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ORIGINAL ARTICLE

SCIENCE & ENGINEERING

MATHEMATICAL MODELING APPLIED TO ASSESS THE DRIVING FACTORS OF INCREASING ROAD ACCIDENTS IN BANGLADESH

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Abstract

In today's world, particularly in developing nations, traffic safety is one of the most pressing challenges. Every single day, traffic accidents alone claim hundreds of lives across all nations and regions. Besides rapid development in the transportation sector, road accidents are also increasing day by day. Therefore, every government has regarded it as one of the crucial issues and as the first big issue that has to be resolved. To lessen the number of accidents on the roads and the harm they cause, necessary actions have been implemented. We discovered that it is impracticable and time-consuming to investigate the contributing elements behind vehicle accidents since vast amounts of statistical data are required. In this paper, we develop a dynamical model to describe the dynamics of the traffic accident. Accidents are mostly caused by negligent or inexperienced drivers and defective machinery. We have described the dynamics in terms of a system of four ordinary differential equations with four state variables and parameters. We have developed a mathematical model that can estimate the number of fatal traffic accidents. In order to validate the model, we looked at the positivity of the solutions, equilibrium points, stability of the equilibrium points, and other relevant analytical studies. In order to demonstrate the validity of our study, we ran a numerical simulation using parameter values that were verified from reliable sources.

Keywords: Basic reproductive ratio, human factor, mathematical model, numerical simulation, road accident

Introduction

Accidents happen when an animated inanimate structure accidentally incurs physical damage. Road accident is the most unwelcome and unexpected event that may happen to a road user, even though they occur relatively frequently. Road traffic accidents have grown to be one of the world's most serious public-health and injury-prevention issues. Around 90% of all fatalities worldwide occur in developing nations, which have a higher than average prevalence of accidents (Azeredo, 2018). Road traffic accidents were not insignificant as a cause of death or disability in 1990, ranking ninth out of over 100 individually documented causes (Alam et al., 2011). The inconvenience of a traffic collision is unavoidable in daily life. It is impossible to completely avoid it. Each year, road accidents result in over 1.3 million fatalities and 50 million injuries globally (International Road Assessment Program, 2010). Road traffic accidents are the third most common cause of death for those

