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HALL EFFECTS ON MHD FREE CONVECTIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE WHEN PLATE TEMPETATURE OSCILLATES IN TIME ABOUT A CONSTANT MEAN

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Abstract: An unsteady MHD free connective flow with Hall current of an electrically conducting incompressible viscous fluid past an impulsively started infinite, vertical porous plate in the presence of a strong transverse magnetic field is studied. The plate temperature is considered to oscillate in time about a constant mean. Hall currents give rise to a cross flow making the flow three-dimensional. Approximate solutions to the coupled non-linear equations occurring in the problem have been obtained. The effects of the various parameters on the mean flow and transient flow have been discussed with the help of tables and graphs.

Key words: Hall parameter; Grashof number; Magnetic parameter; Eckert number; Frequency parameter

Introduction

Soundalgekar (1972) has studied the unsteady free convective flow of an incompressible and viscous fluid past an infinite vertical unmoving porous plate, with constant suction. The plate temperature was considered to oscillate in time about a constant mean. Soundalgekar and Wavre (1977) have extended the above problem, taking into account the effects of mass transfer. However, the flow past plates started impulsively from rest plays an important role. These are particularly important in the design of space ships, solar energy collectors etc. On the other hand the effects of a magnetic field on the flow of an electrically conducting fluid have many technical applications e.g. in the boundary layer flow of high speed air-craft, in the region between the surface of blunt body and its shock wave, etc. However, f the strength of the magnetic field is strong, one cannot neglect the effects of Hall currents.

Hence, the object of the present paper is to study the effects of Hall currents on the MHD free connective flow past an impulsively started infinite, vertical porous plate in the presence of a strong transverse magnetic field; the plate temperature is considered to oscillate in time about a constant mean. The flow is subjected to constant suction through

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the porous plate. The magnetic Reynolds number of the flow is taken to be small enough so the induced magnetic field is negligible. Approximate solutions to the coupled non-linear equations, occurring in the problem have been obtained. The effects of the various parameters on the mean flow and transient flow have been discussed with the help of tables and graphs.

Mathematical Analysis

We consider the unsteady free connective flow of an electrically conducting, incompressible and viscous fluid past an infinite vertical porous plate. The x'-axis is taken along the plate in the vertical upward direction and y'-axis is normal to the plate. Initially the fluid and the plate are at rest but at time $t' > 0$ the plate starts moving impulsively in its own plane with constant velocity U_o. A uniform magnetic field of strength H_o is acting transverse to the plate. The plate temperature is considered to oscillate in time about a constant mean. In the present problem the pressure is assumed to be constant. Since the plate is infinite in extent all physical quantities are functions of y' and t' only. The fluid is subjected to constant suction at the plate and hence if $\overline{V} = (u', v', w')$ is the fluid velocity, the equation of continuity gives $v' = -v_o$ where v_o is the constant suction velocity. Using the relation $\nabla . \overline{H} = 0$ for the magnetic field $\overline{H} = (H_x, H_y, H_z)$ we obtain $H_y = H_o$ everywhere in the fluid (H_o is the constant internally applied magnetic field). If $\overline{J} = (J_x, J_y, J_z)$ is the current density, from the relation $\nabla . \overline{J} = 0$ we have J_y = constant. Since the plate is non-conducting $J_y = 0$ at the plate and hence zero everywhere. Assuming the magnetic Reynolds number to be small, we neglect the induced magnetic field in comparison with the applied magnetic field. The generalized Ohm's law, taking Hall current into account (Cowling, 1957) in the absence of electric field is of the from,

$$\overline{J} + \frac{w_e t_e}{H_o} \overline{J} \times \overline{H} = s \left(m_e \overline{V} \times \overline{H} + \frac{1}{en_e} \nabla p_e \right) \quad (1)$$

Under the usual assumption that the electron pressure (for a weakly ionized gas), the thermoelectric pressure and ion slip are negligible we have from equation 1.

$$J_x - w_e t_e J_z = -s m_e H_o w' \quad (2)$$

$$J_z + w_e t_e J_x = s m_e H_o u' \quad (3)$$

From which we get,

$$J_x = \frac{s m_e H_o}{1 + m^2} (m u' - w') \quad (4)$$

$$J_z = \frac{s m_e H_o}{1 + m^2} (u' + m w') \quad (5)$$

Where,

σ - the electric conductivity,	μ_e – the magnetic permeability
w_e – the cyclotron frequency,	τ_e- the electron collision time
e- the electric charge,	n_e – the number density of electron
p_e- the electron pressure,	$m = w_e\tau_e$ – the Hall parameter

