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DYNAMIC OPTIMIZATION APPLIED TO A CRIMINOLOGICAL MODEL FOR REDUCING THE SPREAD OF SOCIETAL CORRUPTION

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Abstract

Corruption is rapidly affecting any country's economic, democratic, financial, social, and political stability. It has been a consistent social phenomenon that happens in all civilizations. Only in the last 20 years has this phenomenon been given serious attention. It has different forms and different impacts on the economy as well as society as a whole. Economic growth is slowed by corruption, which also has a detrimental effect on business operations, employment, and investment. Additionally, it has a detrimental effect on tax revenues as well as the effectiveness of various financial aid programs. So, it is necessary to reduce this global problem. For this reason, we propose a nonlinear deterministic model for the transmission dynamics of societal corruption in terms of optimal control problem, using two time-dependent controls namely the efforts aimed at preventing corruption through the use of social networks, media, and social organizations; including a strong and effective anti-corruption policy, and also the attempt to encourage the punishment of corrupt people to analyze the model. The goal of this study is to reduce the problem of corruption. The results show that our proposed model can help to alleviate this social issue. Our findings also reveal that by using both control strategies instead of just one we get a more effective result. Overall, this research suggests that the impact of corruption can be reduced by implementing anti-corruption media and advertising campaigns, as well as exposing corrupted people to jail and punishing them.

Keywords: Corruption, mathematical model, boundedness, optimal control, numerical analysis

Introduction

It is alarming that money is earned by breaking the law or moral or social norms. The World Bank (WB) defines corruption as the act of using the government position for the personal benefit [World Bank, 1997]. It is also defined by Transparency International (TI) as the misuse of responsibility for dealing power for the personal benefit [Hathroubi & Trabelsi, 2014]. Colorless, shapeless corruption is collusive, secretive, stealthy, and shameless [Caiden, 2011]. It is a worm and disease to social, economic, and political progress [Abdulrahman, 2014]. Illegal gratitude, conflicts of interest, misuse of public funds, Bribes, and economic exploitation are all examples of corruption [Binuyo, 2019]. It is a significant contributor to poverty globally,

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but especially in Africa [Danford et al.2020], where deep-seated corruption stymies economic development, undermines democracy, and jeopardizes human rights and the legal system [Eguda, 2017]. However, things work differently in Ethiopia, where political power sharing, checks and balances, accountable and transparent institutions, and processes are all lacking [Kebede, 2013]. It is one of the factors that reason for tension and conflict, as evidenced by the current situation in Ethiopia [Legesse & Shiferaw, 2018].

For the establishment of successful intervention methods for reducing corruption as well as corrupt practices, a complete understanding of the corrupt system, as well as prevention and disenchantment programs, is required [Nathan & Jackob, 2019]. In epidemiological studies, mathematical modeling has proven to be an effective tool for comprehending and explaining communicable diseases' dynamic nature [Hattaf, 2020]. The epidemiologic studies corruption analysis technique has been studied as a disease by many researchers. Gweryina et al. [Gweryina, et al. 2019] investigated in detail an epidemiological corruption model with immune defense in Nigeria. Trying to treat corruption in the existence of an immunity clause system will be a hard war to win if the immunity clause stays in place. To investigate the dynamic behavior of corruption as a disease, Eguda et al. [Eguda et al. 2017] created and examined a standard incidence model. A corruption dynamics model was proposed by Hathroubi and Trabelsi [Hathroubi & Trabelsi, 2014]. A simplistic social epidemic model that Crokidakis and Martins [Crokidakis & Sá Martins, 2018] designed and examined illustrates the nature of social impacts between many politicians in a fictitious corrupt legislature. Nathan and Jackob [Nathan & Jackob, 2019] split the whole population into three groups: susceptible (S), corrupt (C), and politically corrupt (PC). They also evaluate strategies for combating the vice of corruption. Abdulrahman [Abdulrahman, 2014] considered corruption transmission dynamics as a disease and created a deterministic mathematical model. Legesse and Shiferaw [Legesse & Shiferaw, 2018] recommended and examined a mathematical model of corruption transmission spread. This model is identical to Abdulrahman [Abdulrahman, 2014] in terms of characteristics, with the exception that a person who loses immune function gained via council through prison does not straightly attend the corrupt class; rather, they are vulnerable due to human attitude, as well as are worried about the effect of anticorruption consciousness.

Optimal control theory is always helpful for any kind of decision making including disease, social problems, etc. [Zhao et al. 2021]. Wang et al. [Wang et al. 2016] developed an optimal control model using the classic Pontryagin maximum principle method that involves converting the ordinary annual incidence to a bilinear only considering discrete time delays for calculation validity. A deterministic model of the spread of corruption was developed by Athithan et al. [Athithan et al. 2018]. They also used differential equations to analyze it. The model was then enlarged to optimal control.

