



UNSTEADY FLOW PAST AN OSCILLATORY PLATE IN A ROTATING SYSTEM

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Abstract: An initial value investigation is made of motion of an incompressible, homogeneous, viscous fluid over a porous plate with uniform suction or blowing. Both the plate and the fluid are in state of solid body rotation with constant angular velocity about z- axis normal to the plate and the plate is assumed to be oscillatory with a given velocity. A solution describing the general feature of the unsteady hydrodynamic boundary layer flow in a rotating fluid with suction has been obtained. With the various values of suction parameter we describe the nature of velocity profile against z-axis considering different transformation and boundary conditions are calculated and describe it in graphically. Special attention has been given to the physical interpretation of mathematical results obtained for steady state solution. The significant effect of the suction parameter on the flow phenomenon in this case has been investigated.

Key words: Oscillatory, suction, blowing

Introduction

The studies on unsteady free convection flows in the porous medium have been of interest to the engineering community and to the investigators dealing with problems in geophysics and astrophysics. Several solutions have been proposed for the oscillatory Ekman boundary layer flow in presence of free convection flows.

Debnath and Mukherjee (1973) have studied the motion in an incompressible homogeneous, viscous fluid bounded by porous plates with uniform suction. Both the fluid and the plates are in a state of solid body rotation with constant angular velocity Ω about z-axis normal to the plate and a non-torsional oscillation of a given frequency ω is imposed on the plate for generation of unsteady flow in a rotating system.

Seth et al. (1981) studied unsteady hydromagnetic flow past a porous plate in a rotating medium with time dependent free stream. They investigated when the fluid is permeated by a transverse magnetic field. They obtained an analytical solution by Laplace transform technique and two cases of interest were discussed i.e., (i) impulsive change in the free

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stream velocity and (ii) an accelerated change in the free stream velocity. Deca et al. (1999) investigated the flow past an accelerated horizontal plate in a rotating fluid. They considered a semi-infinite mass of an incompressible viscous fluid bounded by an infinite flat that was initially rotated with uniform angular velocity about an axis normal to the plate.

In the present paper, the work of Kishore et al. (1981) has been reviewed under different boundary conditions. The plate is assumed to be accelerated with a velocity $Ae^{i\sigma t^m}$ (A and m being the constants). Debnath and Mukherjee (1973) have considered elliptic harmonic oscillations of plate. Here an oscillatory Ekman boundary layer flow has been considered in case of free convection flow.

Materials and Methods

The Navier-Stokes equation and the continuity equation for the unsteady motion of a viscous fluid in a rotating co-ordinate is

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u - 2\Omega k \times u = -\rho^{-1} \nabla p + \nu \nabla^2 u \quad (1)$$

where, $u = (u, v, w)$ is the velocity vector, k is the unit vector along z -axis, p is the pressure including centrifugal term, ρ is the density and ν be the kinematics viscosity.

We assume that the velocity field depends on z and t alone, so that

$$u(z, t) = [u(z, t), v(z, t), w(z, t)] \quad (2)$$

$$\text{we know that, } \text{div } u = 0 \quad (3)$$

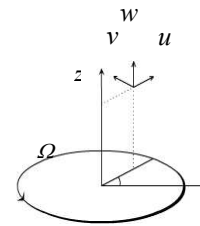


Fig. 1. The flow configuration and the co-ordinate system.