In accordance with the Boussinesq approximation we assume that all fluid properties are considered constant except that the density variation with temperature is considered only in the body force term. The basic equations relevant to the problem are,

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = g\boldsymbol{b}(T' - T'_\infty) + \boldsymbol{n}\frac{\partial^2 u'}{\partial y'^2} - \frac{\boldsymbol{s}\boldsymbol{m}_e^2 H_o^2}{\boldsymbol{r}(1+m^2)}(u' + mw') \quad (6)$$

$$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} = \boldsymbol{n}\frac{\partial^2 w'}{\partial y'^2} + \frac{\boldsymbol{s}\boldsymbol{m}_e^2 H_o^2}{\boldsymbol{r}(1+m^2)}(mu' - w') \quad (7)$$

$$\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'} = \frac{k}{\boldsymbol{r}C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\boldsymbol{n}}{C_p}\left[\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right] \quad (8)$$

Where all the physical quantities have their usual meanings. The initial and boundary conditions are:

$$\begin{aligned} &t' \le o:\ u' = 0,\quad w' = 0,\quad T' = T'_\infty \ \forall\ y' \\ &t' > o:\ u' = U_o,\quad w' = 0,\quad T' = Tw'\left(1 + \boldsymbol{e}e^{in't'}\right) - T'_\infty \boldsymbol{e}e^{in't'}\ at\ y' = 0 \quad (9) \\ &\quad u' = 0,\quad w' = 0,\quad T' = T'_\infty \quad at\ y' = \infty \end{aligned}$$

Introducing the following non-dimensional quantities.

$$\begin{aligned} &y = \frac{y'U_0}{\boldsymbol{n}},\quad t = \frac{t'U_o^2}{\boldsymbol{n}},\quad n = \frac{n'\boldsymbol{n}}{U_o^2},\quad u = \frac{u'}{U_o} \\ &M = \frac{\boldsymbol{s}\boldsymbol{m}_e^2 H_o^2 \boldsymbol{n}}{\boldsymbol{r}U_o^2} \quad \text{(Magnetic parameter)} \\ &s = \frac{v_o}{U_o},\quad w = \frac{w'}{U_o}\quad P = \frac{mC_p}{k} \quad \text{(Prandtl number)} \quad (10) \\ &T = \frac{T' - T'_\infty}{T'_w - T'_\infty},\quad G = \frac{\boldsymbol{n}g\boldsymbol{b}(T'_w - T'_\infty)}{U_o^3},\ (Grashof\ number) \\ &E = \frac{U_o^2}{C_p(T'_w - T'_\infty)} \quad (Ec\ker tnumber) \end{aligned}$$

In equations 6 to 8 we get,

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = GT + \frac{\partial^2 u}{\partial y^2} - \mathbf{d}\left(u + mw\right) \qquad (11)$$

$$\frac{\partial w}{\partial t} - s\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \mathbf{d}\left(mu - w\right) \qquad (12)$$

$$P\frac{\partial T}{\partial t} - sP\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + PE\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \qquad (13)$$

Where, $\mathbf{d} = \dfrac{M}{1+m^2}$

Boundary conditions 9 become in non-dimensional form, $t > 0$

$$u = 1,\ \ w = o,\ \ T = 1 + \mathbf{e}e^{\mathrm{int}}\ \ at\ \ y = 0 \qquad (14)$$

$$u = 0,\ \ w = o,\ \ T = 0,\ \ at\ \ y = \infty$$

The task of solving equations 11–13 under boundary conditions 14 is quite complicated. To simplify the mathematical part, we introduce a complex variable defined as

$$Q = u + iw \qquad (15)$$

which enables us to combine equations 11 and 12 into a single equation of the form

$$\frac{\partial^2 Q}{\partial y^2} + s\frac{\partial Q}{\partial y} - \mathbf{d}\left(1 - im\right)Q - \frac{\partial Q}{\partial t} = -GT \qquad (16)$$

equation 13 with the help of 15 becomes

$$\frac{\partial^2 T}{\partial y^2} + sP\frac{\partial T}{\partial y} - P\frac{\partial T}{\partial t} = -PE\left(\frac{\partial Q}{\partial y} \times \frac{\partial \overline{Q}}{\partial y}\right) \qquad (17)$$

The corresponding boundary conditions assume the form

$t > 0$

$$Q = 1,\ \ \ T = 1 + \mathbf{e}e^{\mathrm{int}}\ \ at\ \ y = 0$$

$$Q = 0,\ \ \ T = 0,\ \ at\ \ y = \infty \qquad (18)$$

Equations 16 and 17 are coupled and non-linear. In order to solve them we can represent the velocity and temperature in the neighborhood of the plate as follows (assuming small amplitude of oscillation)

$$Q(y,t) = q_o(y) + \mathbf{e}q_1(y)e^{\mathrm{int}} \qquad (19)$$

$T(y,t) = T_o(y) + eT_1(y)e^{int}$ where $|e| << 1$.

