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UNSTEADY MHD FLOW BETWEEN TWO PARALLEL POROUS FLAT PLATES

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Abstract: The present paper is concerned with the study of an unsteady flow of a viscous incompressible electrically conducting fluid between two infinite parallel porous flat plates under the action of a uniform transverse magnetic field. The magnetic Reynolds number is taken to be sufficiently large so the induced magnetic field has been taken into account. The Laplace transform technique has been used to obtain the expressions for the velocity field and the induced magnetic field. The effect of the magnetic parameter on the velocity and induced magnetic field are presented graphically and discussed thereafter.

Keywords: Magnetic Reynolds number, Induced magnetic field, Magnetic parameter, Magnetic Prandtl number, Porous flat plates, Suction and injection.

Introduction

Due to the wide-ranging applications of magneto-fluid dynamics in different branches of science many authors have studied the effects of externally applied transverse magnetic field on flows of electrically conducting viscous fluids. The flow of viscous incompressible and electrically conducting fluid between two infinite parallel plates in presence of a magnetic field, when one of the plates starts moving impulsively from rest was studied by Katagiri, (1962). Muhuri, (1963) has studied the flow of a viscous incompressible and electrically conducting fluid between two porous walls, when one of the walls moves with uniform acceleration and there is uniform suction and injection. Katagiri, (1962) and Muhuri, (1963) presented their analysis by assuming the magnetic Reynolds number to be small so the induced magnetic field was neglected. Pande et.al. (1976) investigated the effect of the induced magnetic field for an unsteady hydro-magnetic flow near an oscillating wall. Sattar and Hossain, (1992) has studied the flow of a viscous incompressible and electrically conducting fluid near a moving infinite porous plate in presence of variable suction and variable transverse magnetic field taking into account the induced magnetic field. Rahman and Azad, (1998) has studied the effect of transverse magnetic field on unsteady MHD free convection flow past an impulsively infinite vertical plate when there is constant heat at the plate. In this case the induced magnetic field has been taken into account. The objective of this paper is to study the effects of transverse magnetic field on unsteady flow of a viscous incompressible electrically conducting fluid between two infinite porous flat plates when the lower plate is moving. The magnetic Reynolds number is not small so that the induced magnetic field has been taken into account. The Laplace transform technique has been used to obtain the expression for the velocity field and induced magnetic field.

Mathematical Analysis

The unsteady MHD flow of an electrically conducting viscous incompressible fluid between two parallel, non-conducting infinite porous flat plates at $y' = 0$ and $y' = d$ has been considered. At time $t' \leq 0$ the fluid and the plates are at rest. At time $t'>0$ the lower plate begins to move in its own plane in the x'- direction with velocity $U_0 e^{a' t'}$. A uniform magnetic field of strength H_o is acting perpendicularly to the plates. The magnetic Reynolds number of the flow is not small so that the induced magnetic field has been taken into account. Fluid is being injected into the flow region with constant velocity w_0 through the plate at $y' = 0$ and is being sucked away with the same velocity through the plate at $y' = d$. The flow is in the x' - direction and y'-axis is normal to the plates. Since the plates are infinite in extent all physical quantities are functions of y' and t' only. The pressure gradient is assumed to be zero. The equations of motion taking into account the induced magnetic field are in (Pai, 1962). According to the condition of our problem, the flow is governed by the following differential equations:

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$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial t'^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial H_x'}{\partial y'} \tag{1}$$

$$\frac{\partial H_x'}{\partial t'} + v'\frac{\partial H_x'}{\partial y'} = \frac{1}{\sigma\mu_0}\frac{\partial^2 H_x'}{\partial y'^2} + H_0\frac{\partial u'}{\partial y'} \tag{2}$$

$$\frac{\partial v'}{\partial y'} = 0 \tag{3}$$

where H_x'- the induced magnetic field, H_0 -the constant externally applied transverse magnetic field, μ_0 - the magnetic permeability, σ - the electrical conductivity and the other quantities have their usual meanings.

Equation (3) integrates to $v' = w_0\,(w_0 > 0)$ (4)

where w_0 is the constant velocity of injection at the lower plate and constant suction velocity at the upper plate, the velocity being normal to the plates.