Thus, equation (1) is equivalent to the following equation (in the absence of pressure gradient)

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} \quad (4)$$

$$\frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} \quad (5)$$

Now it follows from equation (3) together with uniform suction (blowing) that $w = -w_0$ is constant. Obviously $w_0 < 0$ for blowing. So, equation (4) and (5) becomes

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} \quad (6)$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} \quad (7)$$

Multiplying equation (7) by i and adding with (6) we get,

$$\frac{\partial}{\partial t}(u + iv) - w_0 \frac{\partial}{\partial z}(u + iv) + 2\Omega(-v + iu) = \nu \frac{\partial^2}{\partial z^2}(u + iv)$$

For $q = u + iv$ and the above equation becomes,

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} + 2i\Omega q = \nu \frac{\partial^2 q}{\partial z^2} \quad (8)$$

And the boundary conditions for the problem are-

$$\left. \begin{aligned} q(z, t) &= 0 \text{ for all } z = 0 \text{ and } t \leq 0, \\ q(z, t) &= Ae^{i\sigma t} t^m, w = w_0 \text{ for all } z = 0 \text{ and } t > 0 \\ q &\rightarrow 0 \text{ or finite as } z \rightarrow \infty \text{ for } t > 0 \end{aligned} \right\} \quad (9)$$

where, A is a constant with the dimension of velocity. The present boundary conditions are generalization of Debnath and Mukherjee (1973) which appear to have some physical significance for the understanding of geophysical and astrophysical problems and is likely to reveal some important and untrusting characteristic features of the hydrodynamic spin up flows as pointed out by Debnath (1975).

Now the dimensionless form of the equation (8) is,

$$\frac{\partial^2 q}{\partial z^2} + s \frac{\partial q}{\partial z} - iqE = \frac{E}{2} \frac{\partial q}{\partial t} \quad (10)$$

where E is the rotational parameter. With the boundary conditions,

$$\left. \begin{aligned} q &= 0 \text{ everywhere for } t \leq 0, q = Ae^{i\sigma t} t^m \text{ at } z = 0, t > 0 \\ q &= 0 \text{ or finite as } z \rightarrow \infty, t > 0 \end{aligned} \right\} \quad (11)$$

To solve the equation (10) let us introduce the Laplace transforms $q(z, p)$ of $q(z, t)$ defined by the integral

$$q(z, p) = \int_0^\infty e^{-pt} q(z, t) dt \quad (12)$$

Now putting these values in equation (10) to evaluate $\partial q / \partial t$ because the other terms become zero as the behavior of constants. Therefore,

$$\frac{\partial q}{\partial t} = \int_0^\infty e^{-pt} q'(z, t) dt = pq(z, p) \quad (13)$$

So the equation (10) becomes,

$$\frac{\partial^2 q}{\partial z^2} + s \frac{\partial q}{\partial z} - (iE + \frac{Ep}{2})q = 0 \quad (14)$$

Again with the help of equation (12) we have,

$$q(z, p) = \int_0^\infty e^{-pt} q(z, t) dt = A \int_0^\infty e^{-(p-i\sigma)t} t^m dt \quad (15)$$

Solving this we get,

$$\left. \begin{aligned} q(z, p) &= \frac{A\Gamma(m+1)}{(p-i\sigma)^{m+1}} \text{ as the boundary conditions as} \\ q(z, t) &= Ae^{i\sigma t} t^m \text{ at } z = 0, t > 0 \text{ and } q(z, t) = 0 \text{ or finite as } z \rightarrow \infty \end{aligned} \right\} \quad (16)$$

Now from equation (14) we have

$$q'' + sq' - (iE + \frac{pE}{2})q = 0 \quad (17)$$

With the boundary conditions

$$q(z, p) = \frac{A\Gamma(m+1)}{(p-i\sigma)^{m+1}} \text{ at } z = 0, q(z, p) = 0 \text{ or finite as } z \rightarrow \infty \quad (18)$$

By using Laplace's transformation we can write,

$$\begin{aligned}
 q(z, p) &= q(0) \left[\frac{(z + s/2)}{(z + s/2)^2 - \sqrt{(s^2/4 + c)^2}} - \frac{s/2}{(z + s/2)^2 - \sqrt{(s^2/4 + c)^2}} \right. \\
 &\quad \left. + \frac{s}{(z + s/2)^2 - (\sqrt{s^2/4 + c})^2} \right] + \frac{B}{(z + s/2)^2 - (\sqrt{s^2/4 + c})^2} \quad (19) \\
 &= q(0) \left[\frac{(z + s/2)}{(z + s/2)^2 - \sqrt{(s^2/4 + c)^2}} + \frac{s/2}{(z + s/2)^2 - \sqrt{(s^2/4 + c)^2}} \right] \\
 &\quad + \frac{B}{(z + s/2)^2 - (\sqrt{s^2/4 + c})^2}
 \end{aligned}$$