Substituting 19 in equations 16 and 17 and equating coefficients of different powers of ε neglecting those of ε^2 and higher powers of ε we obtain the following set of equations:

$$q_o'' + sq_o' - M_1 q_o = -GT_o \tag{20}$$

$$q_o'' + sq_1' - (M_1 + in)q_1 = -GT_1 \tag{21}$$

$$T_o'' + sPT_o' = -PE(q_o'\bar{q}_o') \tag{22}$$

$$T_1''' + sPT_1' - in\ PT_1 = -PE(q_1'\bar{q}_o' + \bar{q}_1' q_o') \tag{23}$$

where $M_1 = d(1 - im)$ and primes denote differentiation with respect to y.

The corresponding boundary conditions are,

$$\begin{aligned} q_o &= 1, \quad q_1 = 0, \quad T_o = 1, \quad T_1 = 1 \quad at \quad y = 0 \\ q_o &= 0, \quad q_1 = 0, \quad T_o = 0, \quad T_1 = 0 \quad at \quad y = \infty \end{aligned} \tag{24}$$

The equations 20 to 23 are still coupled and non-linear and hence difficult to solve analytically. In order to solve them we expand q_o, q_1, T_o and T_1 in powers of E the Eckert number assuming it to be very small as follows (E<<1 for incompressible fluids).

$$\begin{aligned} q_o(y) &= q_{o1}(y) + Eq_{o2}(y) + 0(E)^2 \\ q_1(y) &= q_{11}(y) + Eq_{12}(y) + 0(E)^2 \\ T_o(y) &= T_{o1}(y) + ET_{o2}(y) + 0(E)^2 \\ T_1(y) &= T_{11}(y) + ET_{12}(y) + 0(E)^2 \end{aligned} \tag{25}$$

Substituting 25 in equations 20 to 23 we obtain the following system of equations 26 to 29 and 30 to 32 which govern the mean steady flow and the unsteady one.

$$q_{01}'' + sq_{01}' - M_1 q_{01} = -GT_{01} \tag{26}$$

$$q_{02}'' + sq_{02}' - M_1 q_{02} = -GT_{02} \tag{27}$$

$$T_{01}'' + sPT_{01}' = 0 \tag{28}$$

$$T_{02}'' + sPT_{02}' = -P(q_{01}'\bar{q}_{01}') \tag{29}$$

$$q_{11}'' + sq_{11}' - q_{11}(M_1 + in) = -GT_{11} \tag{30}$$

$$q_{12}'' + sq_{12}' - q_{12}(M_1 + in) = -GT_{12} \tag{31}$$

$$T_{11}'' + sPT_{11}' - inPT_{11} = 0 \tag{32}$$

$$T_{12}'' + sPT_{12}' - in\ PT_{12} = -P(q_{11}'\bar{q}_{01}' + \bar{q}_{11}' q_{01}') \tag{33}$$

subject to the boundary conditions

$$
\begin{aligned}
&q_{01} = 1, \quad q_{02} = 0, \quad T_{01} = 1, \quad T_{02} = 0 \qquad at \;\; y = 0 \\
&q_{01} = 0, \quad q_{02} = 0, \quad T_{01} = 0, \quad T_{02} = 0 \qquad at \;\; y = \infty
\end{aligned} \tag{34}
$$

for the mean steady flow and

$$
\begin{aligned}
&q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 1, \quad T_{12} = 0 \qquad at \;\; y = 0 \\
&q_{11} = 0, \quad q_{12} = 0, \quad T_{11} = 0, \quad T_{12} = 0 \qquad at \;\; y = \infty
\end{aligned} \tag{35}
$$

for the unsteady flow. The solutions of equations 26 to 29 subject to boundary conditions 34 are given by

$$
q_{01} = A_2 e^{-B_1 y} - A_1 e^{-sPy} \tag{36}
$$

$$
\begin{aligned}
q_{02} = {} & L_2 e^{-B_1 y} - A_7 e^{-sPy} + A_8 e^{-\left(B_1 + \overline{B}_1\right)y} - A_9 e^{-\left(sP + B_1\right)y} \\
& - A_{10} e^{-\left(sP + \overline{B}_1\right)y} + A_{11} e^{-2sPy}
\end{aligned} \tag{37}
$$

$$
T_{01} = e^{-sPy} \tag{38}
$$

$$
\begin{aligned}
T_{02} = {} & L_1 e^{-sPy} - A_3 e^{-\left(B_1 + \overline{B}_1\right)y} + A_4 e^{-\left(sP + B_1\right)y} \\
& + A_5 e^{-\left(sP + \overline{B}_1\right)y} - A_6 e^{-2sPy}
\end{aligned} \tag{39}
$$

