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A MATHEMATICAL MODEL OF ALCOHOLISM IN BANGLADESH

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Abstract

In this paper, a POHR mathematical model has been developed and put through its paces to learn more about the developing health and social problems in Bangladesh associated with alcohol where POHR stands four population classes: Potential drinkers, Occasional drinkers, Heavy drinkers, and Recovered and quitters of drinking. The dynamical behavior of this model is explored, and the system's basic properties are determined, as well as its basic reproduction number $R0$. Furthermore, the sensitivity analysis of the system has been discussed. The model's equilibria states are obtained and their local asymptotic stability is established by analyzing the eigenvalues and the Routh-Hurwitz criterion. Sensitivity analysis of the reproduction number indicates that encouraging and supporting potential drinkers to avoid adopting drinking habits and occasional drinkers to quit alcohol consumption is more effective in the long run at controlling the spread of alcoholism than focusing exclusively on alcoholics. Additionally, Bangladesh's alcohol consumption percentages, alcohol-related and natural-cause mortality rates, and other factors are considered to illustrate our numerical findings, which were generated using the MATLAB software and demonstrate the model's practical reliability. The purpose of this study is to identify criteria worthy of further investigation in order to inform and aid policymakers in allocating preventative and treatment resources most effectively.

Keywords: Alcoholism, Compartments, Model, Stability, Reproduction, Sensitivity.

Introduction

Alcoholism, alternatively referred to as 'alcohol use disorder,' is a disease in which an individual engages in a habit of excessive drinking despite the detrimental consequences of alcohol on their job, health, educational, and social lives (Sharma & Samanta, 2015). According to the World Health Organization, alcohol consumption contributes to 3 million fatalities worldwide each year, as well as to disabilities and poor health of millions of people. In total, hazardous alcohol use contributes 5.1% to the worldwide illness burden (WHO, 2021). Apart from destroying people's relationships, impairing their productivity, and degrading their physical and mental health, long-term abuse can also have a detrimental effect on the brain and liver, heart illness, financial waste, poverty, criminality, family breakup, and liver disease are only some of the social, economic, and health consequences of alcohol consumption on people and society as a whole (Islam et al., 2017). Alcoholism is one of the most frequently addressed health risk behaviors due to the high prevalence of severe health and social repercussions (Ma et al., 2015).

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Alcohol consumption is minimal in Bangladesh as alcoholic beverages are restricted in here by law, and consumption is strictly prohibited by Islam, but those who do consume it typically binge drink, posing a public health risk (Islam et al., 2017). Nonetheless, it has been stated that alcoholic drinking is also popular among those with a greater income, and is viewed as a symbol of "modernity" or a "westernized way of life" (Islam et al., 2017). Any beverage containing more than 0.5 percent alcohol is classified as an alcoholic beverage in Bangladeshi legislation(Islam et al., 2017). The most common causes of alcoholism are family history, drinking from an early age, mental health disorder, stressful environments, less opportunity of entertainment, taking alcohol with medication, peer pressure, frequent alcohol consumption over time, failure in academic sector, loneliness, trauma, self-medication, lack of family supervision etc (NIAAA, 2021). Alcohol consumption is more prevalent among university students, truck drivers, sex workers, substance abusers, homeless children, indigenous people, and households with a positive alcohol drinking history (Dewan & Chowdhury, 2015). According to a study on the age-specific frequency distribution (percentage) of alcohol users in alcohol-drinking population of Bangladesh, people aged between 25-34 years consume alcohol the most (WHO, 2019). Though there is a rise in number of cases and arrests made by the Department of Narcotics Control for alcohol-related offenses, the number of DNC issued permits for alcohol consumption are also increasing. Domestic production growth, an increase in the number of drinking permits given, and a large increase in the volume of illegal liquor seized all suggest that actual alcohol use may be far higher than official figures indicate (DNC, 2017, 2018, 2019, 2020).

As stated before, excessive alcohol consumption can lead to a variety of diseases. Heart attacks and liver diseases are the most common diseases in Bangladesh (Dewan & Chowdhury, 2015). Irresponsible drinking is a major cause of domestic violence against women and children, as well as road accidents (Afsari & Rahman, 2018). However, alcoholism and its effect on the abuser's psychology have not been thoroughly explored in Bangladesh. This is partly because alcohol consumers in Bangladesh rarely report their issues to alcohol recovery institutions. In this paper we developed and analyzed a mathematical model of nonlinear differential equations where we considered four population classes to gain a better understanding of the developing health and social problems in Bangladesh associated with alcohol. The aim of this paper is quite specific, to study the suggested model by using stability theory of the nonlinear differential equation with numerical simulations by using MATLAB solver ODE45.