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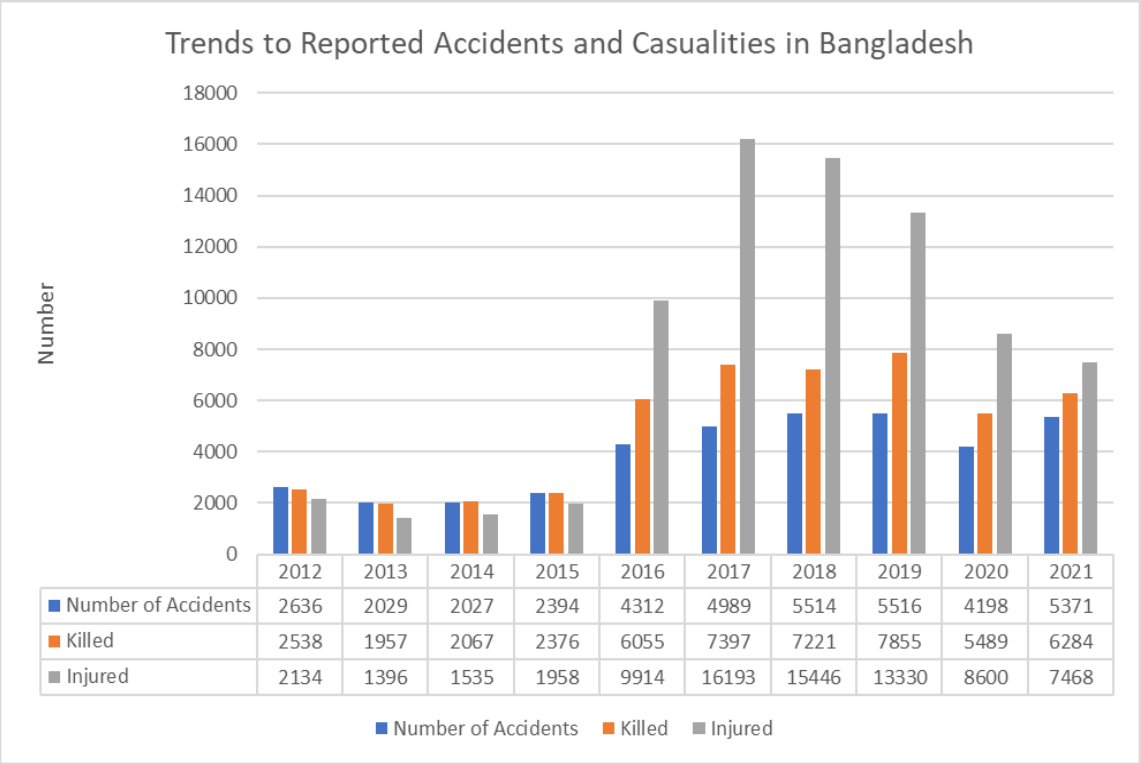
between the ages of 30 and 44 and the second most common cause of death for those between the ages of 5 and 29 (Bangladesh Police, 2021). Vehicles are quickly overtaking diseases and disabilities as the third-leading source of death and disability in emerging nations (WHO, 2021). Due to the country's grave worries over road safety on international routes, vehicle accidents in Bangladesh are increasingly of a concern. According to government figures, more than 60 persons are killed in road accidents in Bangladesh for every 10,000 motor vehicles (Ahsan et al., 2012). The number of fatalities and accidents on the roads is rising across the nation as time goes on. Though Bangladesh is a developing country, most of the people of this country are poor. Their only goal is to make money. For this, they want to earn by driving even though they don't have a license. Every year, roughly 3,500 police-reported incidents result in at least 3,000 fatalities and 3,000 serious and minor injuries on Bangladeshi roadways (Mahmud et al., 2014). Road accidents cause roughly 35,000 injuries and about 12,000 fatalities per year (Labib et al., 2019). According to government statistics, more than 60 persons are killed in road accidents for every 10,000 cars on the road in Bangladesh, which has a comparatively high proportion of these deaths. Approximately 10 individuals every day perish in car accidents (Maniruzzaman et al., 2005). An ongoing issue of top priority in the administration of road safety is the gathering and use of accurate and comprehensive data related to traffic accidents. The analysis of this data can help with a better understanding of operating concerns, as well as setting the stage for a more accurate diagnosis of accident problems. A large database is a necessary requirement for every successful road safety project. The annual road accident rate in Bangladesh for last 10 years, which is initially increasing year by year is shown in Figure 1.



	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Number of Accidents	2636	2029	2027	2394	4312	4989	5514	5516	4198	5371
Killed	2538	1957	2067	2376	6055	7397	7221	7855	5489	6284
Injured	2134	1396	1535	1958	9914	16193	15446	13330	8600	7468

Figure 1. Annual Road Accident Rates in Bangladesh (Bangladesh Police, 2021).

In this paper, we have examined the stability analysis and equilibrium point for our model. In order to better understand the consequences and results, we numerically simulated our model and findings.

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Materials and Methods

Road accidents are mainly caused by the faulty vehicles and misguided drivers. We consider four compartmental model, where $P(t)$ represents the normal drivers, $D_m(t)$ represents misguided drivers, $V(t)$ represents driver with faulty vehicles and $A_c(t)$ represent the driver who made accident. Here, misguided drivers mean who are unskilled and haven't proper training and also don't have any driving license. We consider 8 parameters, where φ describes recruitment rate of driver, α describes the rate at which driver becomes driver with faulty vehicles, β describes the rate at which misguided driver becomes driver with faulty vehicles, μ describes the rate at which driver becomes misguided driver, σ describes the rate at which driver with faulty vehicles becomes driver who made accident, ε describes the death rate of accident and γ describes the rate at which driver becomes driver who made accident.