Taking all these into account we developed a newly proposed model with four compartments namely susceptible, corrupted, jailed, and honest individuals. This work is different from others because jailed individuals can become corrupted again when they are punished. We formulated this model and analyzed it numerically and analytically both below.

Model Formulation

Firstly we divided the whole population into four groups at time t: susceptible $S(t)$, corrupted $C(t)$, jailed $J(t)$, and honest $H_1(t)$ population. People who are susceptible are those who are not corrupt but who are at a high risk of becoming corrupt. On the other hand, corrupted people are those people who are involved in corruption. The people who are involved in corruption and prove guilty due to the law are known as the jail population. Honest people are truthful people who will never be corrupted, but they become vulnerable as a result of social pressure. Individuals who are susceptible, corrupt, or jailed can be decided to convert into honest people through judicial reform or psychotherapy in prison, media, ethical, and religious convictions, and also the effect of consciousness is similar.

The model can be explained as follows, shown in the flowchart in Figure 1. The parameter r represents the recruitment rate into the susceptible class and μ represents the natural death rate for all time

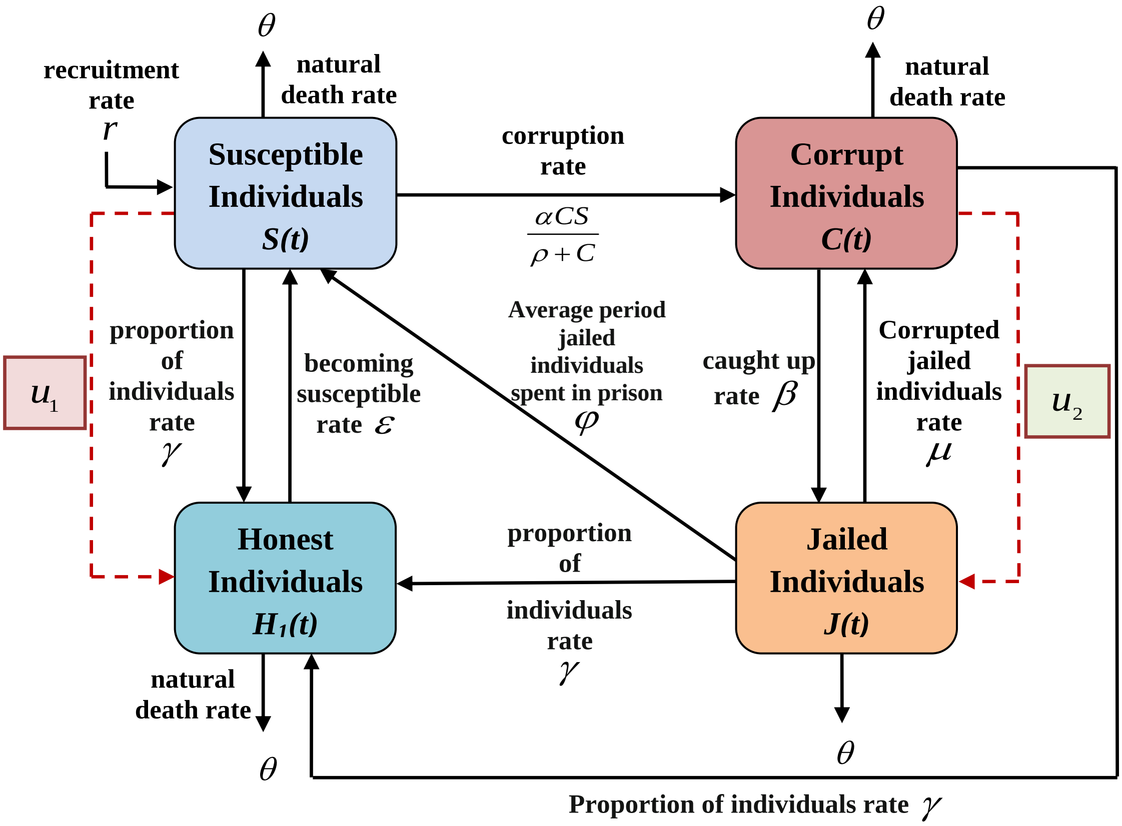

Figure 1. Scheme Diagram for Corruption Dynamics.

under the study. Susceptible individuals can be corrupted at a rapid rate $\frac{\alpha CS}{\rho + C}$ attributed to the influence of corrupt individuals. In addition, other parameters are also defined as: α is the contact rate of effective corruption, ρ is the maximum saturation of corrupt population, is the rate at which corrupt people are arrested and put in prison, μ is the rate at which jail people become corrupted again, γ represents the proportion of individuals that joins H_1 from S, C, and J because of consciousness generated by anti-corruption campaigns or counseling in prison, media, ethical, and religious convictions, ε represents the rate at which social pressure causes honest people to become vulnerable and φ is the average time jailed individuals spent in prison. We also use two controls name u_1 and u_2 in this model. All the assumptions taking into account the differential equation of our model can be expressed as follows:

$$\frac{dS}{dt} = r - \frac{\alpha SC}{\rho + C} - (\gamma + \theta)S + \varphi J + \varepsilon H_1 - u_1 S \tag{1}$$

$$\frac{dC}{dt} = \frac{\alpha SC}{\rho + C} - (\beta + \gamma + \theta)C + \mu J - u_2 C \tag{2}$$

$$\frac{dJ}{dt} = \beta C - (\gamma + \theta + \mu + \varphi)J + u_2 C \qquad (3)$$

$$\frac{dH_1}{dt} = \gamma(S + C + J) - (\varepsilon + \theta)H_1 + u_1 \mathrm{S} \qquad (4)$$

with the initial conditions: $S(0) = S_0 > 0,\ \ C(0) = C_0 > 0,\ \ J(0) = J_0 > 0,\ \ H_1(0) = H_{10} > 0.$ Table 1 lists the description of the non-negative parameters used for the system (1-4).

Table 1. Parameters Description of Corruption Dynamics

Parameters	Description
r	recruitment rate
α	contact rate of effective corruption
ρ	maximum saturation of corrupt population
β	rate at which corrupt people are arrested and put in prison
μ	rate at which jail people become corrupted again
γ	proportion of individuals that joins *H* from *S*, *C*, and *J*
ε	rate at which honest individuals become susceptible
φ	average time jailed individuals spent in prison
θ	natural death rate
u_1	The control parameter, be the efforts aimed at preventing corruption through the use of social networks, media, and social organizations; including a strong and effective anti-corruption policy.
u_2	The control variable, be the attempt to encourage the punishment of corrupt people

Boundedness, Equilibrium Point (Corruption-free), and Basic Reproduction Number

Here we will explain boundedness [Khan & Talukder, 2021, Rudin, 1987], find out the equilibrium [Side et al. 2019] and basic reproduction number [Side et al., 2019] regarding the constant value of control variables u_1 and u_2.

Lemma 1: For model (1)-(4) the region $\Omega = \left\{(S, C, J, H_1) \in R_+^4; 0 \le (S(t) + C(t) + J(t) + H_1(t)) : N(t) \le \frac{r}{\theta}\right\}$ is a positively invariant set.

Proof: Let $N = S + C + J + H_1.$

So, $\frac{dN}{dt} = r - \theta N.$ Now after integrating this equation and taking limit in both sides as $t \to \infty,$ we obtain $\limsup_{t\to\infty} N(\mathrm{t}) = \frac{r}{\theta}.$ Thus Ω is a positively invariant set i.e. all the solutions of model (1)-(4) with initial conditions remain in the region Ω . Thus the above lemma is proved.

The model (1)-(4) has must corruption free equilibrium (CFE)

$$E_1 = (S_1, C_1, J_1, H_{11}) = \left(\frac{r\delta_4}{\delta_4(\delta_1 + u_1) - \varepsilon\gamma}, 0, 0, \frac{r(\gamma + u_1)}{\delta_4(\delta_1 + u_1) - \varepsilon\gamma} \right) \text{with}$$

$\delta_1 = (\gamma + \theta),\ \delta_2 = (\beta + \gamma + \theta),\ \delta_3 = (\gamma + \theta + \mu + \varphi),\ \delta_4 = (\varepsilon + \theta)$ where there is no corruption at this stage. Also, the model (1)-(4) has a basic reproduction number R_0 which represents the quantity of subsequent infection brought on by only one infection.

Lemma 2: Basic reproduction number is $R_0 = r\alpha\delta_3\delta_4 / \rho\{\delta_4(\delta_1 + u_1) - \varepsilon\gamma\}\ \{\delta_3(\delta_2 + u_2) - \mu(\beta + u_2)\}$ for the above model.

Proof: Here we have two infected compartments which are corrupted individuals and jailed individuals. The next-generation matrix $FV-1$ having a small domain is used to define the basic reproduction number R_0 where $F = \begin{pmatrix} \frac{\rho\alpha S}{(\rho + C)^2} & 0 \\ 0 & 0 \end{pmatrix}$ and $V = \begin{pmatrix} \delta_2 + u_2 & -\mu \\ -\beta - u_2 & \delta_3 \end{pmatrix}$.

Thus the basic reproduction number of our model is

$$R_0 = \frac{r\alpha\delta_3\delta_4}{\rho\{\delta_4(\delta_1 + u_1) - \varepsilon\gamma\}\{\delta_3(\delta_2 + u_2) - \mu(\beta + u_2)\}}.$$

Hence Lemma 2 has proved.