Results

Now by applying inverse Laplace's transformation equation (19) becomes,

$$q(z, t) = e^{-sz/2} \left[q(0) \left\{ \frac{e^{kz} + e^{-kz}}{2} + \frac{s/2}{k} \cdot \frac{e^{kz} - e^{-kz}}{2} + \frac{B}{k} \cdot \frac{e^{kz} - e^{-kz}}{2} \right\} \right] \quad (20). \text{ Now, } q(z, p) = 0 \text{ for } z \rightarrow \infty$$

and as such $B = 0$ also simplifying equation (20) we get,

$$q(z, t) = q(0) e^{-sz/2} \cdot e^{-kz} = \frac{\Gamma(m+1)}{(p - i\sigma)^{m+1}} \exp \left[-zs/2 - z\sqrt{E/2} \left(p + \frac{s^2 + 4iE}{2E} \right)^{1/2} \right] \quad (21). \text{ This is the}$$

solution of the problem.

Now we consider the differential equation (14) for an oscillatory boundary layer flow,

$$\frac{\partial^2 q}{\partial z^2} + s \frac{\partial q}{\partial z} - (iE + \frac{pE}{2})q = 0 \quad (22). \text{ With the boundary conditions}$$

$$q(z, t) = 0 \text{ for } t \leq 0, \text{ and } q(z, t) = 0 \text{ as } z \rightarrow \infty, t > 0$$

$$q(z, t) = 1 + \frac{\varepsilon}{2} [e^{iwt} + e^{-iwt}] \text{ at } z = 0 \quad (23). \text{ We consider the solution of the form}$$

$$q(z, t) = q_0(z) + \frac{\varepsilon}{2} [e^{iwt} q_1(z) + e^{-iwt} q_2(z)] \quad (24).$$

Putting these in equation (22) and equating q_0, q_1, q_2 we have as well as their boundary conditions, $q_0''(z) + sq_0' - iEq_0(z) = 0$ (25) having the boundary conditions

$$q_0(z) = 1 \text{ at } z = 0 \text{ and } q_0(z) = 0 \text{ at } z \rightarrow \infty \quad (26) \text{ again, } q_1''(z) + sq_1'(z) - (1 + w)iEq_1(z) = 0 \quad (27).$$

With the boundary conditions, $q_1(z) = 1$ at $z = 0$ and $q_1(z) = 0$ at $z \rightarrow \infty$ (28) and

$$q_2''(z) + sq_2'(z) - (1 - w)iEq_2(z) = 0 \quad (29). \text{ With the boundary condition}$$

$$q_2(z) = 1 \text{ at } z = 0 \text{ and } q_2(z) = 0 \text{ at } z \rightarrow \infty \quad (30).$$

Here we see that the equation (25), (27) and (29) with their boundary conditions are of ordinary differential equation. For the solution of these equations we may consider

$$q_0(z) = e^{mz} \quad (31). \text{ The solution of the equation (25) may}$$

$$\text{be } q_0(z) = C_1 e^{\frac{-s + \sqrt{s^2 + 4iE}}{2} z} + C_2 e^{\frac{-s - \sqrt{s^2 + 4iE}}{2} z} \quad (32). \text{ Where } C_1 \text{ and } C_2 \text{ are the arbitrary constants.}$$

By applying the boundary condition (26) we get, $C_1 = 0$ and $C_2 = 1$, then equation (32) be

of the form $q_0(z) = e^{\frac{-s-\sqrt{s^2+4iE}}{2}z}$ (33). Similarly we get the solutions for q_1 and q_2 as

$$q_1(z) = e^{\frac{-s-\sqrt{s^2+4(1+w)iE}}{2}z} \quad (34) \text{ and } q_2(z) = e^{\frac{-s-\sqrt{s^2+4(1-w)iE}}{2}z} \quad (35)$$

Now for an oscillatory motion of rotating flow with the help of equation (33), (34) and (35) we can get the solution.

