



## MATHEMATICAL MODELING AND ANALYSIS ON WHEAT YIELD IN SOUTHWESTERN COASTAL REGION OF BANGLADESH

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### Abstract

Wheat is one of the most important winter crops which is temperature-sensitive that has been analyzed mathematically. A system of non-linear ordinary differential equations based on agricultural phenomena is to be formed as a mathematical model describing wheat yield, which has been investigated both analytically and numerically. For these temperature-sensitive crops, we've finally demonstrated what would happen to wheat output if the winter season in Bangladesh's southwestern region will be extended a few more days. Also, the effects of salinity on the spike length of wheat as well as yield have been presented. Moreover, the safety wheat production has been reduced by excess use of chemical pesticides.

**Keywords:** Spike length of Wheat, Wheat Yield, , Salinity, Temperature, Chemical Pesticide.

### Introduction

Addressing the temperature for climate change as well as salinity in southwestern coastal area of Bangladesh has been colossal for us, this country is primarily an agriculture based country. The

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country's economic prosperity depends on mainly agriculture. In our country, wheat is the second-most significant grain crop after rice, though, wheat has given the world's diet more calories and protein than any other food crop (Hossain and da Silva, 2013). Furthermore, global wheat production is more than the sum of all other food grains combined. Over the last five decades, the overall harvested area of wheat has grown marginally, from 202.2 million hectares in 1961 to 224.9 million hectare now (Rahman and Hasan, 2009).

Wheat is responsible for 26% of global cereal output and 44% of worldwide cereal consumption. Consumers are expanding their consumption of wheat products as a result of rapid economic and income growth, urbanization, and globalization, particularly in Asia (Pingali, 2007). Wheat output must increase by the middle of the century to fulfill the combined increasing population and increased demand (Tilman et al., 2011). Wheat yield increases are now anticipated to be 0.9 percent per year, much below the 1.5 percent per year needed to fulfill the projected 60 percent rise in world supply by 2050 (Ray et al., 2013).

At the present rate, global wheat output will only rise by 38%, falling well short of expected demand. Furthermore, the impact of climate change, such as less suitable growth circumstances, may cut wheat yield even further (Gammans et al., 2017). If adaptive measures such as enhanced germplasm are not used, yield decreases of up to 6% are expected in wheat for each degree Celsius increase in temperature (Zhao et al., 2017).

While wheat is grown all over the world and suffers a number of biotic and abiotic stresses, heat is the most critical stress and production limiter in South Asia. In temperate zones, where 40% of the world's wheat is grown, terminal heat stress is also a significant concern. Temperatures ranging from 32 to 38 degrees Celsius can reduce grain yields by up to 50% in certain places (Asseng et al., 2011). Heat stress is a physiological phenomenon that affects a variety of plant phenotypes, including canopy temperature (CT) (Ayeneh et al., 2002). Heat tolerance is a physiologically and genetically complicated attribute, according to basic research. It vary by crop (Thomason et al., 2018), species (Kotak et al., 2007), and sowing stages (Tricker et al., 2018).

Temperatures over the optimal threshold are harmful and permanent, with the length and quantity of exposure determining the severity of yield loss. A 3–5% yield loss has been recorded in controlled tests with supra optimal temperatures for every 1°C rise in mean temperature over 15°C (Gibson and Paulsen, 1999). High temperatures can lower each grain mass by up to 23 %, further affecting grain output and quality (Stone and Nicolas, 1994). Many of the world's wheat-growing regions currently have above-average temperatures, and global temperatures are expected to rise by 1.7 to 4.8°C by the end of the century (Teixeira et al., 2013). As a result, boosting grain production under heat stress is a significant global goal, and more effective breeding methods and technologies are required to boost genetic gain in heat-stressed conditions.

In our study, we have showed the variation of temperature and salinity on the total wheat biomass, spike length by using non-linear differential equations and wheat yield is expressed by integral.

### Formulation of the Mathematical Model

To investigate the effects of temperature and salinity on wheat production and present the impacts of organics pesticide on safety wheat yield, the model is proposed consisting three variables such as spike length of wheat, total wheat biomass and pest which are denoted by  $S(t)$ ,  $B(t)$  and  $P(t)$  respectively.