Characterization of the Optimal Control

Two controls have been taken into account in our model where $u_1(t)$ is used to prevent corruption situations, and the other $u_2(t)$ is used to punish corrupted individuals. We consider that both controls are the functions of Differential Equations and Dynamical Systems of time t, which are implemented by the requirements. Our primary goal is to reduce the overall population that is corrupt and the cost due to preventing corruption and punishing the corrupted individuals. As a result, the optimal control strategy is used to reduce both the overall number of corrupted individuals and the cost of implementing the two controls. After accounting for both control variables, the corresponding performance index becomes:

$$\text{Minimize } J\left(u_1(t), u_2(t)\right) = \int_0^T \left(B_1 C(t) + \frac{B_2}{2} u_1^2(t) + \frac{B_3}{2} u_2^2(t) \right) dt \qquad (5)$$

where B_1, B_2 and B_3 represent the balancing cost factor or weight parameters. For convexity, the nonlinear quadratic cost function is used.

For our model, $\left(u_1(\text{t}), u_2(\text{t})\right) \in U$ is considered as Lebesgue measurable, where

$$U = \left\{\left(u_1(\text{t}), u_2(\text{t})\right) : 0 \le a_i \le u_i(\text{t}) \le b_i \le 1, i = 1,2\right\} \forall t \in [0, \text{T}]$$

After that, we get the optimal control problem from [Biswas, 2013] expressed in terms of objective functional like,

$$
(P_1)\begin{cases} \text{Minimize} \quad J(x,u) = \int_0^T L(x(t),u(t))dt \\ \text{subject to} \\ \dot{x}(t) = f(x(t)) + g(x(t))u(t), \forall t \in [0,T] \\ u(t) \in U, \forall t \in [0,T] \\ x(0) = x_0 \end{cases} \tag{6}
$$

where .. $f(x) = \begin{pmatrix} r - \dfrac{\alpha SC}{\rho + C} - (\gamma+\theta)\text{S} + \varphi J + \varepsilon H_1 - u_1 \text{S} \\ \dfrac{\alpha SC}{\rho + C} - (\beta+\gamma+\theta)C + \mu J - u_2 C \\ \beta C - (\gamma+\theta+\mu+\varphi)J + u_2 C \\ \gamma(S+C+J) - (\varepsilon+\theta)H_1 + u_1 \text{S} \end{pmatrix}$, $g(x) = \begin{pmatrix} -\dfrac{\alpha SC}{\rho+C} & 0 \\ \dfrac{\alpha SC}{\rho+C} & -C \\ 0 & C \\ 0 & 0 \end{pmatrix}$, $u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$ and

$$
L(x,u) = B_1 C(t) + \frac{B_2}{2}u_1^2(t) + \frac{B_3}{2}u_2^2(t) \tag{7}
$$

considered as the integrand of the performance index.

The Hamiltonian function [Biswas, 2013, Lenhart & Workman 2007] of systems (1) – (4) w.r.t u_1, u_2 can be written as,

$$
H\left(t, x(t), u(t), p(t)\right) = L(t,x,u) + \sum_{i=1}^{4} p_i f_i^*(t,x,u)
$$

where

$$
x(t) = \begin{pmatrix} S(t) \\ C(t) \\ J(t) \\ H_1(t) \end{pmatrix}, \; p(t) = \begin{pmatrix} p_S(t) \\ p_C(t) \\ p_J(t) \\ p_{H_1}(t) \end{pmatrix}, \; u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}
$$

$f(t,x,u)$ represents the right-hand side of the model (1)-(4).

Here $p(t) = (p_S(t), p_C(t), p_J(t), p_{H_1}(t)) \in \mathbb{R}^4$ indicates the adjoint variable.

If (x^*, u^*) is considered the optimal pair of P_1.

Then the Pontryagin's maximum principle [Pontryagin et al., 1962] is expressed as

i. $\dfrac{\partial H}{\partial u} = 0$ at $u = u^*$, which is the optimality condition.

ii. $p' = -\dfrac{\partial H}{\partial x}$ which is the adjoint equation.

iii. $p(T) = 0$ which is the transversity condition.

So, the standard form of Hamiltonian function H concerning (u_1, u_2) according to our model can be expressed as,

$$H\left(t,x(t),u(t),p(t)\right)=\left[B_1C(t)+\frac{B_2}{2}u_1^2+\frac{B_3}{2}u_2^2\right]$$
$$+p_S[r-\frac{\alpha SC}{\rho+C}-(\gamma+\theta)S+\varphi J+\varepsilon H_1-u_1\mathrm{S}]$$
$$+p_C[\frac{\alpha SC}{\rho+C}-(\beta+\gamma+\theta)C+\mu J-u_2C] \qquad (8)$$
$$+p_J[\beta C-(\gamma+\theta+\mu+\varphi)J+u_2C]$$
$$+p_{H_1}[\gamma(S+C+J)-(\varepsilon+\theta)H_1+u_1\mathrm{S}]$$