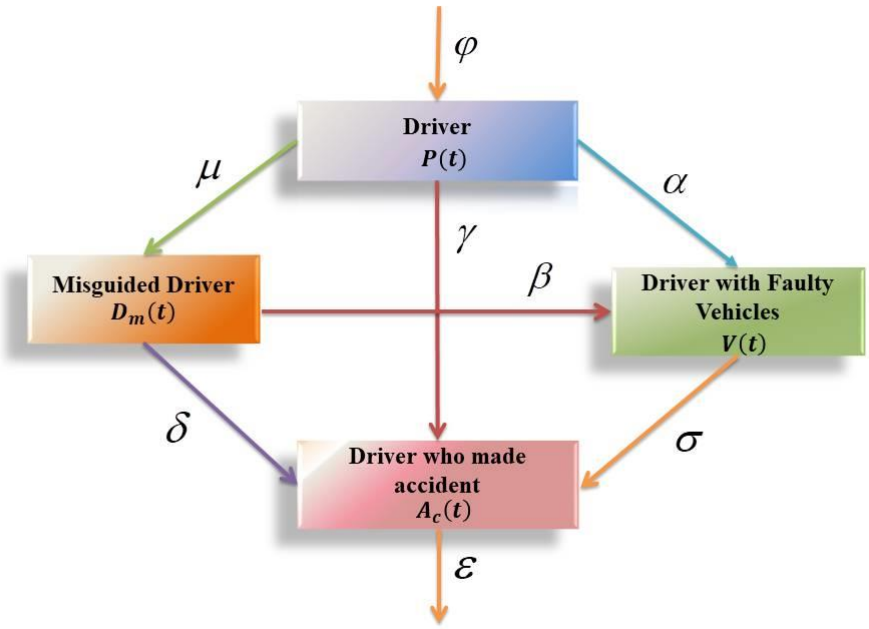

Figure 2. Schematic Diagram of Road Accident Model.

We have formulated the dynamical model system,

$$\frac{dP(t)}{dt} = \varphi P(t) - \mu P(t) D_m(t) - \alpha P(t) - \gamma P(t)$$

$$\frac{dD_m(t)}{dt} = \mu P(t) D_m(t) - \beta D_m(t) - \delta D_m(t)$$

$$\frac{dV(t)}{dt} = \alpha P(t) + \beta D_m(t) - \sigma V(t)$$

$$\frac{dA_c(t)}{dt} = \gamma P(t) + \delta D_m(t) + \sigma V(t) - \varepsilon A_c(t)$$

$P(0) \geq 0, D_m(0) \geq 0, V(0) \geq 0$ and $A_c(0) \geq 0$

The solutions are all non-negative.

Table 1. Description of Parameters and Numerical Values

Descriptions	Symbols	Values	Units
recruitment rate of driver	φ	5.0	$Year^{-1}$
rate at which driver becomes misguided driver	μ	0.56	$Year^{-1}$
rate at which driver becomes driver with faulty vehicles	α	0.3	$Year^{-1}$
rate at which misguided driver becomes driver with faulty vehicles	β	0.6	$Year^{-1}$
rate at which driver becomes driver who made accident	γ	1.0	$Year^{-1}$
rate at which misguided driver becomes driver who made accident	δ	0.9	$Year^{-1}$
rate at which driver with faulty vehicles becomes driver who made accident	σ	0.7	$Year^{-1}$
death rate of accident	ε	1.0	$Year^{-1}$

Positivity analysis

If $P(0) \geq 0, D_m(0) \geq 0, V(0) \geq 0$ and $A_c(0) \geq 0$ then the existing solution $P(t), D_m(t), V(t)$ and $A_c(t)$ of our model system of non-linear equations are all positive.

Proof:

We have from the , $\frac{dP(t)}{dt} = \varphi P(t) - \mu P(t) D_m(t) - \alpha P(t) - \gamma P(t)$

The result of the positivity analysis is given below,

$$P(t) \geq \frac{\varphi}{\alpha + \gamma} + \left[P(0) - \frac{\varphi}{\alpha + \gamma} \right] e^{-(\alpha+\gamma)t}$$

Hence, $P(0) \geq 0$, at $t = 0$ and $t \to \infty$.

Similarly, we can find the possibility of $D_m(t), V(t)$ and $A_c(t)$.

Therefore, it is proved that,

$P(0) \geq 0, D_m(0) \geq 0, V(0) \geq 0$ and $A_c(0) \geq 0$.

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Equilibrium analysis

Trivial Equilibrium Point: When driver, misguided driver, driver with faulty vehicles and driver who made accident are not exists i.e. $P = D_m = V = A_c = 0$ .Thus the equilibrium point is denoted as, $E_0(P, D_m, V, A_c)$ then from the required system we get, $E_0(P, D_m, V, A_c) = E_0(0,0,0,0)$.