Materials and Methods

To describe the population dynamics and analyze the interactions between the drinker's classes, we propose a continuous-time POHR model. The population is divided into four compartments: potential drinkers $P(t)$, occasional drinkers $O(t)$; heavy drinkers $H(t)$; and $R(t)$ recovered and gave up drinking. The graphical representation of the proposed model is shown in Figure 1.

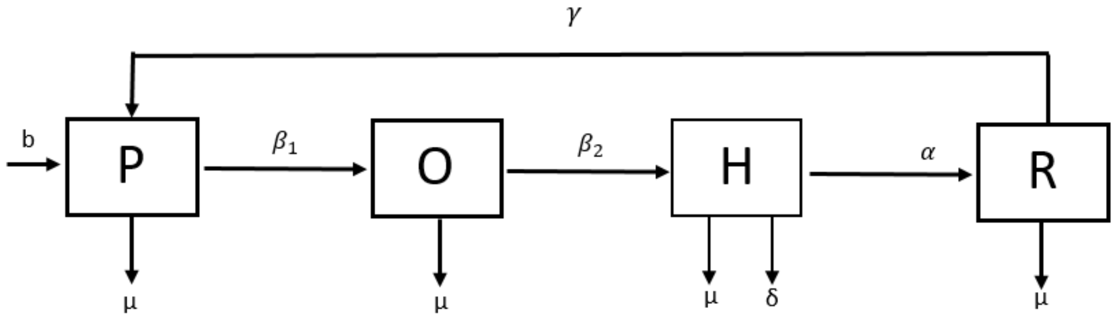

Figure 1. Schematic diagram of the four drinking classes in the model.

We consider the following system of four non-linear differential equations:

$$\frac{dP}{dt} = b + \gamma R - \beta_1 PH - \mu P \tag{1}$$

$$\frac{dO}{dt} = \beta_1 PH - (\beta_2 + \mu)O \tag{2}$$

$$\frac{dH}{dt} = \beta_2 O - (\mu + \alpha + \delta) \tag{3}$$

$$\frac{dR}{dt} = \alpha H - (\mu + \gamma)R \tag{4}$$

where, $N(t) = P(t) + O(t) + H(t) + R(t)$, and $P(0) > 0$; $O(0) \geq 0$; $H(0) \geq 0$ *and* $R(0) \geq 0$.

Model analysis

The model (equations 1-4) has to be analyzed to describe the dynamics of alcoholism. The objective of this analysis is to control the adverse situation from the locality. For the analysis of the model, we consider a closed set with the initial condition

$$\Omega = \left\{ (P, O, H, R) \in \Re_+^4 : 0 < P + O + H + R \leq \frac{b}{\mu} \right\}$$

With the initial conditions P(0) >0 ; O(0) ≥0; H(0) ≥0 and R(0) ≥ 0.

Boundedness of solutions of the model

Theorem 1. The total population N is bounded for all $t \geq 0$.

Proof:

From the model (1-4), solving the rate of change of total population, we find

$$N(t) \leq \frac{b}{\mu} + N(0)e^{-\mu t}$$

where $N(0)$ is the initial value of total number of people. From this solution, it is clear that the total population $N(t)$ will approach $\frac{b}{\mu}$ as $t \to \alpha$. This implies that the initial total population is less than $\frac{b}{\mu}$ if $N(t) \leq \frac{b}{\mu}$. So $\frac{b}{\mu}$ is the upper bound of N.

Positivity of the solutions of the model

Theorem 2. If $P(0) \geq 0$, $O(0) \geq 0, H(0) \geq 0, R(0) \geq 0$, then the solutions of system equations (1-4) $P(t), O(t), H(t), R(t)$ are positive for all $t > 0$.