Theorem 1: Optimal control variables u_1, u_2 and S,C,J,H_1 are the corresponding optimal state variables of our systems (1) to (4). Then there exist adjoint variables $p(t)=(p_S(t),p_C(t),p_J(t),p_{H_1}(t))\in\mathbb{R}^4$ that satisfy the following equations

$$p_S'=\frac{\alpha C}{\rho+C}(p_S-p_C)+(\gamma+\theta)p_S-\gamma p_{H_1}+u_1(p_S-p_{H_1}) \qquad (9)$$

$$p_C'=\frac{\alpha\rho C}{(\rho+C)^2}(p_S-p_C)+(\beta+\gamma+\theta+u_2)p_C-(\beta+u_2)p_J-\gamma p_{H_1}-B_1 \qquad (10)$$

$$p_J'=(\gamma+\theta+\mu+\varphi)p_J-\mu p_C-\varphi p_S-\gamma p_{H_1} \qquad (11)$$

$$p_{H_1}'=(\theta+\varepsilon)p_{H_1}-\varepsilon p_S \qquad (12)$$

with transversality condition $p_i(\mathrm{T})=0,\ i=1,2,3\,\text{and}\,4.$ (13)

Proof: Let u_1^* and u_2^* is optimal control variables and S^*,C^*,J^* and H_1^* is the associated optimal state variables of systems (9) to (12). Then by Pontryagin's maximum principle there exist adjoint variables (9) to (12) which satisfy the following equations,

$$\frac{dp_S}{dt}=-\frac{\partial H}{\partial S},\frac{dp_C}{dt}=-\frac{\partial H}{\partial C},\frac{dp_J}{dt}=-\frac{\partial H}{\partial J},\frac{dp_{H_1}}{dt}=-\frac{\partial H}{\partial H_1} \qquad (14)$$

With transversity conditions,

..

Where Hamiltonian H is defined in (8).

Thus, condition (13) proved **Theorem 1**.

Theorem 2: There exists an optimal control (u_1^*,u_2^*) for optimal control problem (p_1), which minimizes our objective functional $J(u_1,u_2)$ over U is represented by:

$u_1^*=\max\{0,\min(1,\bar{u}_1)\}$ and $u_2^*=\max\{0,\min(1,\bar{u}_2)\}$

with $\bar{u}_1=\dfrac{S^*(p_S-p_H)}{B_2}$ and $\bar{u}_2=\dfrac{C^*(p_C-p_J)}{B_3}$

Proof: Using optimality conditions, we have $\dfrac{\partial H}{\partial u_1}=B_2u_1^*-Sp_S+Sp_H$ and $\dfrac{\partial H}{\partial u_2}=B_3u_2^*-C(p_C-p_J)$.

Following the property of *U*, our both controls (u_1^*, u_2^*) are bounded with (0, 1).
Thus,

$$u_1^* = \begin{cases} 0 & when\ \bar{u}_1 \le 0 \\ \bar{u}_1 & when\ 0 < \bar{u}_1 < 1, where\ \bar{u}_1 = \dfrac{S^*(p_S - p_H)}{B_2} \\ 1 & when\ \bar{u}_1 \ge 1 \end{cases}$$

This can be expressed in compact form as .

.

Similarly,

$$u_2^* = \begin{cases} 0 & when\ \bar{u}_2 \le 0 \\ \bar{u}_2 & when\ 0 < \bar{u}_2 < 1, where\ \bar{u}_2 = \dfrac{C^*(p_C - p_J)}{B_3} \\ 1 & when\ \bar{u}_2 \ge 1 \end{cases}$$

In the same process, this can be displayed as

$$u_2^* = \max\left\{0, \min\left(1, \frac{C^*(p_C - p_J)}{B_3}\right)\right\}$$

Hence **Theorem 2** is proved.

Numerical Analysis

In order to numerically solve the optimality system and confirm the effectiveness of optimal control, we used the forward-backward sweep method. We have set 80 years to solve our problem as well as make the assumption that prevention and punishment will be effective during that time. The simulation that we ran with the initial conditions $S_0 = 60000, C_0 = 1600, J_0 = 0\ and\ H_{10} = 130000.$ and the parametric values is listed in Table 2. All the values of the parameters are taken from Abdulrahman, 2014 and Legesse & Shiferaw, 2018.

Table 2. Values of the model parameters

r	α	ρ	β	μ	γ	ε	φ	θ
12000	0.0234	100000	0.000001	0.143	0.014	0.0021	0.001	0.016

In this paper we use two controls which are $u_1(t)$ is used for preventing the corruption situation and $u_2(t)$ is used for punishment efforts for corrupted individuals respectively. But which control is more effective for reducing corruption? For answering this question we run the program three times for three scenarios. Firstly we show the control profiles in Figure 2. We initially solve the optimality systems when no punishment method is used for our model(1)-(4). So we have taken the control variable $u_1(t) \ne 0$ and $u_2(t) = 0.$ The simulation results are shown in Figure 3. Also, we run the program for our model (1)-(4) when no prevention strategy is employed. So, in this case, we have taken two controls as $u_1(t) = 0$ and $u_2(t) \ne 0$ and the simulation results are shown in Figure 4. After that, finally, employ both control strategy $u_2(t) \ne 0$ and $u_2(t) \ne 0$ we run the program and get the results which are represented in Figure 5.