The expression for mean steady velocity and temperature are given from 25 where q_{01}, q_{02}, T_{01} and T_{02} are given by 36 to 39. If τ_{mu} and τ_{mw} are the components of mean skin friction τ_o at the plate due to mean primary velocity u_o and mean secondary velocity w_o we have

$$
\begin{aligned}
& t_0 = t_{mu} + i t_{mw} = \left. \frac{dq_0}{dy} \right|_{y=o} \\
& = -A_2 B_1 + A_1 sP + E\Big[-L_2 B_1 + A_7 sP - A_8 \left(B_1 + \overline{B}_1\right) \\
& + A_9 \left(sP + B_1\right) + A_{10}\left(sP + \overline{B}_1\right) - 2A_{11} sP \;\; \Big]
\end{aligned} \tag{40}
$$

where the different constants are defined in the appendix. The solution of the equations 30 to 33 of the unsteady flow field under their boundary condition 35 are given by,

$$
q_1 = q_{11} + Eq_{12} \tag{41}
$$

$$= D_1 e^{-h_2 y} - D_1 e^{-h_1 y} + E\Big[x_2 e^{-h_2 y} - D_{10} e^{-h_1 y}$$
$$+ D_{11} e^{-(h_2 + \overline{B}_1) y} - D_{12} e^{-(h_2 + sP) y}$$
$$- D_{13} e^{-(h_1 + \overline{B}_1) y} + D_{14} e^{-(h_1 + sP) y} + D_{15} e^{-(\overline{h}_2 + B_1) y} - D_{16} e^{-(\overline{h}_2 + sP) y}$$
$$- D_{17} e^{-(\overline{h}_1 + B_1) y} + D_{18} e^{-(\overline{h}_1 + sP) y} \Big]$$

and

$$T_1 = T_{11} + ET_{12} \tag{42}$$
$$= e^{-h_1 y} + E\Big[x_1 e^{-h_1 y} - D_2 e^{-(h_2 + \overline{B}_1) y} + D_3 e^{-(h_2 + sP) y}$$
$$+ D_4 e^{-(h_1 + \overline{B}_1) y} - D_5 e^{-(h_1 + sP) y} - D_6 e^{-(\overline{h}_2 + B_1) y} + D_7 e^{-(\overline{h}_2 + sP) y}$$
$$+ D_8 e^{-(\overline{h}_1 + B_1) y} - D_9 e^{-(\overline{h}_1 + sP) y} \Big]$$

where the constants appearing in the solution are defined in the appendix at the end. We obtain the expression for Q and T from 19. The expression of the transient primary velocity, transient secondary velocity and transient temperature at $nt = \frac{p}{2}$ can be obtained respectively as

$$u\left(y, \frac{p}{2n} \right) = u_o(y) - eM_i \tag{43}$$

$$w\left(y, \frac{p}{2n} \right) = w_o(y) + eM_r \tag{44}$$

and $$T\left(y, \frac{p}{2n} \right) = T_o(y) - eT_i \tag{45}$$

Where $q_1 = M_r + iM_i$, $q_0 = u_0 + iw_0$ and $T_1 = T_r + iT_i$ (neglecting the imaginary part for $T\left(y, \frac{p}{2n} \right)$); where u_o, w_o and T_o are the mean primary velocity, mean secondary velocity and mean temperature respectively. The skin friction is given by

$$t = t_x + it_z$$

$$= \frac{\partial Q}{\partial y}\bigg|_{Y=0} = \frac{\partial q_o}{\partial y}\bigg|_{y=0} + \boldsymbol{e}\, e^{\text{int}} \frac{\partial q_1}{\partial y}\bigg|_{y=0}$$

$$= \boldsymbol{t}_0 + \boldsymbol{e} e^{\text{int}} \Big[-D_1 h_2 + D_1 h_1 + E\Big\{ -x_2 h_2 + D_{10} h_1 - D_{11}\left(h_2 + \overline{B}_1\right) + D_{12}\left(h_2 + sP\right)$$
$$+ D_{13}\left(h_1 + \overline{B}_1\right) - D_{14}\left(h_1 + sP\right) - D_{15}\left(\overline{h}_2 + B_1\right) + D_{16}\left(\overline{h}_2 + sP\right)$$
$$+ D_{17}\left(\overline{h}_1 + B_1\right) - D_{18}\left(\overline{h}_1 + sP\right) \Big\} \Big] \qquad (46)$$

where,

$$\boldsymbol{t}_o = \frac{\partial q_o}{\partial y}\bigg|_{y=0}$$ is the mean skin friction.

