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RESEARCH ARTICLE **PHYSICAL SCIENCE**

THE REACTIONS IN THE VISCOELESTIC BOUNDARY LAYER FLOW OF FLUID

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Abstract: The effect of the fluid elasticity on a viscoelastic fluid is considered. A similarity transformation is used which reduces the concentration equation to an ordinary differential equation. An exact solution is adopted for the velocity field and the concentration field is computed numerically. Results show that viscoelasticity tends to reduce the heat transfer from the wall. These effects diminish for high reaction and high schmidt number.

Key words: Walter's 'liquid B', Schmidt number, Newtonian fluid

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Introduction

The studies of the transport of heat, mass, momentum in the laminar boundary layer on moving inextensible or stretching flat surface are considered in electrochemistry and polymer processing. The steady boundary layer flow of a Newtonian fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the sheet has been extended to fluids obeying Non-Newtonian constitutive equations. While Siddappa and Abel (1985) studied the shear-driven flow of the viscoelastic Walter's 'liquid B', Anderson and Dandapat (1992) considered the boundary layer flows of micropolar and power law fluids. The heat and mass transfer problem associated with the Newtonian boundary flow past a stretching sheet was studied by Gupta and Gupta (1977). Unfortunately, the presence of a chemical reaction term in the mass diffusion equation generally destroys the possibilities of finding a similarity solution, except in the case of a stagnation point flow (Chambre and Young, 1958). However Anderson (1993) demonstrated that the mass transfer problem studied by Gupta and Gupta (1977) can be extended to diffusion of a chemically reactive species and still allow for similarity solutions.

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In this research work we have found the solution of the above mentioned problem in form of a series. We have shown the concentration wall gradient in the tabular form and the concentration profile for different values of Schmidt number in the graphically.

Materials and Methods

Consider the flow of a viscoelastic fluid past a flat and impermeable elastic surface. The incompressible prototype fluid, designated as liquid B by Walters (1962), is confined to the half-space y>0 above the surface. By applying two equal and opposite forces along x-axis, the elastic sheet is being stretched with a speed proportional to the distance from the fixed origin x = 0. The resulting motion of the otherwise quiescent fluid is thus caused solely by the moving surface. Bread and Walters (1964) derived the steady two dimensional boundary layer equations for the Walters "liquid B" to first order elasticity i.e. the short memory fluids with short relaxation times. The continuity, momentum and energy equations governing the flow in the viscoelastic boundary layer along the stretching sheet (Anderson, 1993) .

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=o \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}-kc \tag{3}$$

The boundary conditions are-

$$u=ax,\ v=0 \text{ and } c=c_W \text{ at } y=0, \qquad u\to 0,\quad c\to 0 \quad \text{as} \quad \mathrm{y}\to\infty \tag{4}$$

where u and v are the velocity components of the fluid in the x and y directions respectively. Here c is the concentration field governed by the diffusion equation, c_w is the constant prescribed value at the stretching sheet, D is the diffusion coefficient, k is the reaction rate, υ is the limiting kinematic viscosity at small rates of shear.

***Solution of the problem*:** Let us now introduce the following dimensionless variables in to the Eqs. (1) to (4)

$$\bar{x}=\frac{x}{L},\ \bar{y}=\frac{y}{L}\sqrt{\mathrm{Re}},\ \bar{u}=\frac{u}{a},\ \bar{v}=\frac{v}{a}\sqrt{\mathrm{Re}},\ \bar{c}=\frac{c}{c_w}=\theta \tag{5}$$

where Re is the Reynold number.

The nondimensional equations take the following forms (Eqs. 6 to 9)

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}=o \tag{6}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}=\upsilon\frac{\partial^2\bar{u}}{\partial y^2} \tag{7}$$

$$\bar{u}\frac{\partial\theta}{\partial x}+\bar{u}\frac{\partial\theta}{\partial y}=\frac{D}{\upsilon}\frac{\partial^2\theta}{\partial y^2}-k\theta \tag{8}$$

$$\bar{u}\frac{\partial\theta}{\partial x}+\bar{u}\frac{\partial\theta}{\partial y}=Sc\frac{\partial^2\theta}{\partial y^2}-k\theta \tag{9}$$

Let us now consider the following relations satisfying the continuity equation-

$$\psi = x.f(\eta),\ \theta=\theta(\eta),\quad \eta = y \tag{10}$$

The momentum and concentration Eqs. (7) and (9) are then transformed into ordinary differential equations (Anderson, 1993)-

$$f'''+ff''-f'^2=0 \tag{11}$$

$$Sc\theta''+f\theta'-k.\theta=0 \tag{12}$$

The corresponding boundary conditions then become-

$$f=0,\ f_\eta=1 \text{ and } \theta=1 \text{ at } \eta=0 \tag{13}$$

$$f_\eta=0 \text{ and } \theta\to 0 \text{ as } \eta\to\infty$$

Here k = $a\alpha/\mu$ is the elastic parameter, Sc = D/υ is the Schmidt number and $\beta=kc_w/a$ is the reaction rate parameter.