As the plates are non-conducting the initial and boundary conditions of the flow are,

$$t' \le 0;\ u' = 0,\ H_x' = 0 \quad for \quad 0 \le y' \le d \tag{5}$$

$$t' \phi\, 0;\ u' = U_0 e^{a't'},\ H_x' = 0 \quad at \quad y' = 0$$
$$u' = 0,\ H_x' = 0 \quad at \quad y' = d. \tag{6}$$

Introducing the following non-dimensional equations

$$u = \frac{u'}{U_0},\ y = \frac{y'}{d},\ H = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}}\frac{H'_x}{U_0}$$

$$t = \frac{t' w_0}{d},\ a = \frac{a' d}{w_0},\ R = \frac{w_0 d}{v},\ M = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}}\frac{H_0}{w_0},\ P_m = v\sigma\mu_0 \tag{7}$$

in equations (1) and (2) we have

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + M\frac{\partial H}{\partial y} \tag{8}$$

$$\frac{\partial H}{\partial t} + \frac{\partial H}{\partial y} = \frac{1}{RP_m}\frac{\partial^2 H}{\partial y^2} + M\frac{\partial u}{\partial y}. \tag{9}$$

The initial and boundary conditions become in non-dimensional form:

$$t \le 0:\ u = 0,\ H = 0,\ for\ 0 \le y \le 1 \tag{10}$$

$$t\, \phi\, 0:\ u = e^{at},\ H = 0,\ at\ y = 0$$
$$u = 0,\ H = 0,\ at\ y = 1 \tag{11}$$

where, R- Reynolds number, P_m- Magnetic Prandtl number, M- Magnetic parameter. We will use Laplace transform technique to solve the coupled equations (8) and (9) and assume that $P_m = 1$ which is a plausible assumption in most hydromagnetic problems.

Taking Laplace transform of equations (8) and (9) and using (10) we have respectively

$$\frac{1}{R}\frac{d^2\bar{u}}{dy^2} - \frac{d\bar{u}}{dy} + M\frac{d\bar{H}}{dy} - s\bar{u} = 0 \tag{12}$$

$$\frac{1}{R}\frac{d^2\bar{H}}{dy^2} - \frac{d\bar{H}}{dy} + M\frac{d\bar{u}}{dy} - s\bar{H} = 0 \tag{13}$$

where $\bar{u}(y,s) = \int_0^\infty e^{-st}u(y,t)dt,\qquad \bar{H}(y,s) = \int_0^\infty e^{-st}H(y,t)dt.$

The boundary condition (11) is transformed to

$$\bar{u} = \frac{1}{s-a},\ \bar{H} = 0,\ at\ y = 0$$

$$\bar{u} = 0, \quad \bar{H} = 0, \quad at \quad y = 1. \tag{14}$$

In order to uncouple equation (12) and (13) we add them and subtracting (13) from (12) we get,

$$\frac{1}{R}\frac{d^2 X}{dy^2} - (1 - M)\frac{dX}{dy} - sX = 0 \tag{15}$$

$$\frac{1}{R}\frac{d^2 Q}{dy^2} - (1 + M)\frac{dQ}{dy} - sQ = 0 \tag{16}$$

where $X = \bar{u} + \bar{H}$ and $Q = \bar{u} - \bar{H}$, subject to boundary conditions

$$X = \frac{1}{s - a}, \quad Q = \frac{1}{s - a}, \quad at \quad y = 0$$

$$X = 0, \quad Q = 0, \quad at \quad y = 1. \tag{17}$$

Solutions of the equation (15) and (16) under boundary condition (17) yield,

$$\bar{u} = \frac{1}{2}e^{k_1 y}\sum_{n=0}^{\infty}\left[\frac{e^{-a_1(b_1^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2(b_1^2+s_1)^{\frac{1}{2}}}}{s_1}\right] + \frac{1}{2}e^{k_2 y}\sum_{n=0}^{\infty}\left[\frac{e^{-a_1(b_2^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2(b_2^2+s_1)^{\frac{1}{2}}}}{s_1}\right] \tag{18}$$

and

$$\bar{H} = \frac{1}{2}e^{k_1 y}\sum_{n=0}^{\infty}\left[\frac{e^{-a_1(b_1^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2(b_1^2+s_1)^{\frac{1}{2}}}}{s_1}\right] - \frac{1}{2}e^{k_2 y}\sum_{n=0}^{\infty}\left[\frac{e^{-a_1(b_2^2+s_1)^{\frac{1}{2}}}}{s_1} - \frac{e^{-a_2(b_2^2+s_1)^{\frac{1}{2}}}}{s_1}\right] \tag{19}$$

where

$$k_1 = \frac{R(1 - M)}{2}, \quad k_2 = \frac{R(1 + M)}{2}, \quad a_1 = (2n + y)R^{\frac{1}{2}}, \quad a_2 = (2n + 2 - y)R^{\frac{1}{2}}$$

$$b_1^2 = \frac{R(1 - M)^2 + 4a}{4}, \quad b_2^2 = \frac{R(1 + M)^2 + 4a}{4}, \quad s_1 = s - a.$$