Discussion

The solution given by equation (21) of (14) describes the general feature of the unsteady hydrodynamic boundary layer flow in a rotating system. Equation (33), (34) and (35) represent the velocity for the given boundary conditions. The effects of suction and blowing on the steady state solution have been explained graphically.

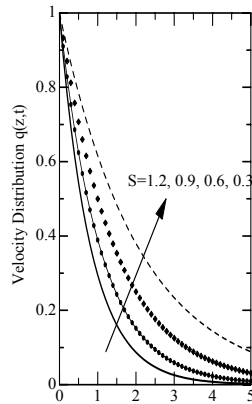


Fig. 1. Velocity distribution against z-axis for different values of suction or blowing (positive).

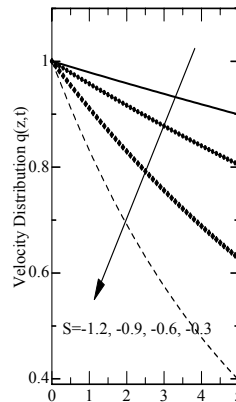


Fig. 2. Velocity distribution against z-axis for different values of suction or blowing (negative).

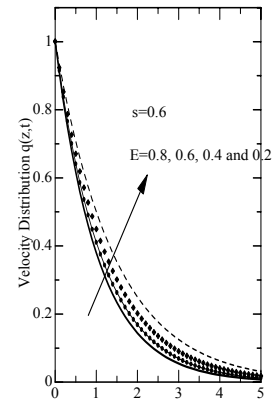


Fig. 3. Velocity profile against z-axis for different values of rotation parameter E and S = 0.06.

Fig. 1 displays the effect of suction on the steady state velocity profile. Here, we have plotted velocity profile $q(z, t)$ against z for different values of suction parameter given by $s=0.3, 0.6, 0.9$ and 1.2 respectively. It is noticed that velocity decreases more rapidly with increase in suction parameter and consequently the boundary layer thickness is reduced with the increase in suction parameter.

Again Fig. 2 represents the effect of suction on the steady state velocity profile for its negative values. Here we have also plotted velocity profile $q(z, t)$ against z on the flow for negative values of suction parameter given by $s = -1.2, -0.9, -0.6$ and -0.3 . In this case we observe that the velocity decreases more rapidly with increase in suction parameter.

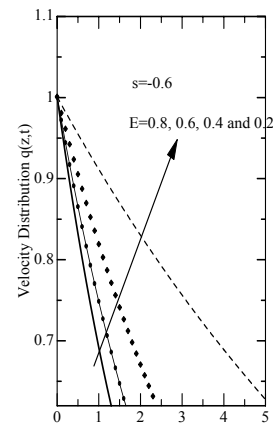


Fig. 4. Velocity profile against z-axis for different values of rotation parameter E and S = 0.06.

Also by considering Fig. 3 and Fig. 4 we see that when the suction parameter is fixed but vary the rotational parameter E then with an increasing values of E the velocity profile decreases for both the positive and negative value of the suction parameter s .

Conclusion

In the unsteady flow past an oscillatory flat plate in a rotating system we can write the conclusion as follows-

(i) The velocity decreases with the increasing values of the suction parameter. (ii) For the fixed values of the suction parameter the velocity profile increases due to the decreasing values of the rotational parameter. (iii) In the case of blowing the velocity profile diverges far away from the surface of the rotating plate.

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