Here the concentration field θ is coupled with velocity field through the dimensionless stream function *f* in equation (12), the momentum equation (11) is uncoupled from the mass transfer equation (12). The momentum boundary layer equation (11) subject to the boundary condition (13) has been solved numerically by Rajgopal *et al.* (1984) and analytically by Siddappa and Abel (1985). Their exact solution was

$$f=[1-\exp(-\eta)] \tag{14}$$

By using Eq. (14), the concentration Eq. (12) can be written in the following form

$$\theta''+\frac{1-e^{-\eta}}{Sc}\theta'-\frac{k}{Sc}.\theta=0 \tag{15}$$

The solution of the concentration equation satisfying the boundary condition Eq. (13) has been obtained in the following form

$$\theta=\exp(-\eta)\left[\begin{array}{l}1-\frac{1}{2}(1-\frac{k}{Sc})\eta^2+(\frac{1}{Sc}-2+\frac{2k}{Sc})\frac{\eta^3}{\Gamma 4}\\ +\left\{-3+\frac{3}{Sc}-(\frac{2}{Sc}-2)\frac{k}{Sc}+\frac{k^2}{Sc}\right\}\frac{\eta^4}{\Gamma 5}+\Lambda\ \Lambda\end{array}\right] \tag{16}$$

For the Newtonian fluid k = 0 and non-reacting fluid β = 0.

Results

The values of concentration wall gradient are obtained for different values of Schmidt numbers, Sc and non reacting parameter β are given in the following tables. The concentration profiles that we obtained are shown in the following figures.

Table 1. The concentration wall gradient for k = 0.0.

Sc β	0.01	0.50	1.00	5.00	20.0
0.01	0.85669	0.77373	0.68922	0.01868	-2.40493
0.50	0.85429	0.65635	0.46007	-0.90328	-2.73941
1.00	0.85148	0.53468	0.21865	-2.03374	-6.11163
5.00	0.83219	-0.50764	-1.98863	-17.97518	-143.4533
20.0	0.75809	-5.50850	-14.63448	-186.96980	*******

Table 2. The concentration wall gradient for k = 0.05.

Sc β	0.01	0.50	1.00	5.00	20.0
0.01	0.06297	0.04681	0.03051	-0.09314	-0.44954
0.50	0.06266	0.03477	0.01332	0.09715	4.45495
1.00	0.06235	0.02013	-0.01358	0.05341	5.71405
5.00	0.05979	-0.18205	-0.56927	-8.26414	-120.4130
20.0	0.04964	-2.28804	-8.04437	-176.67490	********

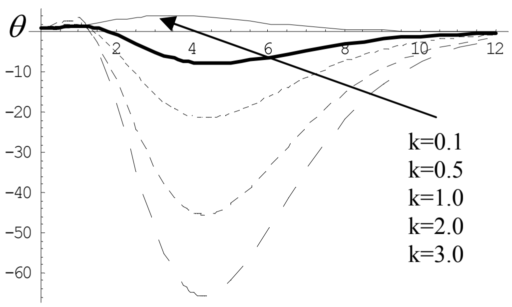


Fig. 1. Concentration profile for different values of k against η where Sc = 0.1.

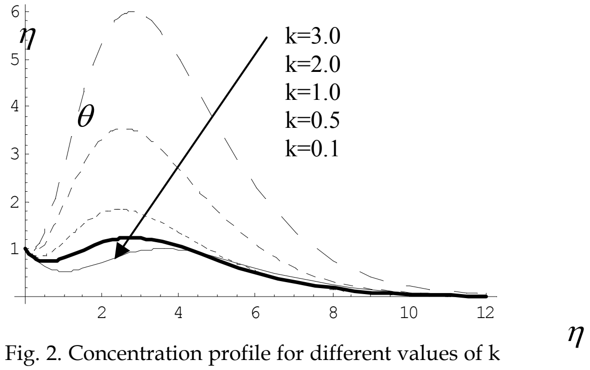
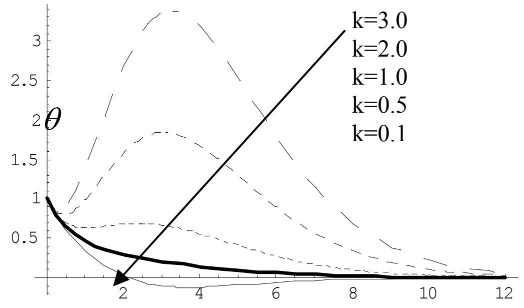


Fig. 2. Concentration profile for different values of k against η where Sc = 0.4.