Results and Discussion

In order to get physical insight into the problem numerical calculations have been carried out for mean flow and transient flow corresponding to different values of the Grashoff number G, suction parameter s, Hall parameter m, magnetic parameter M and frequency parameter n. In order to be realistic the value of the Prandtl number P is chosen to be 0.71 which corresponds to air. In the entire calculation we have taken E=0.003 and $\varepsilon = 0.2$.

Values of the mean primary velocity u_o and mean secondary velocity w_o are given in Table 1. It is seen from the table that u_o increases with increase in m and G. It decreases with increase in s and M. From the same table we conclude that the effects of the various parameters on w_o are similar to their effects on u_o. Table 2 shows the variations of the mean temperature T_o in air (P=0.71). It is clear from the table that the temperature increases with increase in m and G and decreases with increase in s.

Table 3 gives the values of the mean skin friction components τ_{mu} and τ_{mw}. From the table we observe that τ_{mu} the mean skin friction component due to mean primary flow increases with increase in m but decreases with increase in s and M. τ_{mw} the mean skin friction component due to mean secondary flow increases with increase in m but decreases with increase in s. τ_{mw} in general, decreases with increase in M, but increases with increase in M for G=5 and s=1. Both the components of skin friction increase with increase in G. The transient primary velocity profiles $u\left(y, \frac{p}{2n}\right)$ have been displayed in Fig. 1. It is clear from the figure that the transient primary velocity decreases with increase in M and s, but increases with increase in m. We also observe from the figure that near to the porous plate $u\left(y, \frac{p}{2n}\right)$ decreases with increase in n, but away from the plate it increases with increase in n. The transient secondary velocity profiles

$w\left(y, \frac{p}{2n}\right)$ are shown in Fig. 2. From the figure we conclude that the effects of m, M and s on $w\left(y, \frac{p}{2n}\right)$ are similar to their effects on $u\left(y, \frac{p}{2n}\right)$. As for the effect of n we see that near to the porous plate the transient secondary velocity decreases with increase in n, but away from the plate it increases with increase in n; further away from the plate the influence of n is insignificant.

Table1. Values of mean primary velocity u_o and mean secondary velocity w_o (P=0.71, E=0.003).

G	M	m	s	Y→	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3..5	4.0	4.5
5	2	1.0	0.5	u_o	1.0000	1.6225	1.6719	1.5089	1.2912	1.0810	0.8981	0.7451	0.6190	0.5153
				w_o	0.0000	0.6582	0.9620	1.0423	1.0015	0.9050	0.7906	0.6774	0.5741	0.4836
5	2	0.5	0.5	u_o	1.0000	1.4826	1.5076	1.3684	1.1863	1.0078	0.8487	0.7120	0.5964	0.4994
				w_o	0.0000	0.3613	0.5094	0.5380	0.5080	0.4541	0.3942	0.3367	0.2850	0.2401
5	4	0.5	0.5	u_o	1.0000	1.0013	0.8731	0.7349	0.6138	0.5124	0.4282	0.3582	0.2998	0.2510
				w_o	0.0000	0.2850	0.3749	0.3312	0.2912	0.2485	0.2095	0.1759	0.1474	0.1235
5	4	0.5	1.0	u_o	1.0000	0.8194	0.5958	0.4196	0.2933	0.2050	0.1435	0.1005	0.0704	0.0494
				w_o	0.0000	0.2175	0.2239	0.1793	0.1324	0.0947	0.0670	0.0471	0.0331	0.0232
10	4	0.5	0.5	u_o	1.0000	1.6633	1.6380	1.4386	1.2208	1.0253	0.8588	0.7190	0.6020	0.5041
				w_o	0.0000	0.4960	0.6461	0.6381	0.5726	0.4938	0.4187	0.3524	0.2958	0.2479
10	4	1.0	0.5	u_o	1.0000	1.8309	1.7981	1.5478	1.2859	1.0623	0.8804	0.7327	0.6116	0.5115
				w_o	0.0000	0.9236	1.2522	1.2655	1.1484	0.9946	0.8438	0.7098	0.5951	0.4985
10	2	1.0	0.5	u_o	1.0000	2.7717	3.1423	2.9493	2.5756	2.1812	1.8244	1.5200	1.2661	1.0558
				w_o	0.0000	1.2173	1.8351	2.0310	1.9807	1.8093	1.5929	1.3726	1.1681	0.9870
10	2	0.5	0.5	u_o	1.0000	2.5249	2.8323	2.6697	2.3561	2.0205	1.7103	1.4393	1.2080	1.0128
				w_o	0.0000	0.6578	0.9634	1.0424	1.0000	0.9033	0.7897	0.6778	0.5757	0.4860