Interior Equilibrium Point: When driver, misguided driver, driver with faulty vehicles and driver who made accident are exists that means all the state variables of the model system are co-existing i.e., $P \neq 0$, $D_m \neq 0$, $V \neq 0$ and $A_c \neq 0$ .Thus,

$$E_1\left(P^*, D_m^*, V^*, A_c^*\right) = \left(\frac{\beta+\delta}{\mu}, -\frac{\alpha+\gamma-\varphi}{\mu}, \frac{\alpha\delta-\beta\gamma+\beta\varphi}{\mu\sigma}, \frac{\alpha\beta+\alpha\delta-\beta\gamma-\delta\gamma+\beta\varphi+\delta\varphi}{\varepsilon\mu}\right)$$

Stability analysis

Trivial Equilibrium Point: The system will be asymptotically stable at the equilibrium point $E_0(0,0,0,0)$ if $\beta_1 > 0,\ \beta_1\beta_2 - \beta_0\beta_3 > 0,\ (\beta_1\beta_2 - \beta_0\beta_3)\beta_3 - \beta_1^2\beta_4 > 0,\ \beta_4 > 0.$

Interior Equilibrium Point: The system automatically stable at the equilibrium point $E_1\left(P^*, D_m^*, V^*, A_c^*\right)$ if $A_1 > 0,\ A_1A_2 - A_0A_3 > 0,\ (A_1A_2 - A_0A_3)A_3 - A_1^2A_4 > 0 \ \text{and}\ A_4 > 0.$

Results and Discussion

Numerical simulation has been used to validate the results of analytical research.

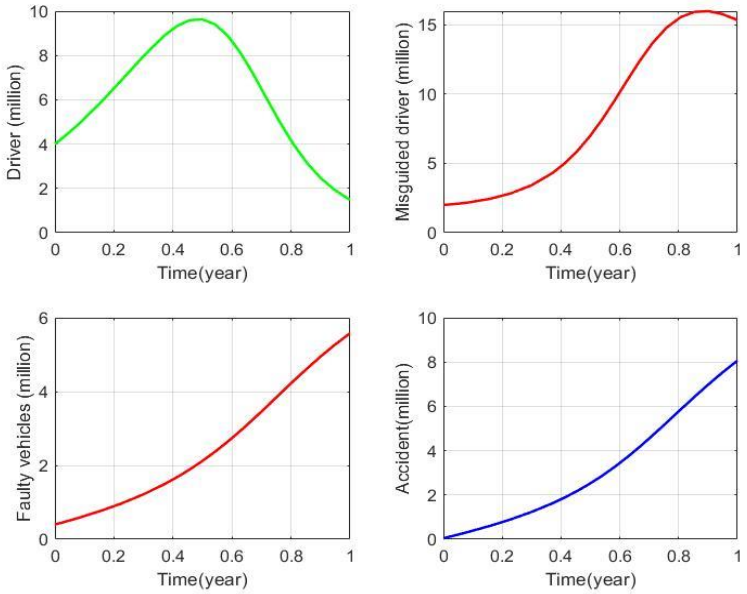

Figure 3. Numerical simulation for road accident model, with time [0 1].

Figure 3 represented driver, misguided driver, faulty vehicles and accident from the given value.

Now for different values of μ, we get different variety of figure as follows:

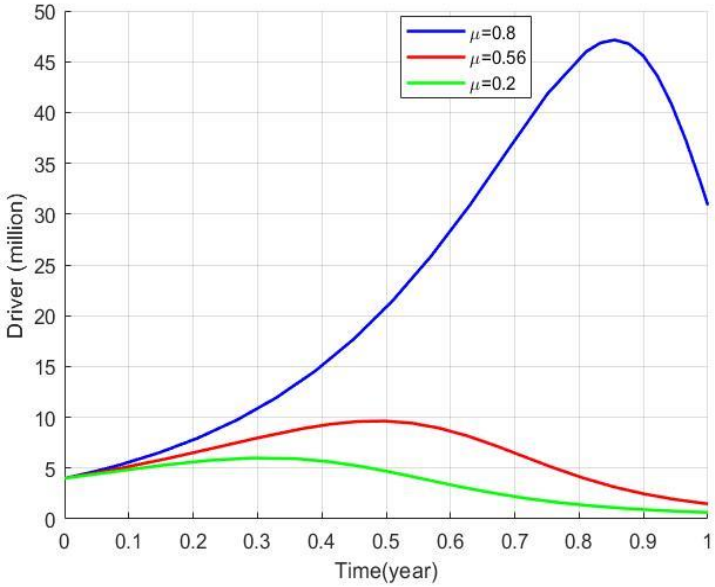

Figure 4. Variation of driver population with time for $\mu = 0.8, 0.56, 0.2$.