Since, wheat production  $W_P$  depends on spike length of wheat crops and side effects of chemical pesticide  $W_E$  is one of sources of human diseases. Here, the effects of using chemical pesticides on safety production which is expressed using integral. So, the following equation can be written for wheat production:

$$W = W_P - W_E = \int_{74}^{104} \beta_3 S - \int_3^{104} \gamma \beta_2 P \quad (1)$$

where,  $\beta_3$  represents the rate at which wheat production increase for growth of biomass and the side effects of chemical pesticide in wheat crops are indicated by  $\gamma$ . When biomass of wheat crops will be good enough, spike length will be increased. Also Spike length of crops changes for soil moisture and temperature. So, we can write the following equation:

$$\frac{dS}{dt} = \beta S B + \sigma S + \rho(T - T_0)(T_f - T)S \quad (2)$$

where,  $\beta$  denotes the increasing rate of spike length for a standard biomass.  $\sigma$  and  $\rho$  represents the rate at which spike length increase for soil moisture and specific temperature respectively. Also, biomass of wheat crop changes when spike length increase. This also depends on temperature and pest. Moreover, soil nutrition supply and salinity are also responsible for change in biomass.

$$\frac{dB}{dt} = \alpha B S + \beta_1(T - T_0)(T_f - T)B + (\xi - \mu)B - \delta P B \quad (3)$$

where,  $\alpha$  express the increase of biomass for spike length. Also,  $\beta_1$  represents the increase of biomass for a specific temperature and  $\xi$  and  $\mu$  indicates the change of biomass for soil nutrition supply and salinity. In addition, decreasing rate of biomass for pest is represented by  $\delta$ .

Taking wheat biomass as foods, pest population will be increased. But they can be eliminated by using chemical and organic pesticides.

$$\frac{dP}{dt} = \kappa P B - \alpha_1 P - \beta_2 P \quad (4)$$

where,  $\kappa$  is the increasing rate of pest because of attacking wheat crops.  $\alpha_1$  and  $\beta_2$  express the death rate for organic pesticide and death of pest by using chemical.

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## Analytical Framework of the Mathematical Model

### Positivity and boundedness

**Lemma 1:** Considering  $S(0) > 0, B(0) > 0$  and  $P(0) > 0$ , it must be proved that  $S(t) > 0, B(t) > 0$  and  $P(t) > 0$  will be positive for all  $t \in [0, T]$  in  $R_+^3$  where  $T > 0$ .

#### Proof:

Taking all parameters of the system and all initial values to be positive, we have to prove that,  $S(t) > 0, B(t) > 0, P(t) > 0$  will be positive for all  $t \in [0, T]$  in  $R_+^3$ . From the equation (2), we can write the inequality as follows:

$$\frac{dS}{dt} = \beta SB + \sigma S + \rho(T - T_0)(T_f - T)S \quad (5)$$

Separating variables and integrating both sides of the above equations with respect to time, we obtain,

$$S(t) = e^{\beta \int B dt + \sigma + \rho(T - T_0)(T_f - T)} > 0 \quad (6)$$

Similarly,

$$B(t) = e^{\alpha \int S dt + \beta_1(T - T_0)(T_f - T) + (\xi - \mu) - \delta \int P dt} > 0 \quad (7)$$

and

$$P(t) = e^{\kappa \int B dt - \alpha_1 - \beta_2} > 0 \quad (8)$$

Therefore, all solutions are positive according to the Equations (6)-(8). Hence, the lemma is proved.