Table 4 displays the values of transient temperature $T\left(y, \frac{p}{2n}\right)$ of air. We observe from the table that the transient temperature increases with increase in m whereas rise in M and s causes a fall in $T\left(y, \frac{p}{2n}\right)$. It decreases with increase in n, near the plate but increases with increase in n away from the plate. Table 5 shows the values of the skin friction components τ_x and τ_z at $nt = \frac{p}{2}$. τ_x decreases with increase in n, M and s and increases with increase in m. τ_z decreases with increase in n and s, but increase with increase in m.

The effect of M on τ_z depends on s. +For s=0.5, τ_z decreases with increase in M but for s=1.0 τ_z increases with increase in M.

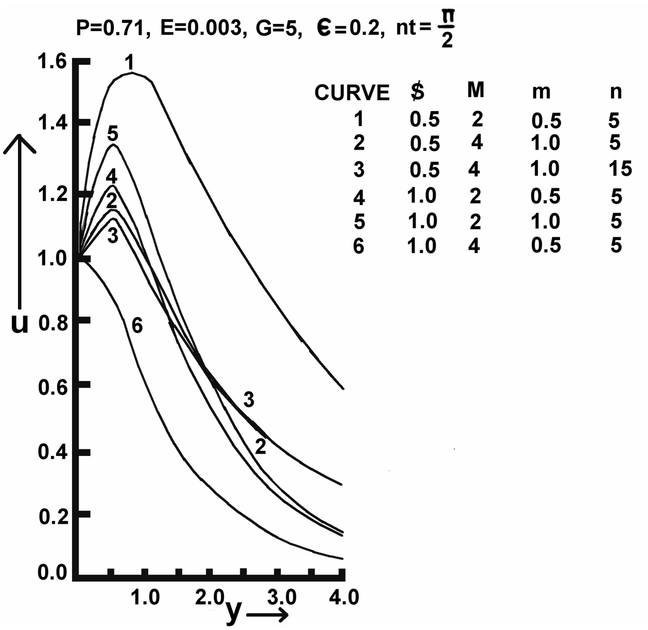

Fig. 1. Transient primary velocity distribution u against y.

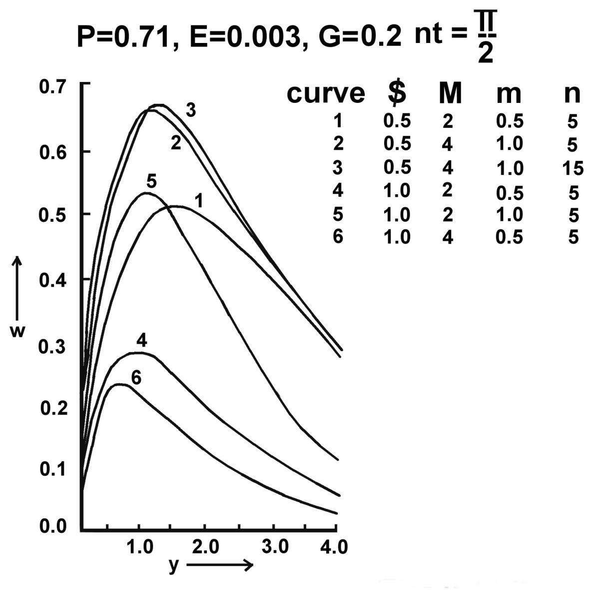

Fig. 2. Transient secondary velocity distribution w against y.

Table 5 shows the values of the skin friction components τ_x and τ_z at $nt = \frac{p}{2}$. τ_x decreases with increase in n, M and s and increases with increase in m. τ_z decreases with increase in n and s, but increase with increase in m. The effect of M on τ_z depends on s. +For s=0.5, τ_z decreases with increase in M but for s=1.0 τ_z increases with increase in M.