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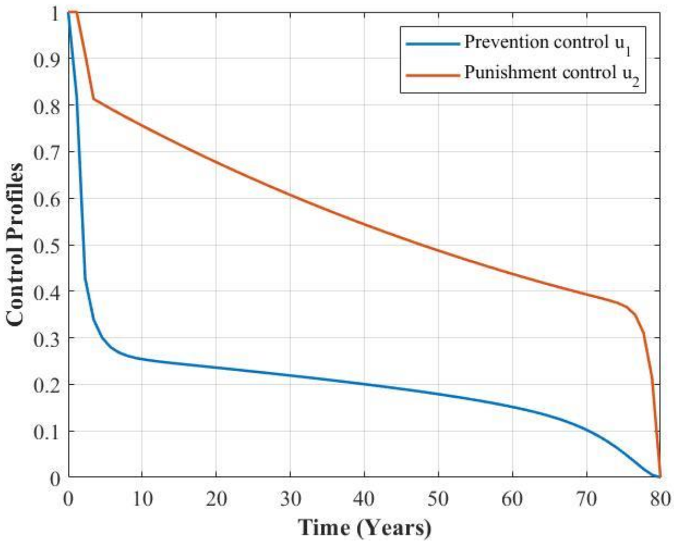

Figure 2. The control profiles of two controls $u_1(t)$ and $u_2(t)$.

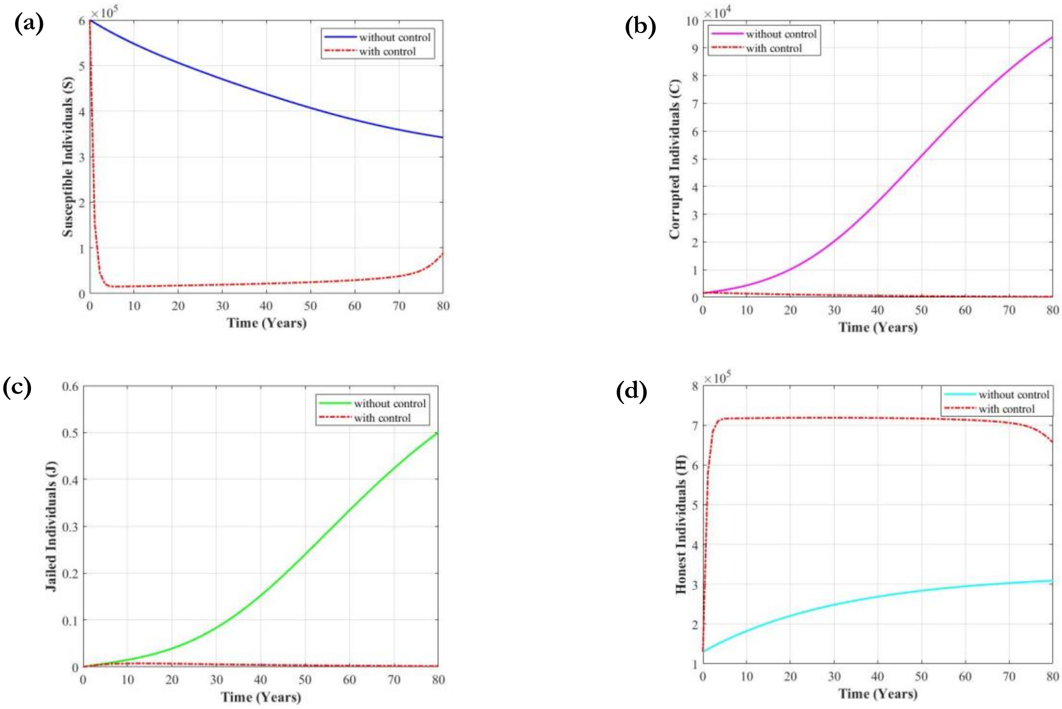

Figure 3. The corruption behavior of (a) susceptible, (b) corrupted, (c) jailed, (d) honest individuals with prevention strategy (i.e. $u_1(t) \neq 0$) but in the absence of punishment control (i.e. $u_2(t) = 0$).