Proof: The solution of Equation (1) is

$P(t) \geq P(0)\exp(\int_0^t -A(s))ds$, where, $A(t) = \beta_1 H(t) + \mu$

So, the solution P(t) is positive. Similarly, the solution of Equation (2) is:

$O(t) \geq O(0)\exp(\int_0^t -B(s))ds,$ where, $B(t) = \beta_2 + \mu$

So, the solution O(t) is positive. Similarly, the solution of Equation (3) and (4) is,

$H(t) \geq H(0)\exp(-(\mu + \alpha + \delta)t) \geq 0$ And,

$R(t) \geq R(0)\exp(-(\mu + \gamma)t) \geq 0$

Therefore, we can see that $P(0) \geq 0, O(0) \geq 0$, $H(0) \geq 0$ and $R(0) \geq 0\ \forall t \geq 0$. This completes the proof.

Drinking free and drinking present equilibrium point

Let us consider, the drinking free equilibrium point is e_0. The drinking-free equilibrium e_0 is achieved in the absence of drinking $(O = H = R = 0)$. Considering, $\frac{dP}{dt} = O = H = R = 0$, we get,

$$\Rightarrow b + \gamma R - \beta_1 PH - \mu P = 0 \Rightarrow P = \frac{b}{\mu}$$

So the drinking free equilibrium point is $e_0 = (P, O, H, R) = (\frac{b}{\mu}, 0,0,0)$.

Let e_1 be the drinking Present equilibrium point of this model which can be obtained by considering

$$\frac{dP'}{dt} = \frac{dO'}{dt} = \frac{dH'}{dt} = \frac{dR'}{dt} = 0$$

So, the drinking Present equilibrium point $e_1 = (P', O', H', R')$ is

$$P' = \frac{\alpha\beta_2+\beta_2\delta+(\alpha+\beta_2+\delta)\mu+\mu^2}{\beta_1\beta_2}, O' = \frac{-\left((\mu+\gamma)(\alpha+\delta+\mu)(\mu^3+(\alpha+\beta_2+\delta)\mu^2+(\alpha\beta_2+\beta_2\delta)\mu-b\beta_1\beta_2)\right)}{\beta_2\eta}$$

$$H' = \frac{-\left((\mu+\gamma)(\mu^3+(\alpha+\beta_2+\delta)\mu^2+(\alpha\beta_2+\beta_2\delta)\mu-b\beta_1\beta_2)\right)}{\eta},$$

$$R' = \frac{-(\alpha\mu^3 + (\alpha^2 + \alpha\beta_2 + \alpha\delta)\mu^2 + (\alpha^2\beta_2 + \alpha\beta_2\delta)\mu - b\alpha\beta_1\beta_2)}{\eta}$$

where, $\eta = (\beta_1\mu^3 + (\alpha\beta_1 + \beta_1\beta_2 + \beta_1\delta + \beta_2\gamma)\mu^2 + (\beta_1\beta_2\gamma + \beta_1\delta\gamma + \alpha\beta_1\beta_2 + \beta_1\beta_2\delta + \alpha\beta_1\gamma)\mu + \beta_1\beta_2\delta\gamma)$

Basic reproduction number

The basic reproduction number R_0 is a threshold parameter for the stability of drinking free equilibrium. If $R_0 > 1$, the drinker population will increase at a higher rate and therefore, alcoholism appears for a long time. Otherwise, alcoholism will die out within the shortest possible time. R_0 measures the average number of new drinkers generated by a single drinker in a population of potential drinkers. The value of R_0 indicates whether the epidemic could occur or not.

For calculating the Basic Reproduction Number R_0, we've used Next Generation Matrix Method. The system has a unique drinking-free equilibrium, with $P_0 = \frac{b}{\mu}$. At the point of drinking free equilibrium, we have,

$F = \begin{bmatrix} 0 & \beta_1 P_0 \\ 0 & 0 \end{bmatrix}$ and $V = \begin{bmatrix} \beta_2 + \mu & 0 \\ -\beta_2 & \alpha + \delta + \mu \end{bmatrix}$

Now, the next generation matrix is given by,

$$FV^{-1} - \lambda I = \begin{bmatrix} \frac{\beta_1 \beta_2 P_0}{(\beta_2 + \mu)(\alpha + \delta + \mu)} - \lambda & \frac{\beta_1 P_0}{(\alpha + \delta + \mu)} \\ 0 & -\lambda \end{bmatrix}$$

So, $|FV^{-1} - \lambda I| = \left(\frac{\beta_1 \beta_2 P_0}{(\beta_2+\mu)(\alpha+\gamma+\mu)} - \lambda\right)(-\lambda)$

Solving the above equation,

we have $\lambda = \frac{\beta_1 \beta_2 P_0}{(\beta_2+\mu)(\alpha+\delta+\mu)}, 0$

So, the basic Reproduction number is

$R_0 = \frac{\beta_1 \beta_2 P_0}{(\beta_2+\mu)(\alpha+\delta+\mu)}$ Where the initial condition is $(\beta_2 + \mu)(\alpha + \delta + \mu) \neq 0$.