Table 2. Values of mean temperature T_o (P=0.71, E=0.003, M=4.0).

y	G=5				G=10			
	m=0.5		m=1.0		M=0.5		m=1.0	
	s=0.5	s=1.0	s=0.5	s=1.0	s=0.5	s=1.0	s=0.5	s=1.0
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	0.8376	0.7015	0.8379	0.7016	0.8384	0.7018	0.8395	0.7023
1.0	0.7016	0.4920	0.7019	0.4922	0.7025	0.4926	0.7038	0.4932
1.5	0.5876	0.3451	0.5880	0.3452	0.5887	0.3457	0.5901	0.3463
2.0	0.4921	0.2420	0.4925	0.2422	0.4933	0.2426	0.4947	0.2431
2.5	0.4121	0.1697	0.4125	0.1698	0.4134	0.1702	0.4147	0.1706
3.0	0.3452	0.1190	0.3455	0.1191	0.3463	0.1194	0.3475	0.1197

Table 3. Values of τ_{mu} and τ_{mw} (P=0.71, E=0.003).

G	M	M	s	τ_{mu}	τ_{mu}
5	0.5	2	0.5	1.7776	1.0110
			1.0	1.1386	0.8327
		4	0.5	0.3250	0.9650
			1.0	-0.1286	0.8724
	1.0	2	0.5	2.2324	1.7549
			1.0	1.5803	1.3967
		4	0.5	0.7378	1.6280
			1.0	0.3312	1.4427
10	0.5	2	0.5	5.1534	1.7285
			1.0	4.1795	1.3818
		4	0.5	2.7608	1.5006
			1.0	2.1495	1.3256
	1.0	2	0.5	5.8812	3.0925
			1.0	4.8754	2.3868
		4	0.5	3.4046	2.6248
			1.0	2.7890	2.2721

Table 4. Variation of transient temperature $T\left(y, \frac{p}{2n}\right)$ in air (P=0.71, E=0.003, G=5,

$e = 0.2$, $nt = \frac{p}{2}$.

S	M	m	N	y→	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.5	2	0.5	5		1.0000	0.8957	0.7447	0.6071	0.4974	0.4122	0.3444	0.2888	0.2424
0.5	2	1.0	5		1.0000	0.8964	0.7456	0.6080	0.4984	0.4131	0.3453	0.2897	0.2431
0.5	4	1.0	5		1.0000	0.8956	0.7445	0.6067	0.4970	0.4117	0.3439	0.2883	0.2419
0.5	4	1.0	15		1.0000	0.8905	0.7142	0.5864	0.4911	0.4123	0.3455	0.2893	0.2423
1.0	2	0.5	5		1.0000	0.7533	0.5271	0.3595	0.2455	0.1696	0.1184	0.0831	0.0584
1.0	2	1.0	5		1.0000	0.7535	0.5274	0.3599	0.2458	0.1698	0.1186	0.0832	0.0585
1.0	4	0.5	5		1.0000	0.7532	0.5269	0.3592	0.2452	0.1693	0.1181	0.0829	0.0583
1.0	4	0.5	15		1.0000	0.7492	0.5023	0.3440	0.2410	0.1695	0.1190	0.0834	0.0585

Table 5. Values of skin friction components t_x *and* t_z *at* $nt = \frac{p}{2}$ (P=0.71, E=0.003, G=5, ε=0.2).

S	M	M	n	τ_x	τ_z
0.5	2	0.5	5	1.9402	1.2061
			15	1.8762	1.1146
		1.0	5	2.4050	1.9485
			15	2.3327	1.8580
	4	0.5	5	0.4665	1.1770
			15	0.4213	1.0730
		1.0	5	0.9502	1.8446
			15	0.8876	1.7350
1.0	2	0.5	5	1.3020	1.0307
			15	1.2381	0.9369
		1.0	5	1.7538	1.5935
			15	1.6815	1.5003
	4	0.5	5	0.0130	1.0869
			15	-0.0314	0.9810
		1.0	5	0.4936	1.6625
			15	0.4319	1.5504

References

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Appendix

$$A_1 = \frac{G}{P^2 s^2 - s^2 P - M_1}, \qquad B_1 = \frac{s + \left(s^2 + 4M_1\right)^{\frac{1}{2}}}{2}, \qquad A_2 = 1 + A_1$$

$$A_3 = \frac{P\left|A_2\right|^2 \left|B_1\right|^2}{\left(B_1 + \overline{B}_1\right)^2 - sP\left(B_1 + \overline{B}_1\right)}, \qquad A_4 = \frac{sP^2 A_2 \overline{A}_1}{B_1 + sP}$$

$$A_5 = \frac{sP^2 A_1 \overline{A}_2}{\overline{B}_1 + sP} = \overline{A_4}, \qquad A_6 = \frac{P\left|A_1\right|^2}{2}$$