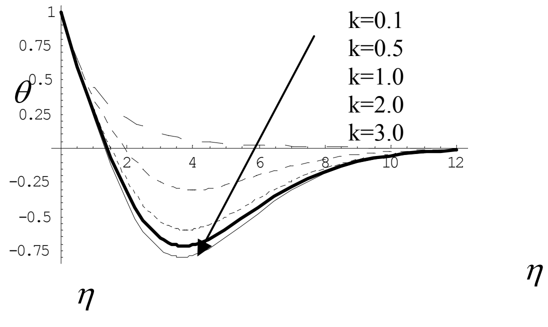


Fig. 3. Concentration profile for different values of k against η where Sc = 1.0.

Fig. 4. Concentration profile for different values of k against η where Sc = 5.0.

Discussion

The present problem is solved numerically for five different values of Schmidt number Sc in the range $0.01 \leq Sc \leq 20$ and for five values of non-reaction rate parameter $\beta \leq 20$ for reaction. First of all, it should be recalled from the analysis of Andersson *et al.* (1992) that the principal effect of a destructive chemical reaction i,e $\beta > 0$ in a Newtonian boundary layer (k = 0) is to reduce the thickness of the thermal boundary layer and increase the magnitude of the temperature gradient $\theta'(0)$ at the stretching sheet. The computed wall gradients for the Newtonian case are summarized in the Table 1 and Table 2 for k = 0.05 and different values of β and Sc. From Table 1 we see that the increasing values of the Sc the wall gradient decreases, while for increasing values of β the wall gradient decrease. For large values of η the magnitude of the concentration decreased to a limiting point. For both the two cases k = 0.0 and k = 0.05 the concentration wall gradient do not exist for β = 20.0 and Sc = 20.0.

From the figs 1 to 4 we see that the concentration profile increases for increasing values of k and these effect is significant near the surface of the plate. But far way from the plate these effects are negligible. When Sc is equal or greater than 5 the graph becomes inverse.

Conclusion

From the studies of the reaction on the viscoelastic boundary layer flow of fluid we can conclude that (i) the effect of viscoelasticity is observed to decrease in a monotone fashion with increasing reaction rate; (ii) for large value of β (i.e, $\beta \geq 20.0$) and Schmidt number the concentration wall gradient does not exist; and, (iii) for large value of η the concentration profiles go to a limiting point.

References

Anderson, H.I. 1993. On approximate formulas for low prandtl number heat transfer in laminar wedge flows. *International Journal of Heat Fluid Flow*, 9**:** 241-243.

Anderson, H.I. and Dandapat, B.S. 1992. Flow of a powe-law fluid over a stretching sheet. *Stability Applied Annual Continuous Media*, 1**:** 339-347.

Beard, D.W. and Walters, K. 1964. Elastico –viscous boundary layer flows 1. Two dimensional flow near a stagnation point. *Proceedings of Cambridge Phil Society*, 60**:**667-674.

Chambre, P.L. and Young, J.D. 1958. On the diffusion of a chemically reactive species in a laminar boundary flow. *Physics Fluid*, 1**:** 45-54.

Gupta, P.S. and Gupta, A.S. 1977. Heat and Mass transfer on a stretching sheet with suction or blowing. *Canadian Journal of Chemical Engineering,* 55**:** 744-746.

Rajgopal, K.R.; Na, T.Y. and Gurta, A.S. 1984. Flow of viscoelastic fluid over a stretching sheet. *Rheol. Acta,* 23**:** 213-215.

Siddappa, B. and Abel, S. 1985. Non-Newtonian flow past a stretching plate. *Zeitschrift für Angewandte Mathematik und Physik* (*ZAMP*), 36**:** 890-892.

Ahmmed, S. F. and Sultana, T. 2007. The reactions in the viscoelestic boundary layer flow of fluid. *Khulna University Studies*, 8(2): 332-336.

Walters, K. 1962. Non-Newton effects in some elastico-viscous liquids whose behaviour at small rates of shear is characterized by general linear equation of state. *Quarterly Journal of Mechanics and Applied Mathematics,* 15**:** 63-76.