The driver population represented in the Figure 4 shows that, the driver becomes increase when μ increases.

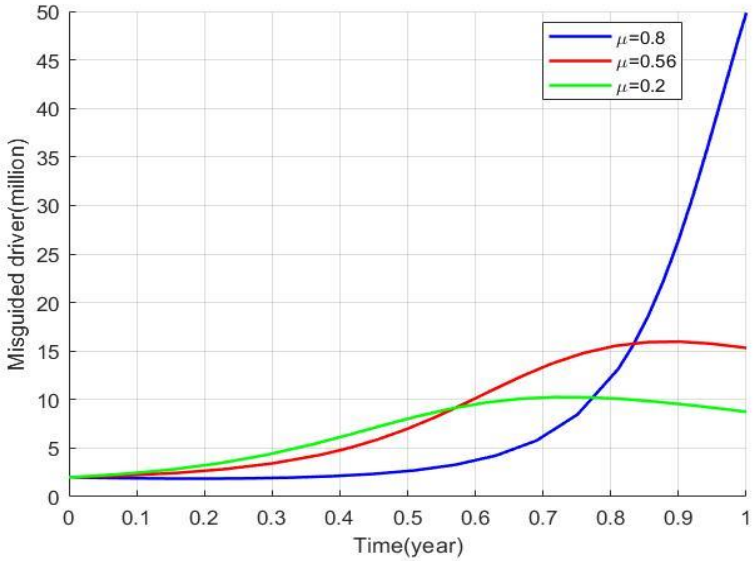

Figure 5. Variation of misguided driver with time for $\mu = 0.8, 0.56, 0.2$.

The misguided driver population represented in the Figure 5 shows that, the misguided driver is initially decreasing and then when μ increase the misguided driver also increase.

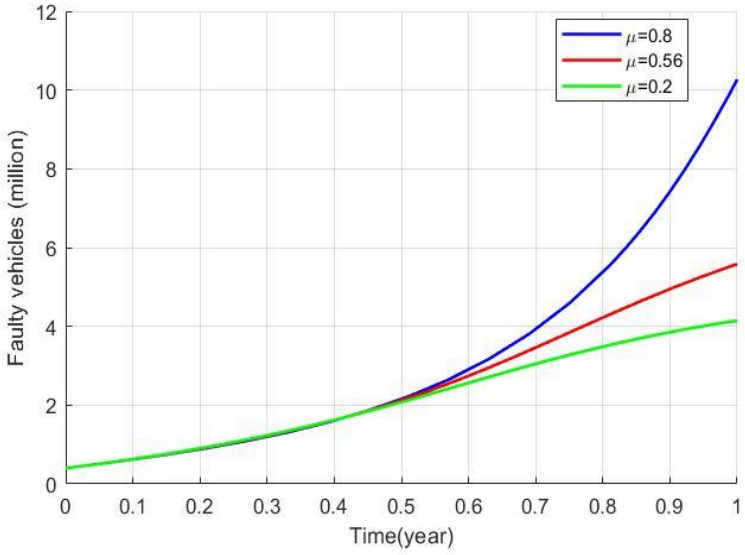

Figure 6. Variation of driver with faulty vehicles with time for $\mu = 0.8, 0.56, 0.2$.

The driver with faulty vehicles represented in the Figure 6 shows that, when a driver with faulty vehicles becomes increase with time when μ increases.