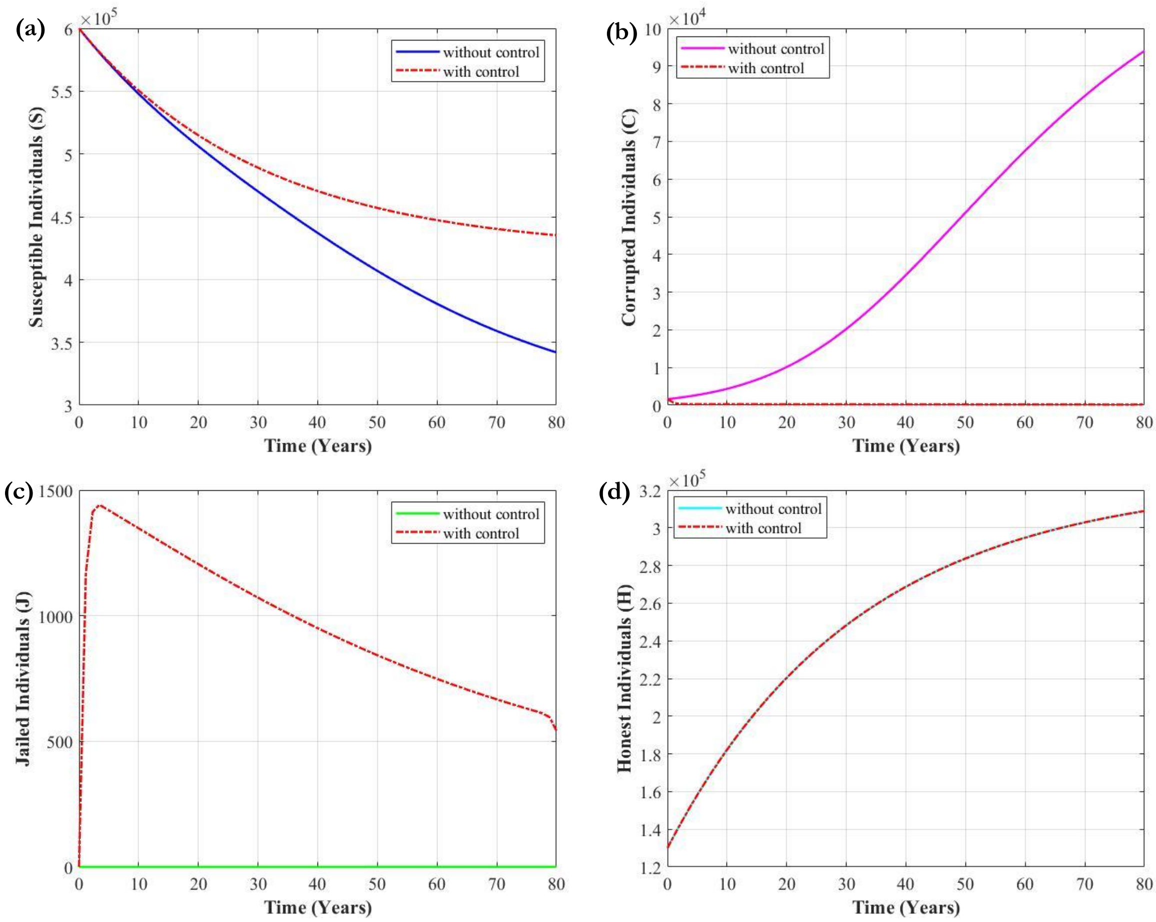

Figure 4. The corruption behavior of (a) susceptible, (b) corrupted, (c) jailed, (d) honest individuals with punishment control strategy (i.e. $u_2(t) \neq 0$) but in the absence of prevention control (i.e. $u_1(t) = 0$).

Figure 2 represents the control status within the time of 80 years. We can see that both controls work all time in this system which is necessary for our goal.

From Figure 3 we observed that when no control strategy (prevention and punishment) is applied, the susceptible, corrupted, and jailed people increase but honest people decrease. After using prevention control initially susceptible population decreased significantly first five years and after then it increases very slowly (Figure 3(a)). Without using control strategy corrupted population increases very highly but after applying the prevention control it becomes zero within 20 years (Figure 3(b)). When no control strategy is applied, the number of jailed individuals increases very slowly but after applying prevention control jailed population tends to zero within 30 years (Figure 3(c)). On the other hand, without using control strategy honest population increase very slowly but after applying control it increases very highly first 4 years and after that, it remains the same within 70 years, and then it decreases slowly (Figure 3(d)).

From Figure 4 we observed that the situation of every compartment using only punishment control and without control. When no control strategy is applied susceptible populations decrease significantly, but applying punishment control susceptible increases (Figure 4(a)). After applying only punishment control

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corrupted population tends to zero initially (Figure 4(b)). That means there is no corruption while the punishment control is implemented into our system. Without punishment control strategy there are no people in jail. But we have to punish corrupted people so that they do not try to do the crime again. So when we punished them then the corrupted population tends to zero and the number of jailed individuals increase first 4 years and then decreases very slowly (Figure 4(b) & Figure 4(c)). But no effect is shown in the honest population with the punishment control strategy (Figure 4(d)).