**Lemma 2:** All solutions  $(S(t), B(t), P(t))$  of the system are bounded.

**Proof:** Adding equation (2) and (3), we can write,

$$\begin{aligned} \frac{d(S + R)}{dt} &= (\alpha + \beta)SB + (\sigma + \rho\tau)S + (\xi - \mu + \tau\beta_1)B - \delta PB \\ \Rightarrow \frac{d(S+R)}{dt} &= (\alpha + \beta)SB + \nu_1(S + R) \quad \text{where } \nu = \max\{\sigma + \rho\tau, \xi - \mu + \tau\beta_1\} \end{aligned}$$

$$\Rightarrow \frac{d(S + R)}{dt} = \frac{(\alpha + \beta)}{2}(S + B)^2 + \nu_1(S + R)$$

$$\Rightarrow \frac{d(S+R)}{dt} = \nu_2\{(S + B) + (S + B)^2\} \quad \text{where } \nu_2 = \max\{\nu_1, \frac{(\alpha + \beta)}{2}\} \therefore$$

$$S(t) + B(t) \leq \frac{1}{e^{\nu_2 t} e^{-c} - 1} \quad (9)$$

where  $c$  is the integrating constant.

According to Lemma 1,  $S(t), B(t) > 0$  for all  $t \in [0, T]$  in  $R_+^3$  and equation (1),  $S(t)$  and  $B(t)$  are bounded.

Now from equation (8), we obtain

$$P(t) = e^{\kappa \int B dt - \alpha_1 - \beta_2} = e^{\kappa B_m a x - \alpha_1 - \beta_2}$$

So,  $P(t)$  is also bounded.

Hence, the Lemma is proved.

### Equilibrium analysis

By solving the equations:  $\frac{dS}{dt}(S^*, B^*, P^*) = 0$ ,  $\frac{dB}{dt}(S^*, B^*, P^*) = 0$ ,  $\frac{dP}{dt}(S^*, B^*, P^*) = 0$ , we obtain trivial equilibrium point  $E_0(0, 0, 0)$  and interior equilibrium points are  $E_1(0, \frac{\alpha_1 + \beta_2}{\kappa}, \frac{\xi - \mu + \tau \beta_1}{\delta})$  and  $E_2(-\frac{\xi - \mu + \tau \beta_1}{\alpha}, -\frac{\sigma + \tau \beta_1}{\beta}, 0)$  where  $\tau = (T - T_0)(T_f - T)$

First two of the equilibrium points will be non-negative if  $\xi + \tau \beta_1 > \mu$  (A)

### Characteristics of states for equilibrium values with respect to $\mu$

The characteristic test of spike length, biomass and pest will be done with respect to  $\mu$ .

Replacing  $P$  from equation (4) in equation (2) and (3), we get two following equations,

$$f(S^*, B^*, \mu) = S^* \sigma + B^* S^* \beta + S^* \rho (T - T_0)(T_f - T)$$

$$g(S^*, B^*, \mu) = B^* S^* \alpha - B^* (\mu - \xi) + B^* \beta_1 (T - T_0)(T_f - T)$$

We know that,

$$\frac{dS^*}{d\mu} = \frac{\begin{vmatrix} \frac{\partial f}{\partial B^*} & \frac{\partial f}{\partial \mu} \\ \frac{\partial g}{\partial B^*} & \frac{\partial g}{\partial \mu} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial S^*} & \frac{\partial f}{\partial B^*} \\ \frac{\partial g}{\partial S^*} & \frac{\partial g}{\partial B^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial B^*} \frac{\partial g}{\partial \mu} - \frac{\partial f}{\partial \mu} \frac{\partial g}{\partial B^*}}{\frac{\partial f}{\partial S^*} \frac{\partial g}{\partial B^*} - \frac{\partial f}{\partial B^*} \frac{\partial g}{\partial S^*}}$$

Finally, we get

$$\frac{dS^*}{d\mu} = \frac{-S^* B^* \beta}{(\sigma + \rho \tau) S^* + (\sigma + \rho \tau + \beta B^*)(\xi - \mu + \beta_1 \tau)}$$

According the condition (A), the denominator of the above fraction is positive and it is also clear that numerator is negative. As a consequence,  $\frac{dS^*}{d\mu}$  is negative which indicates that, for increasing salinity of soil, spike length of wheat crop will be reduced.