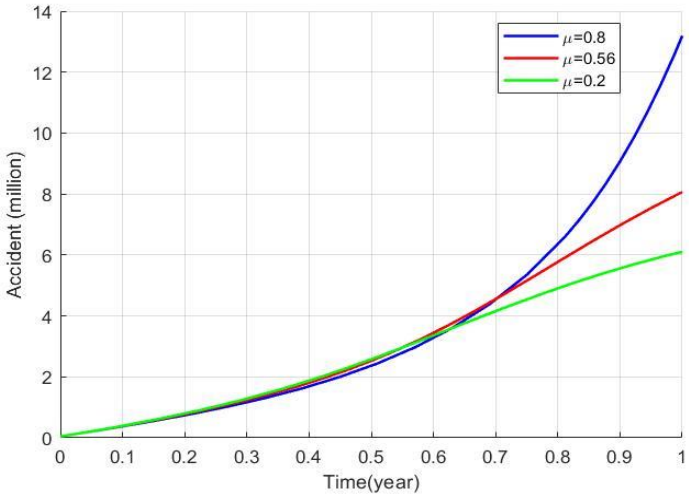

Figure 7. Variation of driver who made accident with time for $\mu = 0.8, 0.56, 0.2$.

The driver who made accident represented in the Figure 7 shows that, when accident becomes initially increase when μ increase, again μ decrease then the total accident becomes decrease with time. Again, we know, γ is the rate at which driver becomes driver who made accident. Now for the different values of γ, we get different variety of figures as follows:

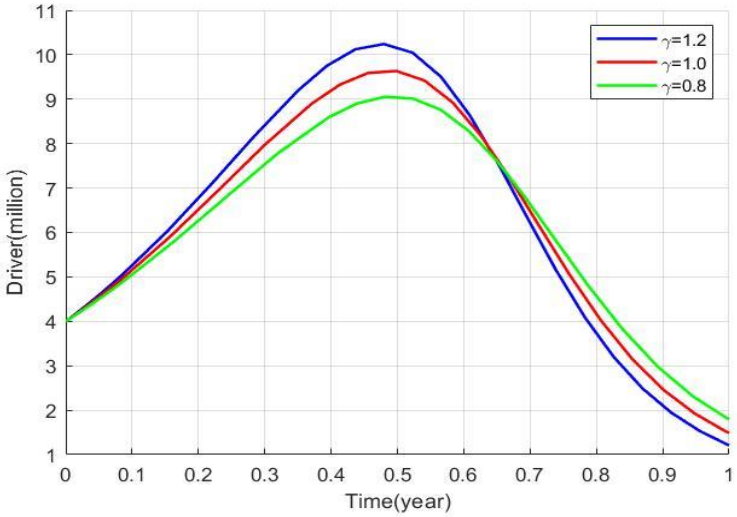

Figure 8. Variation of driver population with time for $\gamma = 1.2,1,0.8$.

The driver population represented in the Figure 8 shows that, the driver becomes initially increase and then decrease as time for the value of parameter γ.

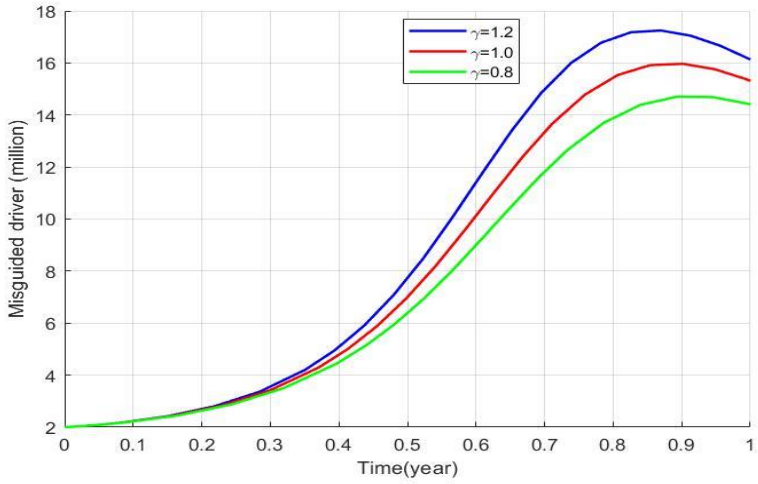

Figure 9. Variation of misguided driver with time for $\gamma = 1.2,1,0.8$.

The misguided driver population represented in the Figure 9 shows that, the misguided driver is initially increasing and then when γ increases.

