. Additionally, this study suggests that the street network seems to be a fractal-like system instead of real fractal structure. The important aspect of multifractal analysis is to diagnose the spatial problem that lies in system structure. The main spatial gap/problem comes from the difference between the development goal and the current status of development (Long and Chen, 2021). To solve the problem, the spatial structure of the street network should be arranged in a way so that it can optimize the geographical space (Masucci et al., 2013). However, because of the hierarchical structure of the organization and the dynamic nature of land use, the process is more complicated. This is where fractal geometry comes into play to solve the problem (Long and Chen, 2021).

## Discussion

In 1969, British mathematician Benoit Mandelbrot encountered difficulty to measure the length of Britain's coastline. One of the primary causes was the uneven shape of the coastline. It is discovered that the coastline's length varies with measurement scale (i.e., from small to large). To measure it accurately, Mandelbrot introduced the concept of fractal geometry. Fractal shape can be defined by three principles: 1) self-similarity 2) scale invariance 3) power-law relation. Fractal geometry simply means a complex geometric figure which is made from an infinite number of parts, each of the parts is exactly similar to the whole or an object which presents repeating pattern in every measurement scale (B. Mandelbrot, 1967; B. B. Mandelbrot, 1989). Various studies have shown that urban formation and its element distribution follow the fractal principle (Batty, 1991; Batty et al., 1993; Batty & Longley, 1994; Bovill, 1996, 2000; Frankhauser, 1998a, 2008; Jevrić et al., 2014a; Man & Chen, 2020; Salingeros, 2003; Shen, 2002; Terzi & Kaya, 2008a). Urban formation basically presents hierarchy (e.g., Street network), self-similarity (e.g., land-use pattern), heterogeneity in its element distribution (Frankhauser, 1998a; Jevrić et al., 2014a). As these are the fundamental characteristics of fractal geometry, fractal can effectively summarize these characteristics in a single value, which is known as fractal dimension (Bovill, 2000; Jevrić et al., 2014a; Kim et al., 2003a; Shen, 2002). The space-filling process of an urban object is also determined by fractal dimension (Kim et al., 2003a; Sreelekha et al., 2017a). After knowing the key advancement of fractal geometry, urban geographer, urban analytics have been tried to fit it in different context of urban area. For example- In 2014, Kowalski et al. first examined whether the water distribution

Table 1. Classification of articles based on the usages of fractal geometry

Field of Study	Research sub-fields	Researchers	Number
Urban water distribution network	Fractal-based planning of urban water distribution system	(Qi et al., 2014)	1
	Fractal geometry in describing the geometrical structures of water supply networks	(Kowalski et al., 2014)	1
	Monitoring of water distribution system effectiveness using fractal geometry	(Kowalski et al., 2015)	1
	Fractal geometry in designing and operating water networks	(Iwanek et al., 2020)	1
Urban formation	The morphology of urban land use	(Batty & Longley, 1988; Feng & Chen, 2010; Purevtseren et al., 2018)	3
	Urban growth and form	(Batty et al., 1989; Batty, 1991; Batty & Longley, 1994; Y. Chen et al., 2017; Man & Chen, 2020)	4
	Urban morphology	(Batty et al., 1993; Y. Chen, 2020; Y.-G. Chen, 2018; Frankhauser, 1998a)	4
	Urban pattern	(Frankhauser, 1998a, 2004, 2008; Jevrić et al., 2014b; Tannier et al., 2012)	5
Space management	Urban border	(Jevric & Romanovich, 2016)	1
	Urban agglomeration	(Frankhauser, 1998b)	1
	Urban sprawl	(Terzi & Kaya, 2008b, 2011)	2
	Urbanized area	(Y. Chen, 2015; Shen, 2002)	2
Street Network	Urban road network analysis	(Benguigui, 1995; Dasari & Gupta, 2020; Kim et al., 2003b; Lu & Tang, 2004; Mo et al., 2015; Mohajeri et al., 2012; Rodin & Rodina, 2000; Sreelekha et al., 2017b; Sun et al., 2012; Wang et al., 2017; Zhang et al., 2021; Zhang & Li, 2012)	10
	Multifractal characterization of road network	(Long & Chen, 2021; Murcio et al., 2015; Pavón-Domínguez et al., 2017, 2018)	4

network does follow the principles of fractal geometry or not. Later on, Qi et al. (2014) implements the concepts for designing better urban Water Distribution System in China. Thereafter, fractal concept used to monitor the urban water distribution system effectiveness (Kowalski et al., 2015), in designing and operating water networks (Iwanek et al., 2020). The advancement of fractal concepts in urban water distribution network

will be continuing. Then, if we look at the urban shape and structure while keeping in mind the principles of fractal geometry, we can easily find the characteristics of fractal in urban shape (Batty et al., 1989; Batty & Longley, 1994; Frankhauser, 1998a, 1998b; Salingaros, 2003). In addition, Fractal dimension can effectively describe the urban shape and structure as well as measure the urban growth in a quantitative manner (Batty, 1991; Batty & Longley, 1987; Feng & Chen, 2010; Shen, 2002). However, after few years later, it was discovered that urban formation follow multifractal characteristics (Y. Chen, 2015; Y. Chen & Wang, 2013; Frankhauser et al., 2018). Multifractal can be exemplified by urban hierarchy, road hierarchy (e.g., local level planning, regional and national level planning). But one of the important questions remains in the discussion: "How do we make decisions based on fractal dimension?" The explanation of urban form elements distribution based on FD is different than street network distribution. Here, the value of FD equal to 0 means the spatial features spreads from a single point, and if the value is equal to 2 which indicates the physical features are distributed uniformly over the city (K. S. Kim et al., 2003). According to Sreelekha et al. (2017), minimum fractal dimension value represents the outer suburban zone, whereas, maximum value represents the CBD area. In the era, where people in a city are trying to fill up open spaces by constructing new housing, city authority provides new services to the housing area. It not only makes a city more congested but also creates noise, air pollution in the city. Fractal based urban design offers more open spaces in urban area. A number of research have shown that an increase of metropolitan area FD and decrease of urban boundary FD indicate more dense, homogeneous development with the tendency to offer more free spaces (Jevrić, Knežević, Kalezić, et al., 2014). The opposite trend indicates the process of urban sprawl. A table is attached below to quickly summarize the uses of fractal in an urban context.

## Conclusion

Fractal geometry means a complex geometric shape which presents self-similarity at different levels of observation. In other words, different levels of zooming show the same shape or pattern over and over again. To measure the fractal characteristics in fractal objects, fractal dimension has been widely used. Empirical studies show that the value ranges between 0 to 2. Fractal dimension varies directly with the spatial pattern or spatial arrangement of objects.

Until now, fractal geometry has been widely used in different domains of urban planning, such as land use pattern analysis, water distribution network, urban pattern analysis, urban morphological analysis, street network, and so on. Almost every case, it is found that fractal geometry is able to explain the complex arrangement of cities and its geometric evolution over the years. Fractal geometry is a potent tool for geographical modeling and spatial analysis, and it has been utilized for a long time in urban studies.

This paper discusses the various applications of fractals in urban planning and provides an overview of how fractal geometry fits into various urban components. In addition, it synthesizes and summarizes the existing knowledge and concepts. It provides easy interpretation of complex terminology widely used in fractal-based research and explain why fractal dimension varies for different spatial pattern of urban area.

In summary, this paper may aid future researchers by introducing them to the fundamentals of fractal geometry and demonstrating how it relates to various aspects of urban planning. In addition, future researchers in this field will gain a comprehensive understanding of fractals and will be able to initiate new studies by identifying knowledge gaps or existing information.

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