Stability analysis

We have established the local stability at the drinking-free and drinking present equilibrium points. To prove this, we used the following theorems.

Theorem 3. The alcohol-free equilibrium point is locally asymptotically stable if all the roots of characteristic equation have negative real parts for the condition $C_1 C_2 C_3 - C_0 C_3^2 - C_4 C_1^2 > 0$.

Proof: The characteristic equation of drinking free equilibrium is

$$|J(e_0) - \lambda I| = \begin{pmatrix} -\lambda - \mu & 0 & \frac{-b\beta_1}{\mu} & \gamma \\ 0 & -\beta_2 - \lambda - \mu & \frac{b\beta_1}{\mu} & 0 \\ 0 & \beta_2 & -\alpha - \delta - \lambda - \mu & 0 \\ 0 & 0 & \alpha & -\gamma - \lambda - \mu \end{pmatrix}$$

Where,

$C_0 = 1, C_1 = \alpha + \beta_2 + \delta + \gamma + 4\mu,$

$$C_2 = 6\mu^2 + 3\delta\mu + 3\alpha\mu + 3\beta_2\mu + \beta_2\delta + 3\gamma\mu + \alpha\gamma + \beta_2\gamma + \delta\gamma + \alpha\beta_2 - \frac{b\beta_1\beta_2}{\mu}$$

$$C_4 = \begin{pmatrix} \alpha\mu^3 + \beta_2\mu^3 + \delta\mu^3 + \gamma\mu^3 + \mu^4 + \alpha\beta_2\mu^2 + \beta_2\delta\mu^2 + \alpha\gamma\mu^2 + \beta_2\gamma\mu^2 + \delta\gamma\mu^2 \\ +\beta_2\delta\gamma\mu - b\beta_1\beta_2\mu + \alpha\beta_2\gamma\mu \end{pmatrix}$$

$$C_3 = \begin{pmatrix} 4\mu^3 + 3\delta\mu^2 + 3\alpha\mu^2 + 3\beta_2\mu^2 + 3\gamma\mu^2 + 2\alpha\beta_2\mu + 2\beta_2\delta\mu + 2\alpha\gamma\mu + 2\beta_2\gamma\mu \\ +2\delta\gamma\mu - b\beta_1\beta_2\gamma - 2b\beta_1\beta_2 + \alpha\beta_2\gamma + \beta_2\delta\gamma - \dfrac{b\beta_1\beta_2\gamma}{\mu} \end{pmatrix}$$

Solving the characteristic equation, we assume λ is negative. Applying Routh-Hurwitz criterion, we get the conditions of being the eigenvalues negative, and the conditions are bellowed:

$$\Rightarrow \frac{C_1C_2C_3 - C_0C_3^2 - C_4C_1^2}{C_1C_2 - C_0C_3} > 0 \Rightarrow C_1C_2C_3 - C_0C_3^2 - C_4C_1^2 > 0$$

All the roots have negative real parts for the condition $C_1C_2C_3 - C_0C_3^2 - C_4C_1^2 > 0$.
Since all values of λ are negative here, then our model is asymptotically stable for drinking free equilibrium.

Theorem 4. The drinking present equilibrium point is locally asymptotically stable if all the roots of the characteristic equation have negative real parts for the condition $C_1'C_2'C_3' - C_0'C_3'^2 - C_4'C_1'^2 > 0$

Proof:

The characteristic equation of the drinking present equilibrium point is

$$|J(e_1) - \lambda I| = C_0'\lambda^4 + C_1'\lambda^3 + C_2'\lambda^2 + C_3'\lambda + C_4' = 0$$

where, $C_0' = 1$, $C_1' = 4\mu + \eta'$, $C_2' = 6\mu^2 + 3\eta'\mu + \varepsilon$, $C_3' = 4\mu^3 + 3\eta'\mu^2 + 2\varepsilon\mu + \rho$,