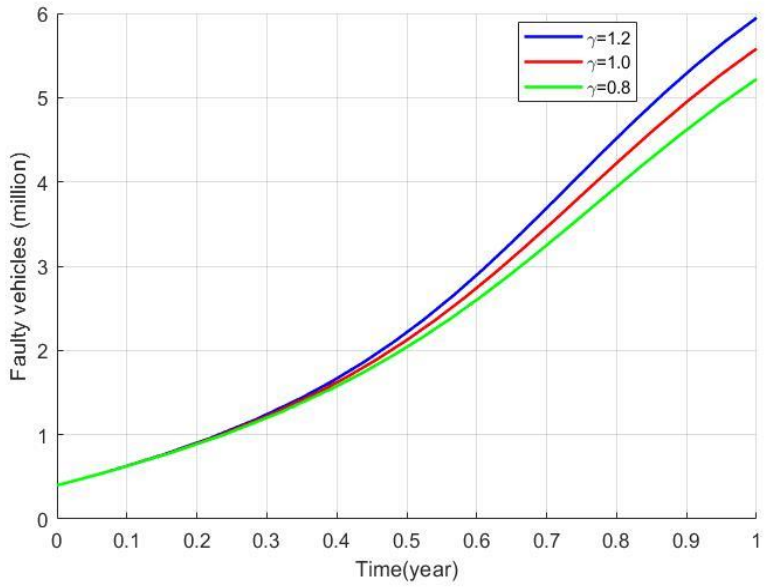

Figure 10. Variation of driver with faulty vehicles with time for $\gamma = 1.2,1,0.8$.

The driver with faulty vehicles represented in the Figure 10 shows that, when a driver with faulty vehicles becomes increase with time when γ increases.

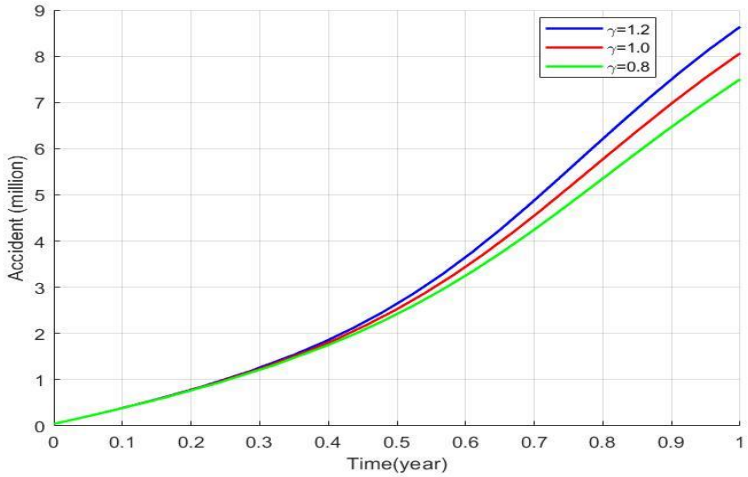

Figure 11. Variation of driver who made accident with time for $\gamma = 1.2,1,0.8$.

The driver who made accident represented in the Figure 11 shows that, when accident becomes initially increase when μ increase, again γ decrease then the total accident becomes decrease with time.

So, we can say that increasing the parameter of μ and γ, the total accident has also increased. Again, at the same process, decreasing those parameter values, the total accident is also decreasing.

Conclusion

On the impact of traffic accidents, we have developed and examined a mathematical model. For the purpose of demonstrating the applicability of the analytical conclusions of the road accident model, numerical simulations based on the relevant model parameters have been carried out. Finally, we have demonstrated that, this model provides the most up-to-date picture of road accidents in Bangladesh. Making and implementing choices for road accident prevention may be beneficial for the government.

Here are the key findings,

- Causes of road traffic accidents
- Number of persons killed and wounded each year on our roads
- Increasing of road accident with increasing the state variables driver, misguided driver and faulty vehicles

Summary

- The model indicates that, the road accident depends on the driver, misguided driver and faulty vehicles
- When misguided driver increases, then the accident rate also increases with time
- The model also indicates that, when the misguided driver interact with the faulty vehicles, then the accident rate is maximum
- We can conclude that, the best decision is to provide road safety information for road users in order to encourage better driving habits

Future works

The following ideas can be recommended for future work. Like we will research how to reduce road accident. For this

- We will monitor our traffic system to decrease this unfavorable situation
- We provide road safety information for road users in order to encourage better driving habits, thus we can reduce the road accident in our country

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