$$L_1 = A_3 - A_4 - A_5 + A_6$$

$$A_7 = \frac{GL_1}{s^2 P^2 - s^2 P - M_1}, \qquad A_8 = \frac{GA_3}{\left(B_1 + \overline{B}_1\right)^2 - s\left(B_1 + \overline{B}_1\right) - M_1}$$

$$A_9 = \frac{GA_4}{\left(s + B_1\right)^2 - s\left(sP + B_1\right) - M_1},$$

$$A_{10} = \frac{GA_5}{\left(sP + \overline{B}_1\right)^2 - s\left(sP + \overline{B}_1\right) - M_1}$$

$$A_{11} = \frac{GA_6}{4s^2 P^2 - 2s^2 P - M_1}, \qquad L_2 = A_7 - A_8 + A_9 + A_{10} - A_{11}$$

$$h_1 = \frac{sP + \left(s^2 P^2 + 4P\ in\right)^{\frac{1}{2}}}{2}, \qquad M_2 = M_1 + in$$

$$h_2 = \frac{s + \left(s^2 + 4M_2\right)^{\frac{1}{2}}}{2}, \qquad D_1 = \frac{G}{h_1^2 - sh_1 - M_2}$$

$$C_1 = \overline{A}_2 \overline{B}_2 D_1 h_1 P, \qquad C_2 = \overline{A}_1 D_1 h_2 s P^2$$

$$C_3 = \overline{A}_2 \overline{B}_1 D_1 h_1 P, \qquad C_4 = \overline{A}_1 D_1 h_1 s P^2$$

$$D_2 = \frac{C_1}{\left(h_2 + \overline{B}_1\right)^2 - sP\left(h_2 + \overline{B}_1\right) - in\ P},$$

$$D_3 = \frac{C_2}{\left(h_2 + sP\right)^2 - sP\left(h_2 + sP\right) - in\ P}$$

$$D_4 = \frac{C_3}{\left(h_1 + \overline{B}_1\right)^2 - sP\left(h_1 + \overline{B}_1\right) - in\ P},$$

$$D_5 = \frac{C_4}{\left(h_1 + sP\right)^2 - sp\left(h_1 + sP\right) - in\ P}$$

$$D_6 = \frac{\overline{C}_1}{\left(\overline{h}_2 + B_1\right)^2 - sP\left(\overline{h}_2 + B_1\right) - in\ P},$$

$$D_7 = \frac{\overline{C}_2}{\left(\overline{h}_2 + sP\right)^2 - sP\left(\overline{h}_2 + sP\right) - in\ p}$$

$$D_8 = \frac{\overline{C}_3}{\left(\overline{h}\ + B_1\right)^2 - sP\left(\overline{h}_1 + B_1\right) - in\ P},$$

$$D_9 = \frac{\overline{C}_4}{\left(\overline{h}_1 + sP\right)^2 - sP\left(\overline{h}_1 + sP\right) - in\ P}$$

$$x_1 = D_2 - D_3 - D_4 + D_5 + D_6 - D_7 - D_8 + D_9$$

$$D_{10} = \frac{Gx_1}{h_1^2 - sh_1 - M_2}, D_{11} = \frac{GD_2}{\left(h_2 + \overline{B}_1\right)^2 - s\left(h_2 + \overline{B}_1\right) - M_2}$$

$$D_{12} = \frac{GD_3}{\left(h_2 + sP\right)^2 - s\left(h_2 + sP\right) - M_2}, D_{13} = \frac{GD_4}{\left(h_1 + \overline{B}_1\right)^2 - s\left(h_1 + \overline{B}_1\right) - M_2}$$

$$D_{14} = \frac{GD_5}{\left(h_1 + sP\right)^2 - s\left(h_1 + sP\right) - M_2}, D_{15} = \frac{GD_6}{\left(\overline{h}_2 + B_1\right)^2 - s\left(\overline{h}_2 + B_1\right) - M_2}$$

$$D_{16} = \frac{GD_7}{\left(\overline{h}_2 + sP\right)^2 - s\left(\overline{h}_2 + sP\right) - M_2}, D_{17} = \frac{GD_8}{\left(\overline{h}_1 + B_1\right)^2 - s\left(\overline{h}_1 + B_1\right) - M_2}$$

$$D_{18} = \frac{GD_9}{\left(\overline{h}_1 + sP\right)^2 - s\left(\overline{h}_1 + sP\right) - M_2}$$

$$x_2 = D_{10} - D_{11} + D_{12} + D_{13} - D_{14} - D_{15} + D_{16} + D_{17} - D_{18}$$