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Again,

$$\frac{dB^*}{d\mu} = \frac{\begin{vmatrix} \frac{\partial f}{\partial \mu} & \frac{\partial f}{\partial S^*} \\ \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial S^*} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial S^*} & \frac{\partial f}{\partial B^*} \\ \frac{\partial g}{\partial S^*} & \frac{\partial g}{\partial B^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial \mu} \frac{\partial g}{\partial S^*} - \frac{\partial f}{\partial S^*} \frac{\partial g}{\partial \mu}}{\frac{\partial f}{\partial S^*} \frac{\partial g}{\partial B^*} - \frac{\partial f}{\partial B^*} \frac{\partial g}{\partial S^*}}$$

Therefore,

$$\frac{dB^*}{d\mu} = -\frac{(\sigma + \beta B^* + \rho\tau)B^*}{(\sigma + \rho\tau)S^* + (\sigma + \rho\tau + \beta B^*)(\xi - \mu + \beta_1\tau)}$$

By applying condition (A) again, we can clearly see that,  $\frac{dB^*}{d\mu}$  is negative. Since, it can be stated that, biomass of wheat crops will be declined for increasing of salinity.

### Characteristics of states for equilibrium values with respect to $T$

Now we will analyse the characterization of the equilibrium values of Spike length, biomass and pest of wheat with respect to  $T$ .

From our mathematical model, we can obtain the following equations,

$$f(S^*, B^*, T) = S^*\sigma + B^*S^*\beta + S^*\rho(T - T_0)(T_f - T)$$

$$g(S^*, B^*, T) = B^*S^*\alpha - B^*(\mu - \xi) + B^*\beta_1(T - T_0)(T_f - T)$$

$$\frac{dS^*}{dT} = \frac{\begin{vmatrix} \frac{\partial f}{\partial B^*} & \frac{\partial f}{\partial T} \\ \frac{\partial g}{\partial B^*} & \frac{\partial g}{\partial T} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial S^*} & \frac{\partial f}{\partial B^*} \\ \frac{\partial g}{\partial S^*} & \frac{\partial g}{\partial B^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial B^*} \frac{\partial g}{\partial T} - \frac{\partial f}{\partial T} \frac{\partial g}{\partial B^*}}{\frac{\partial f}{\partial S^*} \frac{\partial g}{\partial B^*} - \frac{\partial f}{\partial B^*} \frac{\partial g}{\partial S^*}}$$

Therefore,

$$\frac{dS^*}{dT} = \frac{(2T - T_f - T_0)\{S^*\alpha + \xi - \mu + \tau\beta_1 - \beta\beta_1S^*B^*\}}{(\sigma + \rho\tau)S^* + (\sigma + \rho\tau + \beta B^*)(\xi - \mu + \beta_1\tau)}$$

Again,

$$\frac{dB^*}{dT} = \frac{\begin{vmatrix} \frac{\partial f}{\partial T} & \frac{\partial f}{\partial S^*} \\ \frac{\partial g}{\partial T} & \frac{\partial g}{\partial S^*} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f}{\partial S^*} & \frac{\partial f}{\partial B^*} \\ \frac{\partial g}{\partial S^*} & \frac{\partial g}{\partial B^*} \end{vmatrix}} = \frac{\frac{\partial f}{\partial T} \frac{\partial g}{\partial S^*} - \frac{\partial f}{\partial S^*} \frac{\partial g}{\partial T}}{\frac{\partial f}{\partial S^*} \frac{\partial g}{\partial B^*} - \frac{\partial f}{\partial B^*} \frac{\partial g}{\partial S^*}}$$

So,

$$\frac{dB^*}{dT} = \frac{(2T - T_f - T_0)\{B^*\beta_1(\alpha + B^*\beta + \rho\tau) - \alpha\rho S^*B^*\}}{(\sigma + \rho\tau)S^* + (\sigma + \rho\tau + \beta B^*)(\xi - \mu + \beta_1\tau)}$$

For  $T_0 < T < \frac{T_0+T_f}{2}$  then  $2T - T_f - T_0 > 0$

According to the conditions (A) and assuming the conditions:  $S^* \alpha + \xi - \mu + \tau \beta_1 > \beta \beta_1 S^* B^*$  and  $B^* \beta_1 (\alpha + B^* \beta + \rho \tau) > \alpha \rho S^* B^*$

we get,  $\frac{dB^*}{dT} > 0$  and  $\frac{dS^*}{dT} > 0$  Similarly, for  $\frac{T_0+T_f}{2} < T < T_f$  then  $\frac{dB^*}{dT} < 0$  and  $\frac{dS^*}{dT} < 0$

Therefore,  $\frac{T_0+T_f}{2}$  is the suitable temperature for proper increase of biomass and spike length.