$C_4' = \mu^4 + \eta'\mu^3 + \varepsilon\mu^2 + \rho\mu + H'\beta_1\beta_2\delta\gamma$

Where,

$\eta' = \alpha + \beta_2 + \delta + \gamma + H'\beta_1$,

$\varepsilon = \alpha\beta_2 + \beta_2\delta + \alpha\gamma + \beta_2\gamma + \delta\gamma + H'\alpha\beta_1 + H'\beta_1\beta_2 + H'\beta_1\delta + H'\beta_1\gamma - P'\beta_1\beta_2$,

$\rho = \beta_2\delta\gamma + \alpha\beta_2\gamma + H'\alpha\beta_1\beta_2 + H'\beta_1\beta_2\delta + H'\alpha\beta_1\gamma + H'\beta_1\beta_2\gamma + H'\beta_1\delta\gamma - P'\beta_1\beta_2\gamma$

Solving the characteristic equation, we assume λ is negative, we get the conditions of being the eigenvalues negative, and the conditions are bellowed:

By Routh- Hurwitz Criterion, from the characteristic equation, we get

$$\Rightarrow \frac{C_1'C_2'C_3' - C_0'C_3'^2 - C_1'^2C_4'}{C_1'C_2' - C_0'C_3'} > 0$$

$$\Rightarrow C_1'C_2'C_3' - C_0'C_3'^2 - C_4'C_1'^2 > 0$$

All the roots have negative real parts for the condition $C_1'C_2'C_3' - C_0'C_3'^2 - C_4'C_1'^2 > 0$, since all values of λ are negative here, then our model is asymptotically stable for drinking present equilibrium.

Sensitivity analysis of R_0

The basic reproduction number, $R_0 = \frac{\beta_1\beta_2P_0}{(\beta_2+\mu)(\alpha+\delta+\mu)}$

$$A_{\beta_1} = \frac{\frac{\partial R_0}{R_0}}{\frac{\partial \beta_1}{\beta_1}} = \frac{\beta_1}{R_0}\frac{\partial R_0}{\partial \beta_1} = 1,$$

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$$A_{\beta_2} = \frac{\frac{\partial R_0}{R_0}}{\frac{\partial \beta_2}{\beta_2}} = \frac{\beta_2}{R_0}\frac{\partial R_0}{\partial \beta_2} = 1 - \frac{\beta_2}{(\beta_2+\mu)} \Rightarrow |A_{\beta_2}| < 1$$

$$A_{\alpha} = \frac{\frac{\partial R_0}{R_0}}{\frac{\partial \alpha}{\alpha}} = \frac{\alpha}{R_0}\frac{\partial R_0}{\partial \alpha} = -\frac{\alpha}{(\alpha+\delta+\mu)} < 1,$$

$$A_{\delta} = \frac{\frac{\partial R_0}{R_0}}{\frac{\partial \delta}{\delta}} = \frac{\delta}{R_0}\frac{\partial R_0}{\partial \delta} = -\frac{\delta}{\alpha+\delta+\mu} \Rightarrow |A_{\delta}| < 1$$

From the above discussion it is clear that the basic reproduction number R_0 is most sensitive to changes in β_1 and β_2. If β_1 and β_2 increase, R_0 will increase in same proportion and if these parameter decrease, R_0 will also decrease in same proportion. On the other hand, δ and α have an inversely proportional relationship with R_0, i.e., an increase in any of them will cause a decrease in R_0 and a decrease in any of them will cause an increase in R_0.

Results

Numerical simulations are carried out in MATLAB using the parameter values listed in Table 1. We've considered $P = 20, O = 15, H = 10, R = 5.$

Table 1. Description and value of parameters

No.	Parameter	Value (per year)	Description
1	b	6	The recruitment rate of becoming a potential drinker [fitted]
2	μ	0.49	Natural death rate (BBS, 2020)
3	β_1	0.02-0.6	The rate of effective contact of potential drinkers with heavy drinkers [fitted]
4	β_2	0.04-0.8	The rate of occasional drinkers to become heavy drinkers [Estimated]
5	α	0.4	The rate of heavy drinkers become recovered and quitter of drinking [Estimated]
6	δ	0.00047	The death rate induced by the heavy drinking (WHO, 2019)
7	γ	0.4	The rate of recovered people becoming potential drinker again [fitted]

Now, the value of β_1, β_2 and α are changed respectively while keeping the other parameters fixed. The simulation result for these cases are shown below.