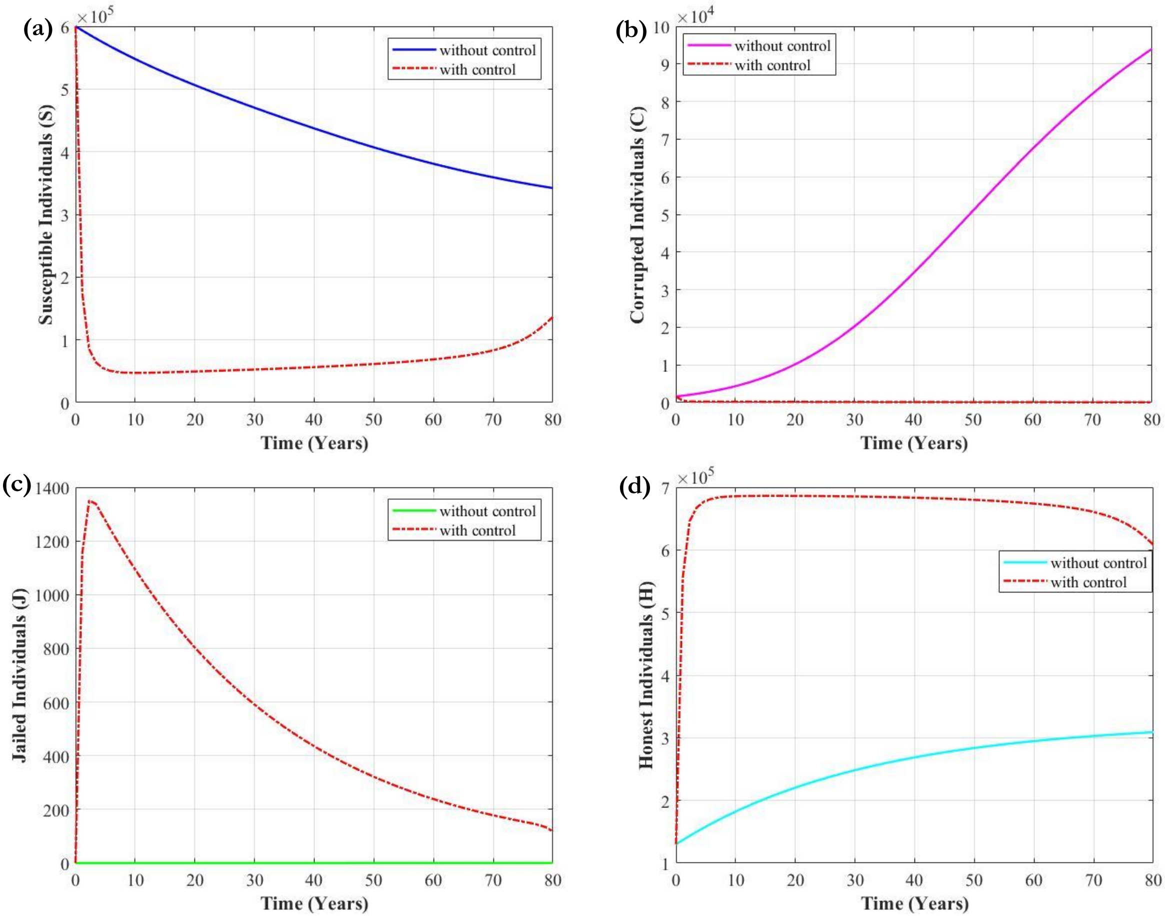

Figure 5. The corruption behavior of (a) susceptible, (b) corrupted, (c) jailed, (d) honest individuals with both control strategies that are prevention control strategy (i.e. $u_1(t) \neq 0$) and punishment control strategy (i.e. $u_2(t) \neq 0$).

We can see from Figure 5 that when both control strategies (prevention and punishment) are applied, the best results are obtained. Because in this case, we get the lowest susceptible individuals (Figure 5(a)). With applying both controls corrupted people moved to jail through punishment thus corrupted individuals tend to zero and jailed individuals increase for the first four years and then it decreases slowly (Figure 5(b) & Figure 5(c)). Since most corrupt people are punished by applying punishment control, so they recovered themselves from jail and become honest. And thus the honest population increases after applying both controls (Figure 5(d)).

Conclusion

We investigated the qualitative behavior of corruption dynamics and the best control strategy in this paper. There have been two control mechanisms employed: one to prevent vulnerable populations from becoming corrupted, while the other aims to control punishment efforts against corrupted populations. In order to explain the complicated dynamics of the solutions for constant controls, we have derived the basic reproduction number R_0, which is essential for our epidemic model. We demonstrated that there is a domain in which the model is both epidemiologically and mathematically sound. We also determine the point of no corruption equilibrium. Then, for our proposed system, we developed and demonstrated the existence and characterization of an optimal control solution. MATLAB was used to run numerical simulations of optimal control (OC). The results show that both control methods, prevention u_1, and punishment u_2, are effective. We concluded that when both controls are implemented into the system, the transmission of corruption into the population is reduced most effectively. From this whole study, it is a suggestion to the researcher and policymaker to apply those both controls to reduce the corruption from spreading.

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Conflict of interests

The authors have declared no conflict of interests.

Author Contribution

Kazi Nusrat Islam: Writing – review & editing, Writing – original draft, Validation, data collection, Methodology, Investigation, Data analysis, interpretation, Conceptualization. Tahera Parvin: Writing – review & editing, Conceptualization. Md. Haider Ali Biswas: Writing – review & editing, Conceptualization, Supervision.

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