### Stability analysis of the system at equilibrium points

**Theorem 1:** The dynamical system is unstable at equilibrium point  $E_0(0, 0, 0)$ .

*Proof:* The jacobian matrix at the equilibrium point  $E_0(0, 0, 0)$  is

$$J(0, 0, 0) = \begin{bmatrix} \sigma + \rho\tau & 0 & 0 \\ 0 & \xi - \mu + \beta_1\tau & 0 \\ 0 & 0 & -\alpha_1 - \beta_2 \end{bmatrix}$$

The characteristic equation of the Jacobian matrix is  $|J-\lambda I|=0$

That is

$$\{\sigma + \rho\tau - \lambda\} \{\xi - \mu + \beta_1\tau - \lambda\} \{-\alpha_1 - \beta_2 - \lambda\} = 0$$

From this equation, we see there are three eigen values. First eigen value is clearly positive, second one is positive under the condition (A) but third eigen value is showing negativity. Therefore, the system will be unstable at the equilibrium point  $(0, 0, 0)$ .

**Theorem 2:** The equilibrium point  $E_1(0, \frac{\alpha_1+\beta_2}{\kappa}, \frac{\xi-\mu+\tau\beta_1}{\delta})$  is unstable.

*Proof:* Again, The jacobian matrix at the equilibrium point  $(S^*, B^*, P^*)$  is

$$J(S^*, B^*, P^*) = \begin{bmatrix} \sigma + B^*\beta + \rho\tau & S^*\beta & 0 \\ B^*\alpha & \xi - \mu + \beta_1\tau - P^*\delta + S^*\alpha & -B^*\delta \\ 0 & P^*\kappa & B^*\kappa - \beta_2 - \alpha_1 \end{bmatrix}$$

where  $\tau = (T - T_0)(T_f - T)$

At the equilibrium point  $E_1(0, \frac{\alpha_1+\beta_2}{\kappa}, \frac{\xi-\mu+\tau\beta_1}{\delta})$ , we have Jacobian matrix,

$$J(S^*, B^*, P^*)|_{E_1} = \begin{bmatrix} \sigma + \beta \frac{\alpha_1+\beta_1}{\kappa} + \rho\tau & 0 & 0 \\ \beta \frac{\alpha_1+\beta_1}{\kappa} & 0 & \frac{\alpha_1+\beta_1}{\delta} \\ 0 & \frac{(\xi-\mu+\tau\beta_1)\kappa}{\delta} & 0 \end{bmatrix}$$

If  $\lambda_1, \lambda_2$  and  $\lambda_3$  are the solutions of the characteristic equation  $|J-\lambda I|=0$  of the jacobian matrix then we have,  $\lambda_1 = \sigma + \beta \frac{\alpha_1+\beta_1}{\kappa} + \rho\tau > 0$ . On the other hand,  $\lambda_2$  and  $\lambda_3$  are purely imaginary. So, the equilibrium point  $E_2$  is unstable.

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## Numerical Simulations

To observe the effects of temperature on wheat production as well as spike length and biomass, we have to simulate our model. For analyzing mathematical model we have used MATLAB(R2021b) using Runge-Kutta method with the help of the values of parameters from Table 1.