Using tables of Inverse Laplace Transform (Bateman, 1954) we get the expression for u and H from (18) and (19) respectively

$$u = \frac{e^{(at+k_1 y)}}{4}\sum_{n=0}^{\infty}\left[e^{-a_1 b_1} erfc\left(\frac{a_1 t^{-\frac{1}{2}}}{2} - b_1 t^{\frac{1}{2}}\right) + e^{a_1 b_1} erfc\left(\frac{a_1 t^{-\frac{1}{2}}}{2} + b_1 t^{\frac{1}{2}}\right) - \right.$$

$$\left.\left\{e^{-a_2 b_1} erfc\left(\frac{a_2 t^{-\frac{1}{2}}}{2} - b_1 t^{\frac{1}{2}}\right) + e^{a_2 b_1} erfc\left(\frac{a_2 t^{-\frac{1}{2}}}{2} + b_1 t^{\frac{1}{2}}\right)\right\}\right] + \frac{e^{(at+k_2 y)}}{4}\sum_{n=0}^{\infty}\left[e^{-a_1 b_2} erfc\left(\frac{a_1 t^{-\frac{1}{2}}}{2} - b_2 t^{\frac{1}{2}}\right) + \right.$$

$$\left. e^{a_1 b_2} erfc\left(\frac{a_1 t^{-\frac{1}{2}}}{2} + b_2 t^{\frac{1}{2}}\right) - \left\{e^{-a_2 b_2} erfc\left(\frac{a_2 t^{-\frac{1}{2}}}{2} - b_2 t^{\frac{1}{2}}\right) + e^{a_2 b_2} erfc\left(\frac{a_2 t^{-\frac{1}{2}}}{2} + b_2 t^{\frac{1}{2}}\right)\right\}\right] \tag{20}$$

$$H=\frac{e^{(at+k_1y)}}{4}\sum_{n=0}^{\infty}\left[e^{-a_1b_1}erfc\left(\frac{a_1t^{-\frac{1}{2}}}{2}-b_1t^{\frac{1}{2}}\right)+e^{a_1b_1}erfc\left(\frac{a_1t^{-\frac{1}{2}}}{2}+b_1t^{\frac{1}{2}}\right)-\right.$$

$$\left.\left\{e^{-a_2b_1}erfc\left(\frac{a_2t^{-\frac{1}{2}}}{2}-b_1t^{\frac{1}{2}}\right)+e^{a_2b_1}erfc\left(\frac{a_2t^{-\frac{1}{2}}}{2}+b_1t^{\frac{1}{2}}\right)\right\}\right]-\frac{e^{(at+k_2y)}}{4}\sum_{n=0}^{\infty}\left[e^{-a_1b_2}erfc\left(\frac{a_1t^{-\frac{1}{2}}}{2}-b_2t^{\frac{1}{2}}\right)+\right.$$

$$\left.e^{a_1b_2}erfc\left(\frac{a_1t^{-\frac{1}{2}}}{2}+b_2t^{\frac{1}{2}}\right)-\left\{e^{-a_2b_2}erfc\left(\frac{a_2t^{-\frac{1}{2}}}{2}-b_2t^{\frac{1}{2}}\right)+e^{a_2b_2}erfc\left(\frac{a_2t^{-\frac{1}{2}}}{2}+b_2t^{\frac{1}{2}}\right)\right\}\right] \qquad (21)$$

Discussion of the results

In order to get physical insight into the problem numerical calculations have been carried out for the velocity u and induced magnetic field H, corresponding to different values of the magnetic parameter M and time t. In the entire calculation we have taken R = 10 and a = 0.4