As illustrated in Figure 2, the number of potential drinkers decreases as the value of β_1 increases. Potential drinkers adopt drinking habits and become occasional drinkers as a result of the increased effective contact rate between potential drinkers and heavy drinkers. This inevitable decline in the class of potential drinkers results in a rise in the class of occasional drinkers. Additionally, as a result of the increase in β_1, we can witne ss increase in the population of heavy drinkers and in the recovered drinkers' class. Now, an increase in β_2 naturally results in a drop in the potential drinker class. As β_2 reflects the rate at which occasional

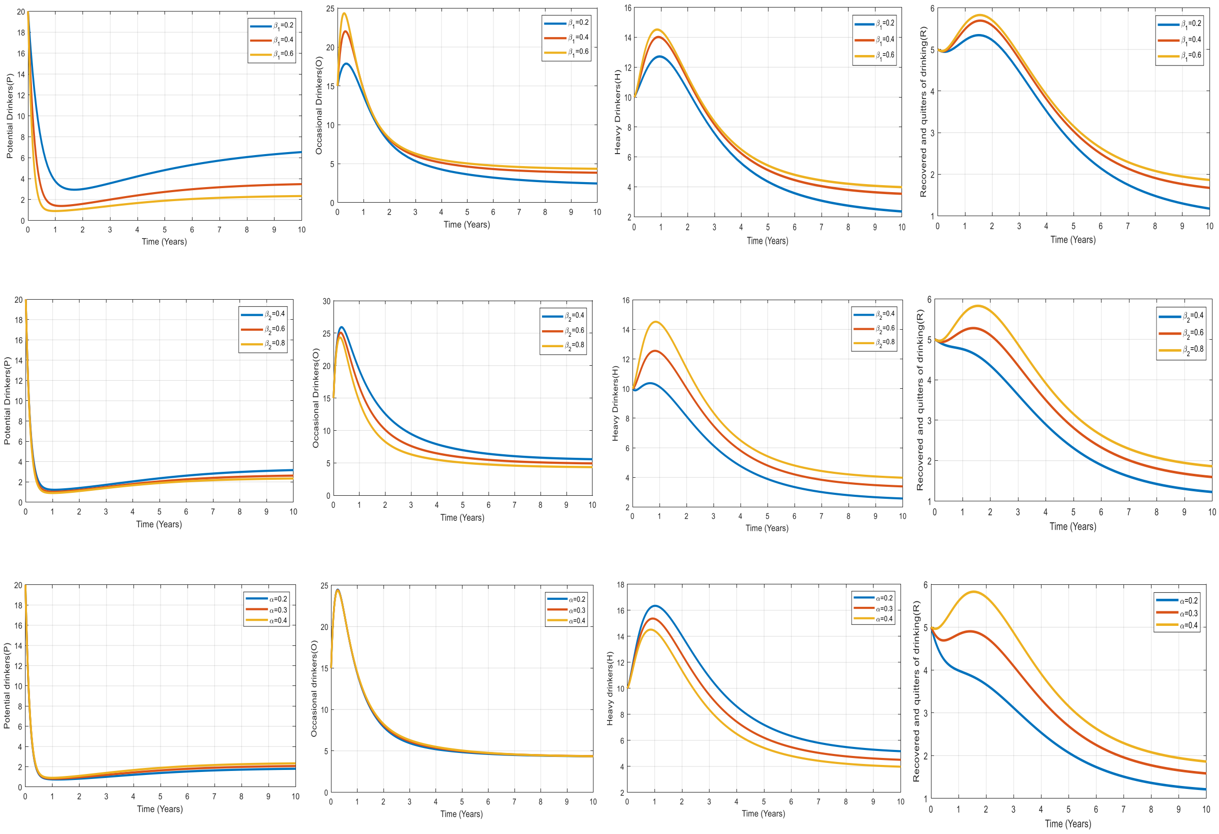

Figure 2. Variation of four drinkers' class population due to different values of β_1, β_2 and α.

drinkers become heavy drinkers, if β_2 increases, the number of heavy drinkers or alcoholics in society will increase as well which results in a drop in the occasional drinker class. Increase in β_2 will also lead to a rise in the recovered class.