**Table 1:** Parameter Values and their Descriptions (Sarker et al., 2022)

Descriptions of the parameter	Notations	Values
Increasing rate of spike length for a standard biomass	$\beta$	$1.35 \times 0.007$
Spike length increasing rate for soil moisture	$\sigma$	$3 \times 0.0186$
Increasing rate of spike length for specific temperature	$\rho$	$2 \times 0.000078$
Minimum value of standard temperature	$T_0$	$12^\circ C$
Maximum value of standard temperature	$T_f$	$22^\circ C$
Increasing rate of biomass for spike length	$\alpha$	$1 \times 0.00028$
Rate of changes of biomass for a specific temperature	$\beta_1$	$6.31 \times 0.000028$
The rate of changes of biomass for soil nutrition supply	$\xi$	$5.39 \times 0.013$
Rate of change of biomass for salinity	$\mu$	$2 \times 0.006$
Decreasing rate of biomass for pest	$\delta$	$2 \times 0.89$
Increasing rate of pest because of feeding wheat crops	$\kappa$	$1.5 \times 0.0045$
Death rate of pest for organic pesticide	$\alpha_1$	$2.0 \times 0.002$
Death of pest by using chemical	$\beta_2$	$2.0 \times 0.006$
The rate at which wheat production increase for growth of biomass	$\beta_3$	$0.0315$
The decreasing rate of wheat because of pest	$\gamma$	$0.095$

Assuming that the seeding time of wheat is 22nd November. At that time, wheat seeds be taken 5 days after sowing to be germinated on average. Plants appear after 5 days from sowing time but they will germinate on 25th November. As a result, we will start with the 4th day of sowing. Plant biomass begins to rise on day 4 from sowing, and insect populations begin to increase in parallel. However, wheat spike length takes time to develop and will be noticeable from day 75. So, there will be no spike after spreading wheat seeds. Therefore, the initial values of spike length, biomass and pest are  $S(0) = 0$  Inch,  $B(0) = 0.03$  ton/hectare and  $P(0) = 0.001$  lac/hectare. We are using pesticide from germination.

The dynamics of state variables for various temperatures are illustrated in Figure 1. where we used three separate data sets of temperature to demonstrate this dynamics. The first one is actual temperature data, while the second one and third one are based on the assumption that the last 15 day's (19th February to 5th March) temperatures were  $23^\circ C$  and  $18^\circ C$  respectively. Particularly this 15 days period is known as the reproductive stage of crop growth. It is mentioned that rest of