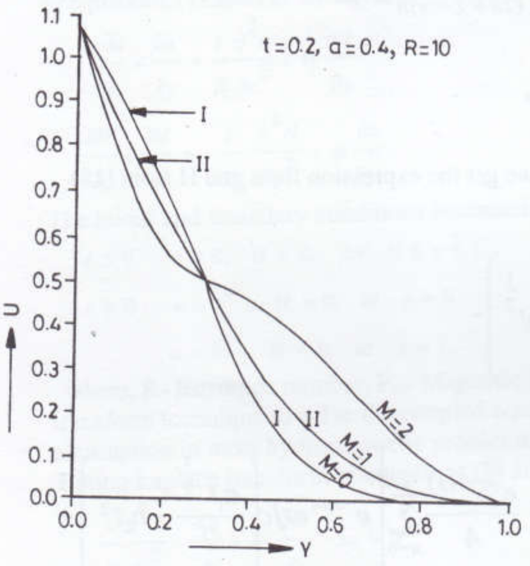

Fig. 1. Velocity variations with distance for M=0, 1, 2

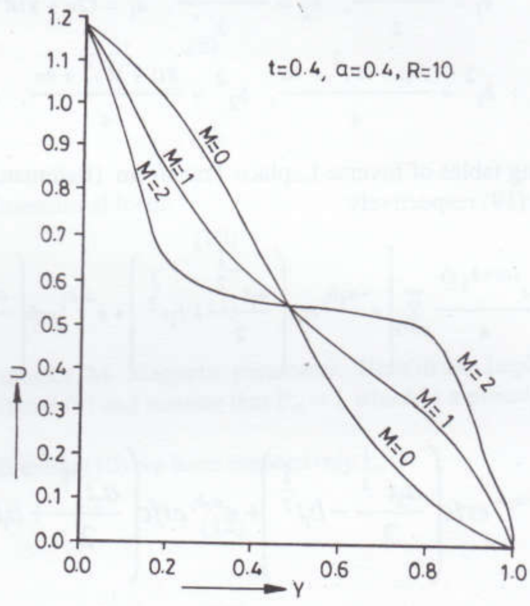

Fig. 2. Velocity variations with distance for M=0, 1, 2

The profiles for the velocity versus distance have been displayed in fig. 1 and fig. 2 for t = 0.2 and t = 0.4 respectively. From both the figures it is clear that the velocity u decreases with increase in M in the lower region between the plates whereas it increases with increase in M in the upper region.

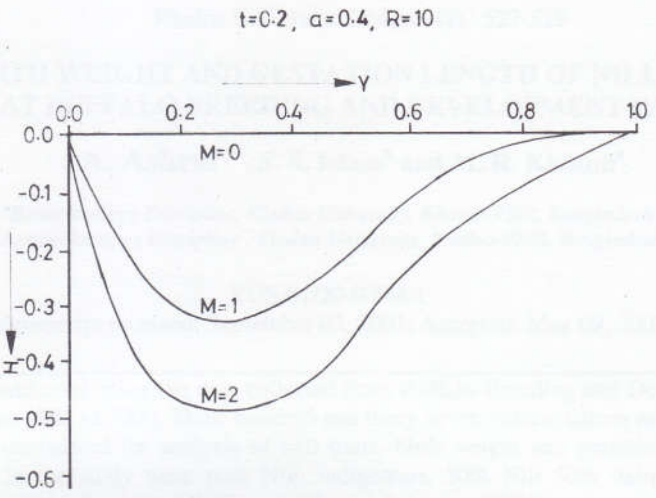

Fig. 3. Induced magnetic field variations with distance for M=0, 1, 2

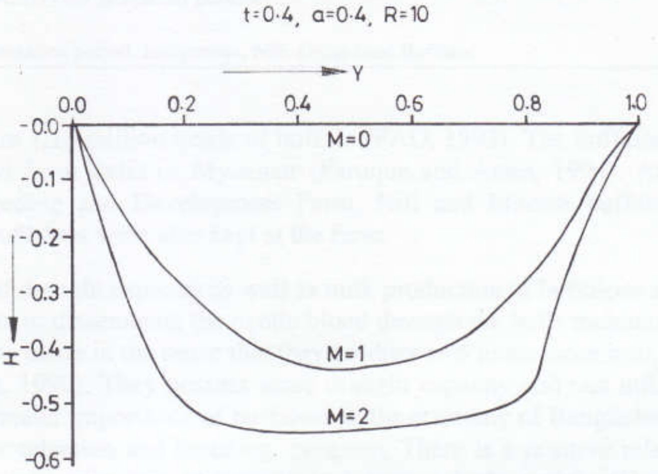

Fig. 4. Induced magnetic field variations with distance for M=0, 1, 2

The profiles for the induced magnetic field versus distance have been displayed in fig. 3 and fig. 4 for $t = 0.2$ and $t = 0.4$ respectively. It is seen from the figures that H takes negative values. It decreases with increase in M. Curves corresponding to $M = 0$ represent the non-magnetic case.

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