Increases in α cause a slight rise in the population of potential drinkers and occasional drinkers. Additionally, increases in α will result in a decline in the population of heavy drinkers as they recover and quit. They will become members of the recovered class, which will result in a rise in the recovered population, which is quite predictable.

Figure 3 illustrates the basic reproduction number vs. the parameters that have impact on it. It is obvious that R_0 is most sensitive to changes in β_1 and β_2. For $\beta_1 = 0.11727$ the reproduction number R_0 is 1. Which suggests that alcoholism exists in the society when $\beta_1 = 0.117$. Also, for $\beta_2 = 0.068602$ the reproduction number R_0 will be 1. Alcoholism spreads throughout society when β_2>0.068602. Additionally, R_0 will be 1 when $\alpha = 4.0659$. As α have an inversely proportional relationship with R_0, an increase in α will cause a drop in R_0 and a decrease in α will cause an increase in R_0.

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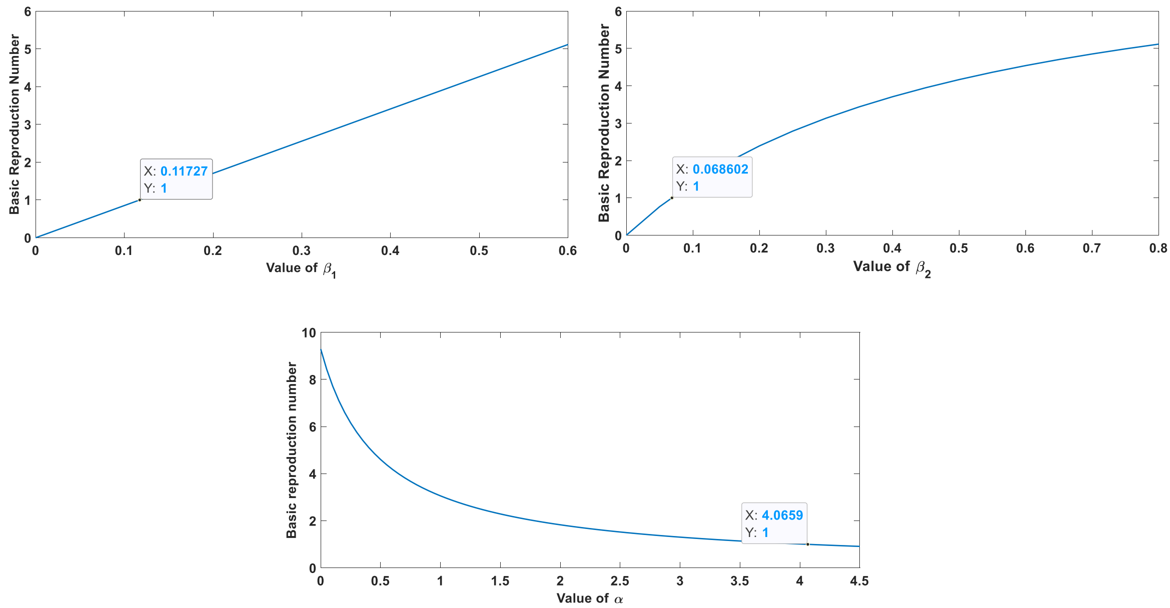

Figure 3. Variation of R_0 due to different values of β_1, β_2 and α.

Conclusion

Sensitivity analysis of the reproduction number indicates that the spread of alcoholism depend on increase of β_1, β_2.To decrease the number of alcoholics in our society we must monitor that the parameters β_1 not to exceed 0.1127 and β_2 not to exceed 0.0686. Our numerical findings are illustrated through MATLAB simulation. Although the model's parameters can be adjusted to reduce alcoholism, a treatment strategy could be implemented in the future to investigate the impact of treatments on other compartments. For now, it can be said that prevention is better than cure. From our perspective, encouraging and helping potential drinkers to abstain from alcohol use and occasional drinkers to abstain from alcohol consumption is more efficient in the long run at containing the spread of alcoholism than focusing entirely on recovering alcoholics. Parents should discuss the risks of drinking with their children, which can assist their children in avoiding alcohol issues. Effective frameworks for monitoring and surveillance activities should be established, such as periodic national surveys on alcohol consumption and alcohol-related harm, as well as a plan for information exchange and dissemination.

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