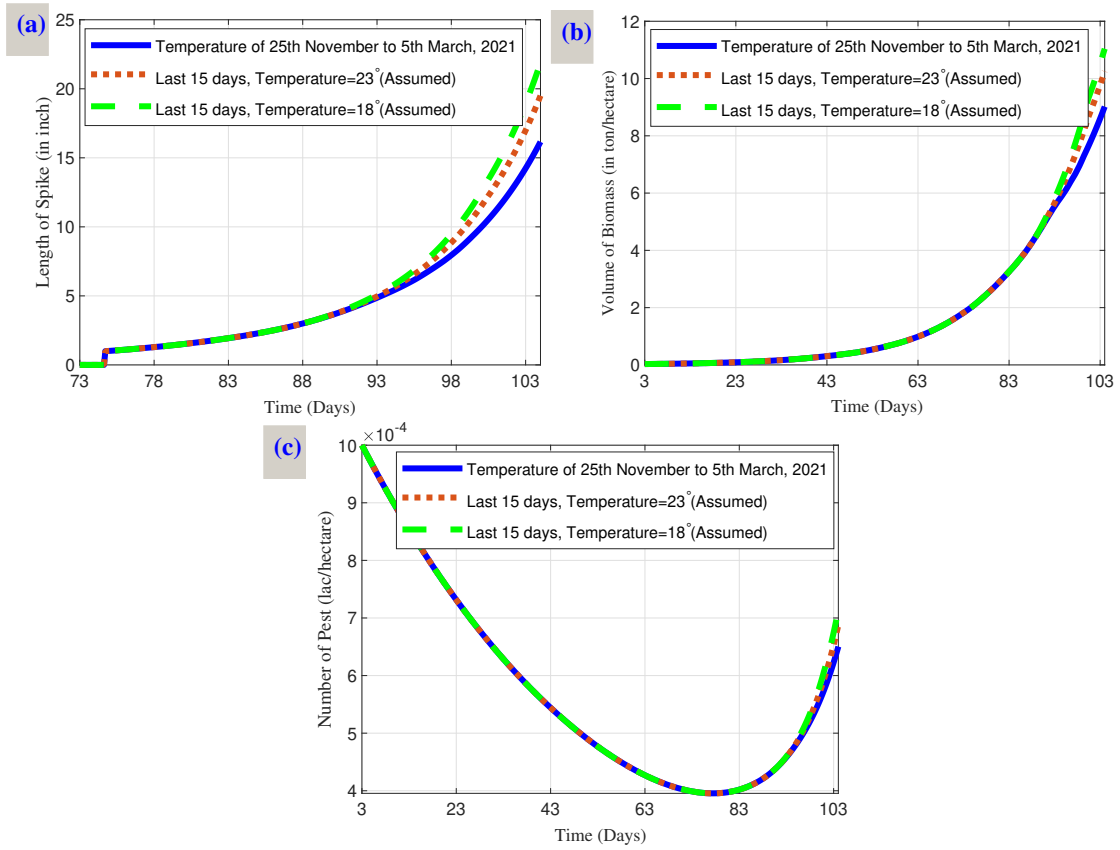


Figure 1: Variation of states variables for different temperature profile.

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all are same on actual temperature. Our major goal is to examine how state variables would have changed if winter had lasted a few more days.

Figure 1(a) shows that spike length increases while temperature decreases. As a result, we achieve the longest spike length by using an 18° C temperature over the last 15 days. As spike length is one of the most essential components of biomass, the biomass of wheat plants will grow as spike length increases. Figure 1(b) illustrates that wheat plants will have more biomass at lower temperature. Again, the amount of pests varies on the biomass of the plants and pesticide. Biomass of wheat plants is increasing in lower temperature but the number of pest is not increasing because of using pesticides regularly. As a result, temperature changes do not have a significant impact on insect numbers (see Figure 1(c)).

Salinity is a common phenomenon in southwestern coastal soil of Bangladesh. Figure (2) depicts the impacts of different parameter values  $\mu$  on state variables, where  $\mu$  is the biomass declination rate due to salinity. We can see that when salinity enhances, the biomass of the wheat plant decreases. Spike length will not be ideal if wheat plant biomass is insufficient. As a result, spike length is reducing while salinity rises. Pests are constantly dependent on plant biomass, hence as biomass decreases, the number of pests decreases. So, For increasing the value of salinity we have seen that, spike length, biomass and the number of pest are decreasing which satisfy the finding of characteristics of states for equilibrium values with respect to  $\mu$  subsection.

## Results and Discussion

Wheat is Bangladesh's second most significant crop that is temperature sensitive. Besides, its production depends on soil nutrients, salinity, soil moisture etc. Therefore, land in all of Bangladesh's regions cannot produce the same quantity of wheat. From characteristic test with respect to salinity, we observed that both spike length and biomass is declining with the increasing of soil salinity. Therefore, soil salinity and temperature are two of the most critical factors affecting wheat productivity in different locations.

Optimum temperature and low salinity are needed for good biomass and spike length, according to our analysis. If the biomass and spike length are good, the total amount of wheat production will be raised. So, in comparison to high temperature and salinity, low temperature and low salinity can produce more wheat. In Table 2, the actual production during the cultivation period is 4.5 tons/hectare, which will be 4.94 tons/hectare hectare if the temperature of the last 15 days will 23 degrees Celsius, and 5.25 tons/hectare hectare if the temperature of the last 15 days will 18 degrees Celsius which indicates that if it had been cold for some more time in the southwestern region of Bangladesh then wheat production will be raised.

Pests are now playing an important role in reducing agricultural productivity. As a result, pesticides must be used to improve wheat output. Chemical pesticides, on the other hand, are extremely detrimental to one's health. As a consequence, organic pesticides will be used to eliminate

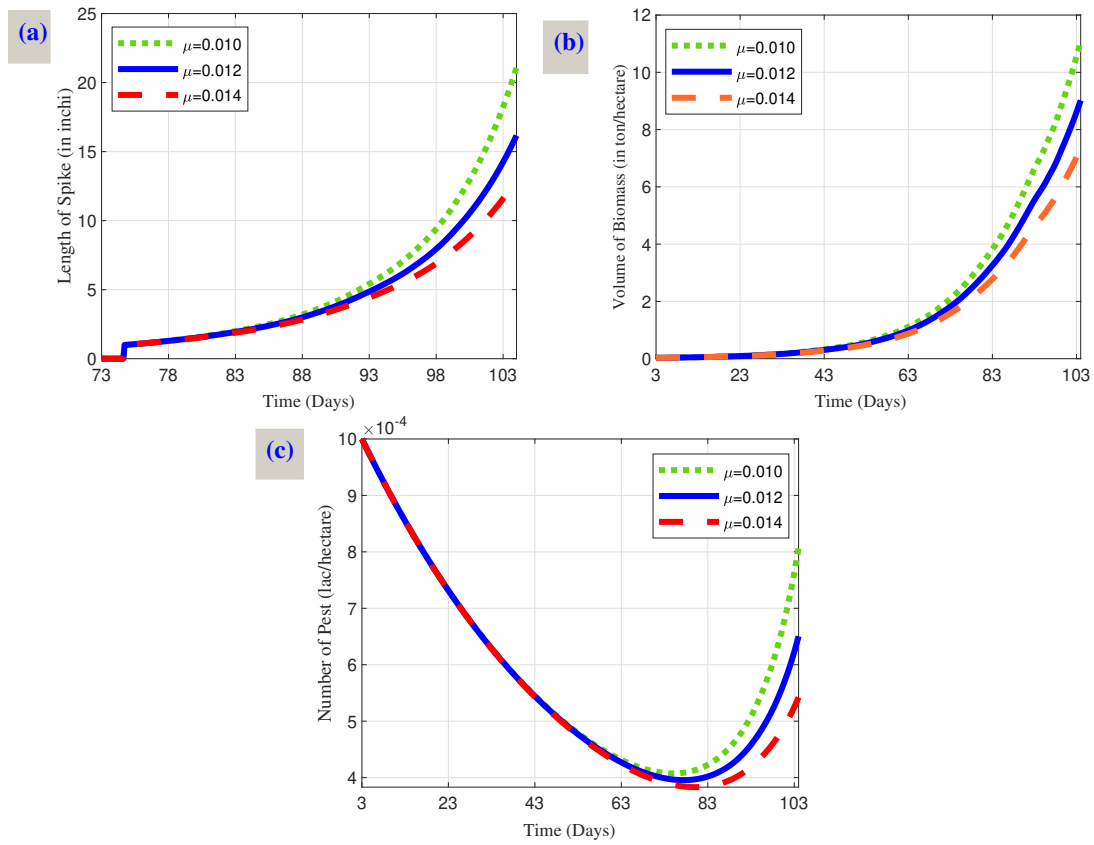


Figure 2: Variation of states variables for different salinity level.

Table 2: Effects of temperature on wheat production

Serial No.	Status of Weather and soil	Production of Wheat (ton/hectare)
1	Temperature of 25th November to 5th March, 2021	4.50
2	Last 15 days (Reproductive Stage), Temperature=23°(Assumed)	4.94
3	Last 15 days (Reproductive Stage), Temperature=18°(Assumed)	5.25

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pests. Then, we obtain some more safety food. Chemical pesticides, but at the other hand, are more effective and aggressive in killing pests than organic pesticides. Table 3 represents the impacts of organic at side of chemical pesticides on safety wheat production where  $\alpha_1$  denotes death rate for organic pesticide and  $\beta_2$  denotes killing rate of chemical pesticide that means these two parameters also represent the amount of organic and chemical pesticides. We can see from the Table 3 that if we use organic pesticide at the side of chemical pesticide, we will get more safety wheat production.

**Table 3: Effects of chemical pesticide on safety wheat production**

Serial No.	$\alpha_1$	$\beta_2$	$W_P$	$W_E$	W
1	$\alpha_1 = 0.004$	$\beta_2 = 0.010$	4.4805	0.2392	4.2413
2	$\alpha_1 = 0.004$	$\beta_2 = 0.012$	4.5055	0.2629	4.2426
3	$\alpha_1 = 0.006$	$\beta_2 = 0.012$	4.5282	0.2416	4.2836
4	$\alpha_1 = 0.006$	$\beta_2 = 0.010$	4.5055	0.2191	4.2864

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